of the seventeen chapters; they are as follows. (1) Point sets. Euclidean space. (2) Measure. Measurable functions. (3) Integration. (4) Additive set functions. (5) The Lebesgue spaces L_p and the Orlicz spaces L_ϕ. (6) Banach space and Hilbert space. (7) Bounded linear transformations. (8) Banach spaces of finite dimension. (9) Bounded linear transformations in Hilbert space. (10) Range, null space and spectral properties of bounded linear transformations. (11) Compact linear transformations. (12) Compact symmetrisable, self-adjoint and normal transformations in Hilbert space. (13) General theory of non-singular linear integral equations. (14) Integral equation with normal kernel. (15) Integral equation with a symmetrisable kernel, expressible as the product of a kernel of finite double-norm and a bounded non-negative function. (16) Integral equation with Marty kernel. (17) Integral equation with Garbe kernel or Pell kernel.

Despite its self-imposed limitations, the book contains so much material, and treats its topics so thoroughly, that it is a welcome addition to the literature of functional analysis; it is recommended as both a reference for the expert and a text for the student.

PAUL R. HALMOS

Principles of numerical analysis. By A. S. Householder. New York, McGraw-Hill, 1953. 10+274 pp. $6.00.

With the age of supersonic aircraft, hydrogen bombs, large automatic control systems, and so on, has come a large increase in the volume and importance of scientific computation. The procession of automatically sequenced digital computers following J. W. Mauchly's construction of the ENIAC during the war is providing an enormous capacity to solve these computing problems and others. But many users and programmers of these machines know relatively little mathematics, while mathematicians are often quite unaware of the mathematical literature on computing methods. Much of the newer literature is found only in journals of diverse fields, or in reports of various research projects.

To cope with the situation, here and there serious mathematicians have been formed into groups with numerically inclined physicists and others, in order to study computing methods and devise new ones. The mathematicians on such a team are likely to call themselves *numerical analysts*. But there has been no agreed definition of numerical analysis as these people use the words, and no standard reference work. Books by Milne, Scarborough, Hartree, and others are primarily oriented toward desk calculating machinery, and most are written for mathematical amateurs or undergraduate students.

Dr. Householder is Chief of the Mathematics Panel at Oak Ridge National Laboratory. Presumably to help fill the gaps mentioned above, he gave a course in Oak Ridge for the University of Tennessee in 1950, and distributed some lecture notes which have been much appreciated. The book now reviewed grew out of these notes, with many additions.

Above all, the contribution of this book is to exhibit a substantial number of the tools of the new numerical analysis, and give them a scholarly mathematician's treatment. This is a large and difficult undertaking, and we congratulate Dr. Householder on its success. In preparing the book, the author has digested a surprising amount of the world literature on numerical methods. Numerical analysts in machine laboratories have now received their first standard reference.

The book is in eight chapters. Each is concluded with a section, Bibliographic Notes, relating the different sections of the chapter to the titles in the book's central bibliography. In the first chapter, The Art of Computation, the author attempts the classification of errors and their estimation. The chapter mainly develops the details of the "rounded-off" arithmetical operations with "digital" (i.e., rounded-off) numbers characteristic of digital computations. While the slavery of these estimates is certainly necessary for precise error bounds, one is thankful that most programming proceeds well enough without it.

Chapter 2 on matrices and linear equations, together with Chapter 4 on matrix eigenvalues, comprise almost half the text. For the most part the discussion of vectors, matrices, linear transformations, metrics, projection operators, norms, and so on, is descriptive and not abstract. Proofs are generally included. The author discusses several iterative methods for solving systems of linear equations and inverting matrices, and unifies them in terms of either projection matrices or a general linear iteration formula. Several methods are presented for the first time in a book—for example, the orthogonalizations of residuals devised by Lanczos, Hestenes, and Stiefel. A brief discussion of round-off errors is followed by a presentation of elimination procedures. The author summarizes the numbers of operations required for different methods.

Next to linear algebra, the most attention is given to the solution of nonlinear equations and systems, treated in Chapter 3. The chapter begins with standard material on polynomials in one variable, symmetric functions, and separation of roots, and carries on to König's theorem on zeros of analytic functions. With these tools the Graeffe and Bernoulli methods for getting polynomial zeros are described. Then follows a long section on functional iteration, from which Newton's method and higher order iterations are developed, both for equations and systems of equations. Aitken's δ^2 process is described here.

The chapter on matrix eigenvalues first sketches the theory of minimal polynomials for general matrices over the real field, and relates eigenvalues to the field of values of a matrix. The rest of the chapter is a gold mine of hard to find information on iterative methods of finding eigenvalues: pure iterations, iterations with Chebyshev polynomials in a matrix, deflating a matrix to make intermediate eigenvalues dominate, separating nearly equal eigenvalues, and the Jacobi rotation method of diagonalizing a matrix. The material on direct methods is just as valuable: solving Newton's identities, escalator methods, minimized iterations, and reduction to triple-diagonal form.

Interpolation, the subject of Chapter 5, is an older and more available field; the author claims to confine himself to a few general principles and some remainder formulas old and new. We also find optimal-interval interpolation. In Chapter 6 least squares approximation, trigonometric interpolation, and other problems are unified from a rather abstract point of view. Chapter 7 treats numerical integration and differentiation with a standard approach. Chapter 8 gives the Monte Carlo method of numerical integration a once over lightly, concluding with the "author's opinion that the method has proved and will prove most useful for the intrinsically stochastic physical problems."

Now follows a consolidated bibliography of over 400 titles, surely a great help to numerical analysts. It is surprising how many of the authors cited are first class mathematicians, considering how unfashionable numerical analysis has been in the memory of most of us. Several pages of exercises and a good index conclude the book.

As indicated, the book abounds with material densely packed into 246 pages of text. A few topics of numerical analysis have been omitted—for example, differential equations, game theory, and inequalities. These are large topics, probably best left to specialized books.

In the preface the author announces that this is a textbook, and that he tried to make the discussion accessible to one who has had a course in calculus and a little probability theory. The reviewer doubts that the author has succeeded in that object—the book is no place for the uninitiated to learn basic mathematical concepts. Many of the descriptions of methods will be perfectly clear to the student. But to be able to comprehend the theory, in the reviewer's opinion, the typical reader will need the equivalent of a master's degree in mathematics, and preferably more. This is partly because the material is fairly difficult in itself, but partly because the theoretical exposition is not lucid. The author goes from topic to topic with few signposts to help the reader, and none for the scanner. Although there are many derivations and proofs, results are nowhere summarized as theorems. The reviewer found much of the theory hard going, and occasionally suspected the author of formal manipulation without any justification within the reader's presumed knowledge (for example, in replacing a polynomial in a scalar by a polynomial in a matrix in equation (2.06.5) and after (4.0.7)).

The publishers have done a fine job in preparing the book. The few misprints noted were minor. The equation numbers might well have been simplified; why have (2.1322.5) or (2.201.10)?

In summary, the reviewer considers the book as a first definitive treatise on the subject matter and spirit of modern numerical analysis. As such, it is unique in a new area, and gives background, essential ideas, and references for a great number of methods never before brought together in a book. It is oriented toward the exploitation of automatic digital computers. While hard to read in spots, it will be a welcome addition to the library of every one interested in digital computation.

GEORGE E. FORSYTHE

Tables of integral transforms. Prepared under the direction of A. Erdélyi. New York, McGraw-Hill, 1954. 20+391 pp. $7.50.

This is the fourth of a series of five volumes prepared in part from notes left by the late Harry Bateman. The first three are entitled *Higher transcendental functions* and are devoted to a description of the properties of such functions; the volume under review together with a fifth to follow form a table of integrals involving such functions and intended as "companions and sequel" to the first three. The whole work is dedicated to the memory of Harry Bateman, and was prepared under the direction of A. Erdélyi with the collaboration of Research Associates: W. Magnus, F. Oberhettinger, F. G. Tricomi; Research Assistants: D. Bertin, W. B. Fulks, A. R. Harvey, D. L. Thomsen, M. A. Weber, E. L. Whitney; and Vari-typist, R. Stampfel. The project was performed at the California Institute of Technology, supported by a grant from the Office of Naval Research.

The integrals of the present volume are classified in seven chapters under the following types of transforms: Fourier cosine, Fourier sine, exponential Fourier, Laplace, inverse Laplace, Mellin, inverse Mellin.