"nd" at the end of a line. Yet on p. 169 of this book, $H_1(S-P, Q)$ is broken into $H_1(S-$ and $P, Q)$; on p. 149–150, a similar formula is similarly broken between one page and the next; on p. 121, the two factors of a product occur, one on line 22 and the other on line 23; several dozens of such instances could easily be given. It is difficult enough to follow such a text in detail without having constantly to reconstruct in one's mind what has been separated on paper; and, apart from all aesthetical considerations, such practices, which in this country are fast becoming the rule rather than the exception, may soon make many of our mathematical texts intolerably hard to read. It is high time that a reaction should set in against the tendency to cram as much text as possible into each page at the lowest possible cost, regardless of the effect on the reader; this will require a coordinated effort on the part of authors, editors and the printing-presses. The authors, who undoubtedly bear some responsibility for the present situation, should be more mindful of such matters in the preparation of their manuscripts; editors and editorial assistants should cooperate with them to a greater extent than sometimes happens now. As to the typesetters, who are doing an extraordinarily good job of setting the most complicated formulas, they could very easily be trained to avoid broken formulas, if their attention were drawn to it by the presses; they could well be trusted to use their judgment in displaying some long formulas, even in the absence of an indication from the author or editor; as to short formulas, all that is mostly required is some adjustment in the spacing of words; this might sometimes take more time than mechanically running along, but would still be far less expensive than later corrections which may affect a whole paragraph of type. Possibly, at least in the transitional period until typesetters acquire experience in such matters, the average cost of the printed page in mathematical texts would increase slightly; possibly the number of pages to be printed every year by mathematical journals would have to be somewhat cut down. Maybe the gain would be greater than the loss.

A. WEIL

Projektive Differentialgeometrie. Part I. By G. Bol. (Studia Mathematica, vol. 4.) Göttingen, Vandenhoeck and Ruprecht, 1950. 8+365 pp. 20 DM.

The present book is the first part of a treatise on projective differential geometry. This volume is divided into four sections: I. *Plane curves*; II. *Introduction to space geometry*; III. *Space curves*; IV. *Surface strips* (*Flächenstreifen*). The second (and last) volume will deal more specifically with the theory of surfaces. As the author points out in his preface, the main difference between his treatise and preceding ones on projective differential geometry is in the method employed. The general idea is that of choosing the arbitrary elements in the analytic representation of the quantities involved (proportionality factor in the homogeneous projective coordinates of a point; change of parameter) so that the formulas are as simple as possible. As an example of this process, let us denote a point in the projective plane by a vector x, so that a (regular) arc of a curve is obtained by giving the (three) components of x (defined, of course, only up to a factor different from zero) as functions of a parameter t (with all the regularity conditions required by the subsequent developments). If the arc under consideration has no inflexion points, then the determinant $D=(x, x', x'')$ (the primes indicating differentiation with respect to t) is different from zero, and we can always normalize the vector x so that along our arc: (1) $D=\text{const.} \neq 0$. For a transformation of parameter: $t^*=t^*(t)$, (1) is not preserved. The author associates any transformation of parameter with such a normalization of the vector x that (1) still holds. Analogous methods are followed for space curves and surfaces. Also the duality principle is extensively used to simplify the formulas.

In the first section, after the above mentioned specification for the representation of a plane curve, we find the usual developments about local projective frames, local expansions, projective arc and projective curvature. The last section is devoted to questions of projective differential geometry in the large; for example, the author proves that on any closed curve without inflexions, and cut by any straight lines in no more than 2 points, there are at least 6 sextactic points (that is, points where the osculating conic is stationary).

In the second section, the author presents an introduction to space geometry; we begin with curves, ruled and developable surfaces. A number of paragraphs are devoted to the geometry of the line in R_5, and its applications to ruled surfaces, linear complexes, null systems, and so on. The following paragraphs contain the first formulas of the theory of surfaces. This subject will be dealt with in more detail in the second volume; however, we find here, in addition to the ideas of asymptotic lines and of asymptotic parameters, the quadrics of Lie and Darboux, the tangents of Darboux and Segre, the most important results about conjugate nets, and the successive Laplace-transforms of a given surface, with the cases in which a Laplace-chain breaks down; here some extensions to hyperspaces are also given. After presenting the main theorems of the theory of line congruences, the section ends with a very accurate analysis of the ideas of analytic and geometric contact between curves or surfaces; the fact that the idea of geometric contact is independent of the analytic representation, as well as its topological rather than projective character, are presented in the clearest way.

In section III, the author's method of treating questions of normalization yields its best results, making possible the introduction, with a minimum of effort, of the most important configuration attached to a space curve. By attaching a suitable projective frame to any point of the curve, we arrive quite easily at the C^3's of Fubini (in particular, the harmonic C^3), the osculating linear complex, and the characterization of the curves belonging to a linear complex, to a quadric, and so on, by means of the ordinary differential equation of which the components of the vector x (associated with a point of our curve) are a set of independent solutions.

Section IV is the most original and presents in detail many of the author's recent results about the geometry of surface strips. Here too a suitable projective frame is associated with every point of the strip-curve, and by using the duality principle the author succeeds in giving a particularly simple form to most of his results. Among the many results of this section, we mention the ideas of osculating conic, cone, and quadric, as well as the surfaces generated or enveloped by them, the ruled surfaces belonging to a strip, in particular those for which the strip-curve is flecnodal, and so on. The section ends with some results about the strips of the second order (defined by assigning for every point of the strip curve, and in every strip plane, the asymptotic tangents of any surface containing the strip), and their connections with pangeodesic strips.

As one may see from the preceding survey, the book is mainly concerned with the projective differential geometry of ordinary space, although some topics of the projective differential geometry of hyperspaces are dealt with; this limitation is, in the reviewer's opinion, the only adverse criticism to be made of this excellent book. The reviewer would have liked very much, for instance, an exposition of the connection between curves in a projective n-space and linear ordinary differential equations. The proofs are very clear and rigorous; the geometrical point of view is never lost, and this will certainly help to make Dr. Bol's book very welcome. Very little previous knowledge is required from the reader, who will find after the last section a summary of the most important results and theorems which were needed in the text. In addition, at the end of every paragraph the author presents a collection of exercises and additions.

The book concludes with a very complete bibliography; this is intended to be the continuation of the one appearing at the end of the *Introduction à la géométrie projective différentielle* (Paris, 1931), by Fubini and Čech; the author's list includes almost all papers and books on this subject from 1931 to 1950, and will certainly be of the greatest aid to everyone interested in projective differential geometry.

V. DALLA VOLTA

Στοιχεῖα θεωρητικῆς γεομετρίας. (*Elements of theoretical geometry*). By N. Sakellariou. Athens, Pountza, 1950. Vol. I, 224 pp.; vol. II, 208 pp.; vol. III, 208 pp.

One approaches the task of reviewing this work with more than usual interest since here at the middle of the twentieth century appears a new *Elements* of geometry written some twenty-two and a half centuries after the *Elements* of Euclid (c. 300 B.C.) and in the same language. Although this work appears in modern Greek, one cannot help but marvel at the fact that most of it can be read with the help of the same dictionary that unlocks the treasures of Euclid.

The author of the present work is a Professor of Mathematics at the University of Athens; Euclid was the head of the mathematics group at the Museum in Alexandria. Both works cover essentially the same material although the modern *Elements* includes many theorems discovered in the centuries since Euclid, such as the circle of Lemoine, the triangle and circle of Brocard, Tucker's circle, and similar complicated constructions. It is also worthy of observation that students who purchased the original *Elements* paid for it in coins of the same denomination as those used to buy the modern text, namely, drachmas, although the values have slipped a bit during the centuries; for while the original might have been purchased, perhaps, for two or three drachmas (3 drachmas = one bushel of wheat) the publisher's announcement states that the first volume of the new *Elements* will cost around 20,000 drachmas.

It is also an interesting matter to compare the contents of the present work with that of Euclid. One difference is the frequency of historical comment which illuminates the text since such material obviously was not available to the father of the subject, although modern historians would have welcomed information on the part of Euclid as to the origin of much of his material, his debt to Pythagoras and Eudoxus among others. The modern work contains pictures of Euclid, Pythagoras, Archimedes, Pierre de Fermat, and K. F. Gauss, with brief biographical sketches of these mathematicians. The first book begins with a short history of geometry starting with Thales