

by 350 authors. The author says that in order to retain something of the encyclopedic character of the first edition, he selected the titles in such a way that the reader interested in some topic will always find at least a few titles which can lead him on to more references. The bibliography indicates where each paper is quoted in the text, which is very convenient for the reader.

The main topics in differential geometry which are not treated in this book are the geometries of Finsler and Cartan spaces and their generalizations, the geometries of paths and  $K$ -spreads and their generalizations, projective and conformal geometries with the use of supernumerary coordinates, the geometry under contact transformations, the geometry in almost complex spaces, the theory of general geometric objects, the differential geometry of fibre bundles, etc. The reviewer sincerely hopes that the author may soon have another opportunity to give a survey of these interesting topics.

This excellent book by an author who has since 1918 always been a pioneer of research in the fields of differential geometry will serve not only as "an introduction to tensor analysis and its geometrical applications" but also as an encyclopedia for the differential geometers of the front line and it will give all the information in the small necessary for the development of differential geometry in the large.

KENTARO YANO

*Lectures on partial differential equations.* By I. G. Petrovsky. Trans. by A. Shenitzer. New York, Interscience, 1954. 10+245 pp. \$5.75.

This book is intended to be an introductory and self-contained text in the theory of partial differential equations. That it fulfills this purpose is a result not only of the excellent presentation of the author but also of the excellent translation of A. Shenitzer which preserves the easy, flowing style of the original.

The book is divided into four chapters: the classical trichotomy, hyperbolic, elliptic and parabolic equations, and an introductory chapter devoted to motivation and notation.

The first chapter begins with a derivation of the various classes of equations mentioned above from prototype physical problems. Beginning the study of existence and uniqueness, the author presents the Cauchy-Kowalewski theorem for the Cauchy problem with analytic initial conditions. This leads to the concept of characteristics, and to the question of existence and uniqueness of solutions with nonanalytic initial conditions. There is a very thorough discussion of this topic, in which the author brings the reader up to date on some of the latest developments in the field. It is characteristic of the book

that the author continually emphasizes the latest advances as well as the classical results and singles out the outstanding open problems. The remainder of the chapter is devoted to the question of the reduction of equations to canonical form, thus preparing the way for the division of the three remaining chapters into the study of hyperbolic, elliptic and parabolic equations.

The second chapter is devoted to the study of hyperbolic equations, a subject particularly dear to the author's heart. It begins with an example due to Hadamard illustrating the discontinuous dependence on the initial values of the solution of the Cauchy problem for the Laplace equation. This motivates a discussion of the generalized solutions of Sobolev. Existence and uniqueness theorems for the wave equation and hyperbolic systems are then presented, followed by the derivation of the explicit formulas for the solution due to Kirchhoff, Poisson and d'Alembert. Using these, the stability of the solution as a functional of the initial values is demonstrated for the particular equations above. Again using the explicit formulas, the concept of diffusion and its dependence upon dimension is treated. Pulling over to a siding temporarily, the author discusses the Lorentz transformation, and the mathematics of the special theory of relativity. This part of the second chapter concludes with a survey of the basic results for the Cauchy problem for general hyperbolic systems and a discussion of the finite difference approach of Courant, Friedrichs and Lewy.

In the second part of the chapter we meet the wave equation with boundary as well as initial conditions. Uniqueness and continuous dependence upon the initial values are again demonstrated under simple conditions on the region. To illustrate the application of the method of Fourier, based upon separation of variables and the solution of eigenvalue problems, a one-dimensional problem is treated in detail. In order to keep the text self-contained, a large section is now devoted to a derivation of the properties of eigenfunctions and eigenvalues, including the minimax principle of Courant. Under simple conditions, the author establishes Parseval's identity and the convergence of the Fourier series obtained formally. To illustrate an alternate approach, it is shown how a Green's function may be used to convert the boundary value problem into that of solving a Fredholm integral equation. A multi-dimensional problem is now treated to demonstrate the points of similarity in the basic approach by means of Fourier's method, and some of the new difficulties that arise. Bessel functions occur in the particular example given. The chapter concludes with a short sketch of the variational techniques of Rayleigh, Ritz, and Galerkin.

The topic of elliptic equations occupies the third chapter. After a short proof, due to Privalov, of the minimum-maximum theorem for solutions of the Laplace equation, the author turns to the Dirichlet problem for the circle. Using the Poisson integral representation, the standard sequence of theorems concerning harmonic functions is derived. The existence of a solution for general regions is then proved using the Poincaré-Perron concept of superharmonic functions. In order, the author discusses the exterior boundary value problem, the Neumann problem, potential theory, and the application of potential theory to the solution of boundary value problems. Following these results, there is a brief discussion of the approximate solution of the Dirichlet problem by means of finite differences, following an approach due to Lusternik. As in the previous chapters, this chapter closes with a survey of some of the most important results for elliptic equations.

The last chapter is a very brief one, and sketches the application of some of the techniques developed in the previous chapters to the solution of some simple problems in the theory of heat conduction.

The only fault in the book is a small one—there is no index. The bibliography is given in footnotes, and this also is not completely desirable. Apart from these minor items, the book is highly to be recommended. It is printed very attractively, reads very smoothly, and all in all is to be regarded as an elegant introduction to an attractive field of mathematics.

RICHARD BELLMAN

*Vorlesungen über Approximationstheorie.* By N. I. Achieser. Berlin, Akademie-Verlag, 1953. 10+309 pp., 10 figures. 29.00 DM.

This splendid book (translated from the Russian edition of 1947) gives much more than the title promises. Besides a discussion of specific problems of approximation it provides also an introduction to many different parts of analysis, as can be seen from the following list of topics in Chapter I: Elements of functional analysis. Chapter II: Approximation in  $C$  (Chebyshev approximation). Chapter III: Fourier analysis:  $L^2$ -theory, Fejér's theorem, Watson transforms, conjugate functions. Chapter IV: Entire functions of exponential type bounded on the real axis, S. Bernstein's inequalities for the derivatives of these functions and generalizations. Chapter V: Problems of best approximation by trigonometric polynomials and by entire functions of exponential type. Chapter VI: Wiener's Tauberian theorem.

The presentation is very clear and interesting. The author has achieved a happy mixture of the abstract and the concrete. Defini-