of coverings of T such that, for any two, U_1, U_2, there is a third, U_3, such that, for each $u_0 \in U_3$, the sum of the u's in U_3 meeting u_0 lie in a set of U_1 and in a set of U_2. Such a structure exists if and only if T is completely regular. This notion generalizes the notion of metric. A given space may have various inequivalent uniformities; a compact space has only one. The concept of completeness is extended, and it is proved that any space with a uniform structure may be completed. Chapter VII carries over these notions from a collection of spaces to their product. An extension of Tychonoff's embedding theorem is proved. Chapter VIII presents a number of examples.

An extensive symbolism makes the reading difficult—especially so when meeting one of the numerous misprints or omissions.

N. E. STEENROD

Formal Logic. By Albert A. Bennett and Charles A. Baylis. New York, Prentice-Hall, 1939. 17+407 pp.

It is well known that logic, which for a long time remained in practically the same form in which Aristotle left it, has lately been undergoing a revolution. The cause of this phenomenon is, essentially, the impact of mathematics upon logic. It has been realized for some time that, by using mathematical methods, the statement of logical principles can be greatly simplified; likewise their scope can be extended so as to take in types of reasoning—such as those dealing with relations—which were only awkwardly handled by the traditional methods. The result has been the creation of an entirely new formal framework of great power and elegance.

As in many such revolutions a long time has had to elapse between the discovery of the new ideas and their appearance in elementary textbooks. Although the revolution in logic has been going on for nearly a century, yet it is only within about the last decade that textbooks of general logic have shown appreciably the effects of the new ideas. But since 1930 there have appeared a number of books of that character in which this effect has been marked. These include books by Stebbing, Cohen and Nagel, Eaton, Chapman and Henle, and a recent book by Churchman. None of these books contains a complete revamping of the subject, but every one is something of a compromise between the old and the new; nevertheless the amount of information concerning the new doctrine is considerable.

The present book belongs to the category just described. It aims "to provide a simple and clear survey of the field of formal logic synthesizing classical and modern developments into a unified treatment." It is the joint work of a mathematician and a philosopher, both of whom identified themselves with the new movement to the extent of becoming officers in the Association for Symbolic Logic. Under these circumstances a deeper penetration by the new ideas is to be expected.

The book begins with a chapter on the nature of logic. This is extremely well done; there is nothing comparable to it in any of the other books at the reviewer's disposal. A feature of it is the inclusion of examples of relatively complicated reasoning processes to be carried out informally. This is followed by a chapter on logical symbolism, in which the nature of symbols, and also of propositions, is discussed. After this follow two chapters containing a philosophical discussion of individuals, concepts, relations, classes, etc. The next two chapters are devoted to the traditional analysis of propositions and syllogisms. Boolean algebra, with a class interpretation, is then taken up in a chapter entitled "Formal Non-syllogistic Class Inference." After a chapter on classification, definition, and descriptions there follow three chapters, with a distinctly modern flavor, on the propositional algebra, the theory of propositional functions, and the formulation of logical deductive systems. The book closes with two chapters on probability and scientific method, occupying together about fifty pages.

The above outline will give an idea of the scope and general plan of the book. Let us now consider a few criticisms.

The reviewer is somewhat disappointed in regard to the treatment of the traditional logic. The procedures of the Boolean class algebra are simpler than those of the traditional logic and include them as special cases. Consequently, with the introduction of the former logic the latter has become practically useless. Yet in spite of this the authors devote a considerable portion of their book to developing the traditional logic along essentially traditional lines; in particular they establish the valid moods by giving a table showing the possible conclusions in each of 64 possible cases, and then saying that the reader may check the table by means of the Veine diagrams. There should be a Society for the Prevention of Cruelty to Students to stop this sort of thing!

The foregoing must not be supposed to mean that traditional logic has no uses whatever. There are in fact two main uses, neither of them practical. In the first place a book like this will be studied largely by philosophers; and in philosophy it seems that the history of the subject is especially important. From this point of view Aristotelian logic will be interesting to philosophers for a long time to come. Again, the derivation of traditional logic on the basis of an underlying Boolean algebra is a useful illustration of the technique of the latter. The first of these purposes will be equally well served, and the second vastly better, if traditional logic is treated after Boolean algebra, rather than before. Then the syllogistic rules can be derived, e.g. in ways suggested by the reviewer in *Mind*, vol. 45 (1936), pp. 209–216.

In connection with the above criticism there is another related one. The traditional hypothetical syllogism, dilemma, etc., are treated by the authors in the chapter on syllogisms. It is evident, however, that these topics have to deal with propositions as unanalyzed wholes, and should therefore be treated in an earlier part of the discussion. It would be useful to have all the relations between propositions as such, including those discussed here in Chapter 2, treated together under the heading Propositions. So far Churchman is the only author who has done this (see his Chapter 2).

The discussion should be preceded by a general discussion of significance of formal methods. The present authors postpone this to Chapter 11, at which time the student has become sufficiently sophisticated to tackle the difficulties of applying this idea to the calculus of propositions. But an idea of the nature of formal reasoning, as exemplified in the class calculus, can be presented to the reader at a much earlier stage; just as a quite adequate idea of the nature of logic as such is presented in Chapter 1. Such a discussion is the natural sequel to Chapter 1 as presented; and the reviewer was conscious of a hiatus at that point. A discussion of the sort stated is necessary in order to answer the question of how we can use formal methods to establish logical theorems, such as would be necessary for a development of the theory of the syllogism according to the above lines.

Let us now turn to a criticism of a different kind. In the discussion of concepts and classes the authors introduce at the start two levels of equivalence (and hence of inclusion), viz., strict or logical equivalence, and material or factual equivalence. The distinction is in keeping with a conceptualist-idealist point of view which the authors maintain throughout the work. The reviewer sympathizes with this point of view, and recognizes that for some purposes it may be necessary to consider simultaneously not only two, but even three or more such levels of abstraction. But if such distinctions are introduced into the calculus of classes the result is a mathematical system of great complexity. It seems better, when setting up the formal calculus, to introduce it first with only equality relation; to point out that this equality relation may be interpreted in either way; and to reserve the consideration of a calculus with two or more equality (or inclusion) relations until a later stage of the inquiry. Generalizations of this kind are not considered in any other elementary text.

To sum up, we have in this book by Bennett and Baylis a textbook on logic, designed for presentation to beginners, and intended as an introduction to modern mathematical, as well as to traditional formal logic. This is a difficult expository problem; and one for which a thoroughly satisfactory solution has not yet been found by anyone. That the book should, under these circumstances, be something of a compromise, is perhaps inevitable. The reviewer has criticized it from an ideal point of view, with reference to the goal to be attained—which, by the way, is of some importance for mathematics; these criticisms are to be taken not as pointing out defects in this book but as suggesting ways in which the next approximation to the goal can be improved. The text is one of great merit; most of the criticisms here made would apply to any similar book the reviewer knows of.

HASKELL B. CURRY

The Theory of Group Characters. By D. E. Littlewood. Oxford University Press, 1940. 8+292 pp.

The theory of group representations and group characters has already been treated very recently in the comprehensive expositions of Hermann Weyl's *The Classical Groups*,[1] and F. D. Murnaghan's *The Theory of Group Representations.* We now have a third treatment. All three differ not only in emphasis but also in the spirit[2] of their approach to the subject.

Littlewood's book is intended by him to give "a simple and self-contained exposition of the theory[3] in relation to both finite and continuous groups, and to develop some of its contacts with other branches of pure mathematics such as invariant theory and the theory of symmetric functions." Thus the first fifty-two pages of his text are devoted to an exposition designed to make it self-contained. Chapter I consists mainly of a discussion of the classical canonical form of a matrix under similarity transformations, the properties of unitary, orthogonal and real orthogonal matrices, the reduction of Hermitian matrices under unitary transformations, and the definition of direct product. Chapter II presents the concept of an algebra and its regular representations, the consequent definition of trace, and the further topics necessary for an understanding of the property

[1] Reviewed in this Bulletin, vol. 46 (1940), pp. 592–595.

[2] Cf. Footnote 4.

[3] Of group *characters*, not of group representations.