writer excluded from consideration the equation $1/x_1+1/x_2+\cdots+1/x_n+\lambda/x_1x_2\cdots x_n=b/((c+1)b-1)$, in which b, c are any positive integers, and λ is an integer greater than 1. The present paper identifies a class of maximum numbers relative to this equation. Using the terms E-solution and Kellogg solution, and the symbol $\Sigma_{n,r}(x)$ in senses previously employed, it is proved that if w is the Kellogg solution and if $X\neq w$ is any E-solution of the equation, then $\Sigma_{n,n-1}(w)>\Sigma_{n,n-1}(X)$ and $\Sigma_{n,n}(w)>\Sigma_{n,n}(X)$. On account of these two inequalities, it is possible to associate with this equation a class of infinitely many maximum numbers. It is also shown that $w_n>X_n$, which might be called the Kellogg property. (Received October 17, 1940.)

87. L. I. Wade: *Certain quantities transcendental over the field $GF(p^n, x)$.*

In a previous paper (abstract 46-1-129) the author proved certain quantities transcendental over the field $GF(p^n, x)$. The present paper includes certain additional transcendental quantities. For example, $\sum_{k=1}^{\infty}1/(x^{p^{nk}}-x)$ is transcendental, $\sum_{k=0}^{\infty}1/x^{q^k}$ $(q>1)$ is algebraic if q is of the form p^s and transcendental otherwise, and $\sum_{k=0}^{\infty}1/x^{k^q}$ $(q>1)$ is transcendental. (Received November 25, 1940.)

88. Max Zorn: *Idempotency of infinite cardinals.*

This paper contains a simple proof of the theorem that the product of two infinite numbers is equal to the maximal factor. The application of ordinal numbers is replaced by the use of the maximum principle. (Received October 28, 1940.)

Topology

89. Ben Dushnik and E. W. Miller: *On the dimension of a partial order.*

Let A be any set. Let K be any collection of linear orders, each defined on all of A. A partial order P on A is defined as follows: For any two elements a_1 and a_2 of A, $a_1<a_2$ (in P) if an only if $a_1<a_2$ in every linear order of the collection K. A partial order so obtained will be said to be realized by the linear orders of K. With the aid of a result due to Szpilrajn on the linear extensions of a partial order (Fundamenta Mathematicae, vol. 16 (1930), pp. 386–389), it is seen that if P is any partial order on a set A, then there exists a collection K of linear orders on A which realize P. By the dimension of a partial order P defined on a set A is meant the smallest cardinal number m such that P is realized by m linear orders on A. It is shown that if n is any natural number, there exists a finite partial order of dimension n, and if m is any transfinite cardinal, there exists a partial order of dimension m defined on a set of power m. Reversible partial orders (see abstract 46-5-266) are those of dimension not greater than 2. (Received October 25, 1940.)

90. Samuel Eilenberg: *Imbedding of spaces into euclidean spaces.* Preliminary report.

Given a metric space X, let $P(X)$ be the subset of the cartesian product $X\times X$ consisting of all points $(x, y)\in X\times X$ such that $x\neq y$. If every i-dimensional compact Vietoris cycle mod 2 of $P(X)$ bounds in $P(X)$ for $i=1, 2, \cdots, n-1$, then X is not imbeddable topologically into the euclidean n-space. Many other results of this type are obtained. Analogous theorems hold if the join $X\circ X$ is considered instead of the product $X\times X$. (Received November 25, 1940.)

91. Samuel Eilenberg: *Monotone families of manifolds.*

Given a closed n-manifold M in the euclidean space E^{n+1}, $D(M)$ will denote the bounded component of $E^{n+1}-M$. A family $\{M_t\}$ of n-manifolds depending on a real parameter t is called: (a) monotone if $D(M_{t_1}) \subset D(M_{t_2})$ for $t_1 < t_2$, (b) strictly monotone if $M_{t_1} \subset D(M_{t_2})$ for $t_1 < t_2$, (c) continuous if $\lim M_{t_n} = M_t$ whenever $\lim t_n = t$. If $\{M_t\}$ $(0 < t \leqq 1)$ is monotone continuous and $\delta(M_t) \to 0$ as $t \to 0$, then $D(M_t) + M_t$ is contractible to a point and M_t is a homology-sphere. If moreover $\{M_t\}$ is strictly monotone, then also $D(M_t)$ is contractible to a point. (Received November 25, 1940.)

92. O. G. Harrold: *On a class of continuous maps.*

In this note the class of continua having the property of Čech (that is, of admitting a map of finite sections into the interval) is identified as those Peano continua M such that every dendrite D in M has the Čech property. For dendrites the problem was solved by Mazurkiewicz (Fundamenta Mathematicae, vol. 17, pp. 88–98). (Received November 23, 1940.)

93. F. B. Jones: *Topologically flat spaces.*

A nondegenerate continuous curve (in a complete Moore space) will be called topologically flat provided that (1) it contains no cut point, (2) the Jordan curve theorem holds true in it, and (3) it is locally remotely connected. Conditions are obtained under which a topologically flat continuous curve is a subset of a plane (complete separability being sufficient) and certain of these subsets are characterized by means of upper semicontinuous collections in the plane. (Received November 23, 1940.)

94. F. B. Jones: *Totally discontinuous linear functions whose graphs are connected.*

An example of a real function $f(x)$ of a real variable is constructed with the following properties: (1) $f(x+y) = f(x) + f(y)$; (2) $f(x)$ is defined for all real values of x and is discontinuous for each value of x; (3) the graph of $y = f(x)$ is a connected subset M of the number plane; (4) the set M is dense in the plane but contains no continuum; (5) considered as a space, (a) M is metric and linearly ordered, (b) M is the sum of a countable number of arbitrarily small totally disconnected domains, and (c) M contains a totally disconnected closed set which contains a domain. Other related problems are considered. (Received October 24, 1940.)

95. J. P. LaSalle: *Pseudo-normed linear sets over valued rings.* II.

A generalized concept of a linear function on a pseudo-normed linear set over a valued ring T (a P_1-space) to a P_1-space T' is introduced where the valued rings need not be the same, though a sort of generalized homogeneity is assumed. Bounded sets are defined for P_1-spaces, and it is shown that a linear function maps bounded sets into bounded sets. These generalized linear functions are shown to have others of the properties which linear functions possess in less general spaces. A differential with the usual properties can be defined for functions with arguments and values in P_1-spaces. When the P_1-spaces are in particular the "topological abelian groups" considered by Michal (*First order differentials of functions with arguments and values in topological abelian groups*, to be published in the Revista de Ciencas, University of Lima), the existence of the M_1-differential implies the existence of the differential considered in this paper. By replacing the valued ring by a valued division ring in which the valuation satisfies $M(\alpha\beta)=M(\alpha)M(\beta)$ (it is not required that the valuation be archimedean), additional theorems concerning linear functions, and so on, may be proved. In particular it can be shown that the differential, if it exists, can be calculated as a limit. (Received November 8, 1940.)

96. A. N. Milgram: *Extensible and inextensible decompositions.* Preliminary report.

If A is a closed subset of a topological space S, and if A is decomposed into a finite number of closed sets $A=A_1+A_2+\cdots+A_r$, then we shall call the decomposition extensible if $S=S_1+S_2+\cdots+S_r$ is a decomposition of S into closed sets such that $S_i\supseteq A_i$ and the decomposition of S has the same nerve as that of A. If the decomposition of A is not extensible it will be called inextensible. (See Menger's *Dimensionstheorie*, pp. 312–313.) This paper is devoted to a study of extensible and inextensible decompositions of topological spaces. For example, it is shown that a compact space is of dimension not less than n if and only if it contains a closed subset having an inextensible decomposition into $n+1$ closed sets, each n of which intersect. Criteria to determine when a continuous mapping of a subset of a space can be extended to the whole space are given in terms of extensible decompositions. Further results will be reported in a later paper. (Received October 24, 1940.)

97. A. N. Milgram: *Extensions of decompositions.*

The system of sets $B_1, B_2, \cdots, B_s$ is said to govern the system $A_1, A_2, \cdots, A_s$, if $B_{i_1}\cdot B_{i_2}\cdot \cdots \cdot B_{i_k}=0$ implies $A_{i_1}\cdot A_{i_2}\cdot \cdots \cdot A_{i_k}=0$. For each absolute neighborhood retract R, there exists a decomposition into closed sets $R_1, R_2, \cdots, R_n$ which in a sense determines whether mappings of subspaces into R can be extended to mappings of the whole space into R. Namely, if $f\in R^A$, where A is a closed subset of the metric space S, then f can be extended to a mapping $f^*\in R^S$ if and only if there exist closed sets $S_i\supseteq f^{-1}(R_i)$, where $i=1, 2, \cdots, n$, which cover S and such that the system $S_1, S_2, \cdots, S_n$ is governed by the system $R_1, R_2, \cdots, R_n$. If R is not an absolute neighborhood retract, the theorem is shown to be false by an example. A space S is called connected between n of its closed sets $A_1, A_2, \cdots, A_n$ (see Menger, *Dimensionstheorie*, pp. 312–313) if for each decomposition of S into closed sets $S_1, S_2, \cdots, S_n$ such that $S_i\supseteq A_i$ we have $\prod S_i\neq 0$. It is shown that a separable metric space is of dimension not less than n if and only if it is connected between some $n+1$ of its closed sets. (Received November 26, 1940.)

98. Harlan C. Miller: *Concerning certain types of end points of compact continua.*

H. M. Gehman made a study of various definitions of end points of bounded plane continua. The following definition was not included: The point P of a compact continuum M is said to be a terminal point of M if every irreducible subcontinuum of M which contains P is irreducible from P to some point. In this paper it is shown that this definition is not equivalent to any of those considered by Gehman, except for the case where M is a continuous curve. Among others the following results are established: A compact nondegenerate continuum every subcontinuum of which is unicoherent and decomposable has two terminal points. A compact continuum every subcontinuum of which is unicoherent and decomposable is irreducible about the set consisting of all its terminal points. If M is a compact hereditarily decomposable continuum and K is a subset of the set of all its terminal points, then $M-K$ is connected. (Received November 25, 1940.)

99. Harlan C. Miller: *On the existence of a certain type of irreducible continuum.*

In this paper the following theorem is proved: if M is a closed and compact subset of a line L lying in a plane E, there exists a compact continuum N lying in E such that (1) one of the complementary domains of L has no point in common with N, (2) the common part of N and L is M, and (3) if x and y are points of different components of M then N is irreducible from x to y. (Received November 25, 1940.)

100. Harlan C. Miller and R. L. Swain: *Properties of two intersecting arcs.*

Let space satisfy Axioms 0–5 of R. L. Moore (American Mathematical Society Colloquium Publications, vol. 13). The following definition is due to R. L. Moore: The arc AB is said to cross the arc CF provided there exist intervals $A'B'$ and $C'F'$ of AB and CF, respectively, and a simple closed curve J containing $C'F'$ such that A' and B' belong to different complementary domains of J and such that the arc segment $C'F'$ contains the common part of J and $A'B'$. Some necessary and sufficient conditions are given for crossing, and the crossing relation is shown to be reciprocal. Two arcs may cross at a point or interval, may touch at a point or interval, or may cross in every neighborhood of a point or interval. Among other results obtained in this paper are (1) if two arcs cross, they cross in every neighborhood of some component of their common part, (2) if two arcs cross, but cross at no component of their common part, and K is the set of all components H of their common part such that they cross in every neighborhood of H, then K is a perfect set of point sets. (Received November 25, 1940.)

101. S. B. Myers: *Compact groups whose elements are of finite order.*

Denote by G a metrizable topological group all of whose elements are of finite order, and by G_N a G which satisfies the postulate that if a sequence of elements g_i converges to the identity, then the order of g_i becomes infinite. (This postulate is satisfied by any G which is a group of periodic homeomorphisms of a manifold into itself.) In this note it is proved that a compact G_N is 0-dimensional, and a compact abelian G_N is finite. (Received November 25, 1940.)

102. J. W. Odle: *Non-separating and non-alternating transformations modulo a family of sets.*

In this paper the type of generalization of non-alternating and non-separating transformations introduced by E. P. Vance (Duke Mathematical Journal, vol. 6 (1940), pp. 66–79) is carried still further, and the most general possible transformations of this kind are shown to have the same characteristic properties as Vance's transformations. A systematic analysis of product and factor theorems involving monotone, non-separating, non-alternating, weakly non-separating, and weakly non-alternating transformations is made, and with the addition of the two new theorems in this paper, all possible theorems of this type are now known. The relationships between locally non-alternating and 0-regular transformations are studied, and it is proved that any 0-regular transformation is locally non-alternating. An example is given to show that the converse is not true in general. However, on certain special sets the transformations are equivalent. (Received October 17, 1940.)

103. N. E. Rutt: *Sets with a certain sort of derived set.*

Let D be a set of objects. Suppose that corresponding to each subset K_a of D there is a second subset K_a' of D such that, when K_a' is nonvacuous and at most a finite number of K_a are not in K_b, then the product of K_a' and K_b' is nonvacuous. The sets K_a' exhibit a few of the simpler aspects of derived sets and the nature of these is the subject of this paper. (Received November 18, 1940.)

104. G. E. Schweigert: *Border transformations.*

The nature of these transformations may be inferred from the fact that for $B=T(A)$ a perfect set, where A is compact and metric, the transformation is interior if and only if T is a (continuous) border transformation such that the inverse image of a point contains no open set. Questions as to what extent open or connected open sets are preserved are considered in detail. These considerations yield necessary and sufficient conditions that T be border. On locally connected continua, theorems holding for T interior are generalized and a factor theorem shows that a border T is quasi-monotone. Some results showing the difference between border and interior transformations are also obtained. (Received November 25, 1940.)

105. R. H. Sorgenfrey: *Concerning triodic continua.*

It is the purpose of this paper to present several definitions of the term triodic, first defined by R. L. Moore in Fundamenta Mathematicae, vol. 13 (1929), p. 262, and to establish the following theorem: If the compact continuum M contains three continua which have a point in common and such that no one of them is a subset of the sum of the other two, then M contains a continuum N, three points P, Q, and O not belonging to N, and continua H, K, and L irreducible from N to P, Q, and O, respectively, each two of the continua H, K, and L having only N in common. (Received November 23, 1940.)

106. R. L. Swain: *Distance axioms in Moore spaces.*

A Moore space satisfies Axiom 0 and Axiom 1 (condition (4) omitted) of R. L. Moore (American Mathematical Society Colloquium Publications, vol. 13); a weak Moore space satisfies Axiom 0 and Axiom 1′, where Axiom 1′ is Axiom 1 except that the condition $\bar{g}\subset (R-B)+A$ is replaced by $g\subset (R-B)+A$. In the following axioms, X, Y, X_n, Y_n are points of a space S: (A) $d(XY)=d(YX)$ is a real number; $d(XY)=0$ if $X=Y$; $d(XY)>0$ if $X\neq Y$; (B) if $\lim d(YY_n)=0$, then $\liminf d(XY_n)\leqq d(XY)$; (C) if $\lim d(XX_n)=0$ and $\lim d(YY_n)=0$, then $\lim d(X_nY_n)=0$; (D) given X and given a positive number ϵ, there exists a positive number $\delta_{p\epsilon}$ such that if $\lim d(YY_n)=0$ and $d(XY_n)<\delta_{p\epsilon}$, then $d(XY)<\epsilon$. The point X is said to be a limit point of a point set M if, for each n, there exists a point X_n of M such that $\lim d(XX_n)=0$. In order that S be a weak Moore space, it is necessary and sufficient that S satisfy Axioms A, B, C. For S to be a Moore space, it is necessary and sufficient that S satisfy Axioms A, B, C, D. (Received November 25, 1940.)

107. R. L. Swain: *Linear metric space.*

A metric space S is called linear if for each three points X', Y', Z' of S there exist three points X, Y, Z such that $X+Y+Z=X'+Y'+Z'$ and $d(XZ)=d(XY)+d(YZ)$, where d is a metric distance function for S. If S does not consist of just four points, it is shown that there exists a reversibly continuous transformation which preserves order and distance and which maps S into some subset of the x-axis. (Received November 25, 1940.)

108. P. M. Swingle: *An abstraction of types of connected sets.*

Let U and V be subsets of a set M such that $M=U+V$ where not both U and V are M. Let $S(X)$ denote that set X has property S, $U\ r\ V$ denote that U has relation r to V. A set M is of type (I, B, W) in P and r if and only if M has property P and, for every pair U and V with property P, $U\ r\ V$. Let M be a set of type (I, B, W). Then M is an SPQ-set of type B if and only if $U\ r\ V \equiv [S(U)\cdot S(V)$ has property $Q]$, M is an SPQ-set of type I if and only if $U\ r\ V\equiv$ [either $S(U)$ or $S(V)$ has property Q], and M is an SPQ-set of type W if and only if $U\ r\ V\equiv$ [both $S(U)$ and $S(V)$ have property Q]. Certain general theorems are obtained concerning these sets and particular theorems obtained assigning various properties to S, P and Q. One can take these such that types B, W and I are respectively biconnected, widely connected and indecomposable closed connected sets. (Received November 18, 1940.)

109. P. M. Swingle: *Indecomposable connexes.*

Let M be a connected point set. Then M is an indecomposable connex if and only if for every two connected subsets H, K of M such that $M=H+K$ either both H and M or both K and M have the same closure. And M is an irreducible joining connex closure between a and b if and only if there exists a connected subset N of M such that $N+a+b$ is connected and for all such N's the sets N and M have the same closure. Examples are given of these sets and among others the following theorem is proven: If M is connected and the closure of M is compact, then in order that M be an indecomposable connex it is necessary and sufficient that there exist three points x, y, z such that M is an irreducible joining connex closure between each two of these points. (Received October 26, 1940.)

110. A. W. Tucker: *Barycentric mappings.*

To each cell x of an augmentable complex X let there correspond a vertex $v(x)$ of a simplicial complex Y in such a way that $v(x), v(x'), \cdots, v(x^{(k)})$ are all vertices of some single simplex of Y if $x<x'<\cdots<x^{(k)}$ ($<$ means "is a face of"). Then $Tx=v(x)TFx$ defines a chain mapping of X into Y satisfying the usual requirement that $FTx=TFx$. And if Y is a closed subcomplex of X such that each x is joined in X to the simplex formed from the vertices $v(x), v(x'), \cdots, v(x^{(k)})$ where $x<x'<\cdots<x^{(k)}$, then $Dx=v(x)\{x-DFx\}$ defines a chain deformation such that $DFx+FDx=x-Tx$. The correspondence $x\rightarrow v(x)$ may be thought of as a simplicial mapping of the barycentric subdivision of X into Y—hence the name barycentric mapping. It has many convenient properties and a wide variety of uses, particularly in comparing one homology system with another. (Received October 26, 1940.)

111. C. W. Vickery: *A new proof of a theorem of Chittenden.*

It is shown that if S is a space $(\mathcal{E})$ in which the écart is regular (terminology of Fréchet's *Les Espaces Abstraits*, p. 219), there exists an equicontinuous collection G of real valued point functions defined over S and satisfying the conditions of Theorem 1.6 of the author's paper *Spaces of uncountably many dimensions* (this Bulletin, vol. 45 (1939), p. 459). Thus S is homeomorphic with a subset of a space D^{α} and hence is metric. This establishes a theorem of Chittenden (Transactions of this Society, vol. 18 (1917), p. 161). (Received October 12, 1940.)

112. R. L. Wilder: *Characterizations of those euclidean domains whose boundaries are lc^k.*

The central problem is to find positional properties which characterize those domains in the euclidean n-sphere whose boundaries are lc^k $(k \geqq 0)$. The only solution previously given was for the case where $n=2$ and $k=0$ (R. L. Moore, Fundamenta Mathematicae, vol. 3 (1922), pp. 232–237). A complete solution is given in this paper, using the properties S_i defined in an earlier paper (American Journal of Mathematics, vol. 61 (1939), pp. 823–832) and a weak property S_i (equals WS_i) defined exactly as was S_i except that carriers of chains need not be self-compact: In order that the boundary of a domain D in the euclidean n-sphere should be lc^k, it is necessary and sufficient that D have (1) property WS_0^k and (2) property S_{n-k-1}^{n-1} and property S_{n-k-2} relative to bounding cycles. For $n=2$, $k=0$, this is equivalent to the result of Moore cited above, since S_0 implies WS_0. Weak uniform local i-connectedness (equals i-*wulc*) is also investigated, and among other results it is found that in order for a common boundary of two domains in the euclidean n-sphere to be lc^k it is necessary and sufficient that the domains be $wulc_0^k$. (Received November 23, 1940.)

113. R. L. Wilder: *On the domains complementary to continua having certain avoidability properties.*

G. T. Whyburn showed (American Journal of Mathematics, vol. 61 (1939), pp. 733–749) that in the plane the boundary of a domain complementary to a compact semi-locally connected continuum is peanian. The present paper employs the property of almost local i-avoidability (equals i-*ala*; see American Journal of Mathematics, vol. 61 (1939), p. 832, footnote 18) which for $i=0=p^0$ is equivalent to semi-local connectedness. Among the results obtained are the following: Let M be a subcontinuum of S^n which is ala_0^{n-2}. Then (1) if only finitely many of the "small" i-cycles $(i \leqq n-2)$ of a complementary domain D are linearly independent relative to homologies (*lirh*) in D, $F(D)$ is peanian; (2) if the complementary domains of M form a null sequence and only finitely many "small" i-cycles $(0<i \leqq n-2)$ of S^n-M are *lirh* in S^n-M, then M is lc^{n-2}; (3) under condition (2) if M is i-avoidable for $i \leqq n-2$, all but a finite number of complementary domain boundaries are g.c. $(n-1)$-m.'s. If M is an $(n-2)$-*ala* Peano continuum in S^n, then the complementary domain boundaries are all peanian (since every Peano continuum is 0-*ala*, this is exactly the well known Torhorst theorem when $n=2$). (Received November 23, 1940.)

114. R. L. Wilder: *The duality between the S-properties of closed sets and their complements in the euclidean n-sphere.*

Let property B_i denote property S_i relative to bounding cycles (American Journal of Mathematics, vol. 61 (1939), pp. 823–832). The following duality is established: In order that a closed subset M of the euclidean n-sphere S^n should have property B_i, it is necessary and sufficient that S^n-M have property B_{n-i-2}. Thus property S_i of S^n-M and finiteness of $p^{i+1}(S^n-M)$ are dual to property S_{n-i-2} of M and finiteness of $p^{n-i-1}(M)$, since for any set K to have property S_i is equivalent to having property B_i and finite $p^i(K)$. The duality is a consequence of the following results: (1) If M is a closed subset of S^n, then a necessary and sufficient condition that S^n-M have property B_i is that the closed subsets of M be almost completely $(n-i-2)$-avoidable by bounding cycles of M; and (2) a necessary and sufficient condition for a compact metric space to have property B_i is that the closed subsets of M be almost completely i-avoidable by bounding cycles. (Received November 23, 1940.)