ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

152. V. W. Adkisson: *Plane peanian continua with homeomorphisms extendable in the sense of Antoine.*

Let M be a peanian continuum which lies in a plane S, has a finite number of cut points of power greater than 2, and does not separate S. Let M' be any topological map of M in a plane S'. The following conditions are necessary and sufficient that there exist at least one homeomorphism T such that $T(S)=S'$ and $T(M)=M'$: (1) for each cut point P of power 2, at least one of the two components of $M-P$ is symmetric, (2) for each cut point of power greater than 3 the power is finite, and all the components of $M-P$ are homeomorphic with each other except perhaps one. If M is a planar graph, the homeomorphism T exists if and only if M is one of the graphs characterized in the author's thesis, with the one exception noted there (Comptes Rendus des Séances de la Société des Sciences et des Lettres de Varsovie, vol. 23 (1930), pp. 164–193). (Received March 16, 1939.)

153. H. A. Arnold: *Note on completely continuous differentials.*

Given a completely continuous operator $F(x)$ on a Kantorovitch space S to S, suppose $F(x)$ to have a completely continuous differential $F(x_0; dx)$ at x_0. Then if the topological index of the operator $x-F(x_0; x)$ exists, so will the topological index of the transformation $x-F(x)$, and they will be equal. The differential used is one that was first defined by the author, using the idea of approximation to the first difference by means of (abstract) infinitesimals of higher order. The proof of the theorem in question is by means of a metric defined over a single convergent sequence. (Received March 18, 1939.)

154. A. A. Aucoin and W. V. Parker: *Diophantine equations whose members are homogeneous.*

Desboves (Nouvelles Annales de Mathématiques, (2), vol. 18 (1879), p. 481) implies that for $ax^m+by^m=cz^n$ to have a solution in integers, c must always be an integer of a particular kind. If f and g are homogeneous polynomials with integral coefficients, of degrees m and n, respectively, where m and n are relatively prime, then the equation $f(x_1, x_2, \cdots, x_r)=g(y_1, y_2, \cdots, y_s)$ always has solutions in integers. Expressions for these solutions in terms of $r+s$ parameters are obtained. The method of solving is also shown to be applicable to certain nonhomogeneous equations. As an example, solutions are obtained for the equation $ax^2y+bxz+cyz^2=pu^3+qv$, where a, b, c, p, q are integers. (Received March 14, 1939.)

155. Reinhold Baer: *Duality and commutativity of groups.*

Two groups are said to be duals of each other if there exists an anti-isomorphism between their lattices of subgroups. Hamiltonian groups and groups containing elements of infinite order do not possess duals. To be a dual and to be abelian are equivalent properties of finite groups, generated by elements of order a prime number p; and this statement is but one in a larger class of theorems connecting duality and commutativity. If G is an abelian group, then the following three statements are equivalent: (1) there exists a dual to G; (2) the orders of the elements and the orders of the primary components of G are finite; (3) G is self-dual. Finally, fairly large classes of groups are characterized where the existence of a dual implies that the group is "almost abelian." (Received March 7, 1939.)

156. I. A. Barnett and Otto Szász: *On a certain Diophantine equation.*

The authors determine all the solutions of the equation $\cos n\alpha+\cos n\beta=0$ for which $\cos\alpha$ and $\cos\beta$ are rational. It is shown that except for the trivial solutions $\cos\alpha=-\cos\beta$, which exist for all odd n, there are rational solutions if and only if n is contained in one of the two arithmetical sequences $4k-2$ or $6k-3$. Corresponding to each value of k, there are infinitely many rational solutions. However, if $n=4k-2$ where k is of the form $3k'-1$, there are additional trivial solutions. The case $n=4k-2$ leads to the Pythagorean equation $x^2+y^2=1$. Certain generalizations to systems of equations in more than two variables are discussed. (Received March 9, 1939.)

157. H. G. Barone: *Limit points of sequences and their transforms by methods of summability.*

In a paper given before the Mathematical Association of America at Charleston, S.C., March 25, 1939, the following theorem was proved: If $\{s_n\}$ is a bounded complex sequence such that $\lim |s_{n+1}-s_n|=0$, then the set of its limit points is connected. In the present paper, this theorem is applied to sets of limit points of the transforms of bounded complex sequences by the Hölder, Cesàro, Riesz, de la Vallée Poussin, and Euler methods of summability. The set of limit points of the transform of a bounded complex sequence is connected for the Hölder transformation of order r, where r is any positive integer; for the Cesàro transformation of order r, where $Re(r)>0$; for the Riesz transformation of order r, where $Re(r)>0$; for the de la Vallée Poussin transformation; and for the Euler transformation of order r, where $0\leqq r<1$. The set of limit points need not be connected for the Cesàro transformation when $Re(r)\leqq 0$; for the Riesz transformation when r is real and less than or equal to 0; and for the Euler transformation except when r is a real number less than 1. (Received March 8, 1939.)

158. Walter Bartky: *Solution of systems of differential equations by definite integrals.*

A fundamental set of solutions of $dx_i/dt=(A_{ij}+t^{-1}B_{ij})x$, $(i,j=1,\cdots,n)$, with matrix $B_{ij}=a_ib_j$ of rank one, is given by $\int_c e^{\lambda t}(\lambda I-A)^{-1}\phi d\lambda$ where $\phi=I-Be^{-\alpha(\lambda)}\cdot\int e^{\alpha(\lambda)}(\lambda I-A)^{-1}d\lambda$ and $\alpha(\lambda)=b[\log(\lambda I-A)]a$. The contour C has extremities at infinity and loops the characteristic values of A. Similar results are obtainable for $(A_{ij}+tI_{ij})dx_j/dt=B_{ij}x_j$. (Received March 17, 1939.)

159. E. F. Beckenbach and Maxwell Reade: *A characterization of plane isothermic maps.*

Continuing the work reported in abstracts 44-3-92, 44-7-309, and 45-1-76, the authors prove the following theorem: If the functions $x_j(u, v)$, $(j=1, 2, 3)$, are of class C_3 (that is, all derivatives of order less than or equal to three are continuous) in a simply connected domain D, then a necessary and sufficient condition that these functions map D isothermically on a plane surface is that $\sum_{j=1}^{3}[\int_\gamma x_j(u, v)(du+idv)]^2=0$ for all closed rectifiable Jordan curves γ lying in D. (Received March 17, 1939.)

160. Clifford Bell: *Solution of numerical equations.*

The usual methods of approximating to an irrational root of a numerical equation are such that the accuracy of a result is not easily obtained, at least by the particular method used alone. However, combinations of methods have been used which will give a measure of the accuracy of any given approximation. In this paper osculatory parabolas are used to obtain approximations, and conditions are stated under which the intercepts of such parabolas bound the irrational root of the given numerical equation. Thus, without the aid of any other devices, the maximum numerical error is known at each step. (Received March 24, 1939.)

161. B. A. Bernstein: *Groups and abelian groups in terms of negative addition.*

The author defines postulationally groups and abelian groups in terms of the "negative" of $a+b$ in (additive) groups and abelian groups. (Received March 16, 1939.)

162. Garrett Birkhoff and John Dyer-Bennet: *A note on real algebraic functions.*

Let $f(x)$ denote any real algebraic function, and let $p(n)$ denote the proportion of the first n terms of the series $f(1), f(2), f(3), \cdots, f(n), \cdots$ which are integers. It is shown that unless $f(x)$ is a polynomial, $p(n)\rightarrow 0$ as $n\rightarrow\infty$. In other words, the density of integral values assumed by a non-polynomial algebraic function for integral arguments is zero. If $f(x)$ is polynomial of degree d, and has more than d integral values, then it is trivial that its coefficients are rational, and the pattern of its integral values is periodic. Further special results are given. (Received March 31, 1939.)

163. L. M. Blumenthal: *Metric methods in determinant theory.*

This paper is a resumption of the general program, initiated by the writer in earlier articles, of applying the methods of abstract metrics to determinant theory. The behavior of several types of symmetric determinants is studied when certain of their principal minors are subjected to various conditions. Of the chains of new theorems obtained, some were conjectured by the writer in previous papers. One of these chains of theorems (the proof of which is based upon the characterization of pseudospherical sets) is the following: Let $\Delta=|r_{ij}|$, $-1<r_{ij}<1$, $(i\neq j)$, $r_{ii}=1$, $(i, j=1, 2, \cdots, m)$, be a symmetric determinant of order $m>n+3$, where n is any positive integer. If (i) every principal minor of order less than $n+2$ is nonnegative, while every such minor of order $n+2$ vanishes, (ii) at least one principal minor of order $n+3$ does not vanish, then (1) upon multiplying appropriate rows and the same numbered columns of Δ by -1, each element outside the principal diagonal has the value $-1/(n+1)$, and (2) each kth order principal minor of Δ equals $(n+2)^{k-1}(n-k+2)/(n+1)^k$, $(1 \leqq k \leqq m)$. Thus the hypotheses (i), (ii) serve to fix (essentially) the value of *every* element of a determinant of type Δ. (Received February 21, 1939.)

164. Richard Brauer: *On groups whose power is divisible by the first order of a prime.* Preliminary report.

Let G be a group of order $g = pg'$ where the prime p does not divide g'. The irreducible representations of G are studied, and a large number of results concerning the group characters of G are obtained. Three applications of this theory may be mentioned. The degrees of the irreducible representations of numerous groups can be calculated. Further, it can be shown that if the commutator group of G has an order divisible by p, then the degrees of the irreducible representations of G are greater than or equal to $(p-1)/2$ (excluding, of course, the 1-representation). This improves, for this class of groups, a result of Blichfeldt. Finally, it follows that a group of order pqr^c, $(p, q, r$ distinct primes), can be simple only if $r=2$, and the difference of p and q is a power of 2. (Received March 10, 1939.)

165. R. H. Cameron and W. T. Martin: *Reciprocals of Fourier-Stieltjes transforms analytic in a strip.*

Conditions are given under which the reciprocal of an analytic Fourier-Stieltjes transform in a strip is itself an analytic Fourier-Stieltjes transform in the strip. The theorm furnishes an extension of the Wiener-Pitt theorem on reciprocals of nonvanishing Fourier-Stieltjes transforms on a line. (Duke Mathematical Journal, vol. 4 (1938), pp. 420–436.) The theorem has applications to integro-difference equations and to difference equations of infinite order. (Received March 10, 1939.)

166. C. C. Camp: *A multiple series expansion.* Preliminary report.

In a former paper the author treated the system $X_j' + [\sum_{i=1}^{p} \lambda_i a_{ji}(x_j)]X_j = 0$, $(j=1, 2, \cdots, p)$, with corresponding auxiliary conditions at more than two points in each independent variable. In order to make the denominators in the contour integrals separate after appropriate transformations on the λ's, it was assumed that each $a_{ji}(x_j)$ maintains its average value when integrated over every subinterval involved in the auxiliary condition in x_j. The purpose of the present paper is to remove this restriction. To do this one makes use of an article by H. Poincaré (*Sur les residus des intégrales doubles*, Acta Mathematica, vol. 9 (1886), pp. 321–380) and extends it to the case of p-fold contour integrals for the case of simple poles in each complex variable. With suitable restrictions on the constants of the auxiliary conditions and the extension of certain lemmas from simple to p-fold contours, it is possible to establish the convergence of the appropriate multiple series expansion for the usual type of arbitrary function $f(x_1, x_2, \cdots, x_p)$. (Received March 8, 1939.)

167. H. H. Campaigne: *Product hypergroups.*

The product $A \times B$ of two hypergroups A and B is defined to be the set of all pairs $a \times b$, a in A, b in B. Products are defined thus, $(a' \times b')(a'' \times b'') \supset a \times b$ if and only if $a'a'' \supset a$ and $b'b'' \supset b$. $A \times B$ is a hypergroup. A subproduct $A \circ B$ is a set of such pairs in which each element a of A appears in at least one pair, each element b of B likewise, and $A \circ B$ is a hypergroup. The product as defined is a special case of subproduct. $A \circ B$ is regular (or semiregular) if and only if both A and B are regular (or semiregular). If A has an idemfactor a_1 such that the set a_1^2 contains only a finite number of elements, then $A \times B$ contains a subhypergroup abstractly identical with A if and only if B contains an idempotent element b_0. An example shows that the theorem cannot be strengthened. If A and B have conjugations among their elements, then there exists a conjugation among the pairs of $A \circ B$ such that $\{A \circ B\} = \{A\} \circ \{B\}$. (Received March 18, 1939.)

168. R. V. Churchill: *On an extension of the Sturm-Liouville development.*

In the problem of determining the temperatures in a slab with one face in contact with a fluid, a homogeneous system of ordinary differential equations arises in which the parameter appears not only in the differential equation but also in one of the boundary conditions. Several other problems in partial differential equations lead to such systems, notably problems in torsional vibrations of shafts with ends subjected to elastic restraints. The temperature problem has been treated quite fully by R. E. Langer in the Tôhoku Mathematical Journal, vol. 35 (1932), pp. 260–275. The characteristic functions do not form an orthogonal set. But by modifications of methods used when the characteristic functions are orthogonal, Langer found the required expansion of the arbitrary initial temperature function and established a theorem of equiconvergence with the Fourier expansion. It is shown in the present paper that by restricting the problem somewhat and requiring greater regularity of the arbitrary function the system can be transformed by a suitable substitution to a standard Sturm-Liouville system, so that the classical theory furnishes the required expansion. (Received March 17, 1939.)

169. L. W. Cohen: *On the mean ergodic theorem.*

It is shown that if T is a linear transformation on a Banach space B to B such that the norms of the iterates T^n of T are bounded, a_{nj} is a regular matrix such that $\sum_{j=k}^{\infty}|a_{n,j+1}-a_{nj}| \to 0$ as $k \to \infty$ uniformly in n, and $L_n x = \sum_{j=1}^{\infty} a_{nj} T^j x$ is a weakly compact set, then there is an $x_0 \varepsilon B$ such that $L_n x \to x_0$ and $Tx_0 = x_0$. The matrices of (C, r) with $r > 0$ satisfy the uniformity condition. The case $r = 1$ with Hilbert space as B is the mean ergodic theorem of von Neumann. (Received March 2, 1939.)

170. Harald Cramér: *On the representation of a function by certain Fourier integrals.*

This paper contains necessary and sufficient conditions in order that a complex-valued function $f(t)$ of a real variable be representable as the Fourier-Stieltjes transform of a function $F(t)$ having one of the following properties: (i) $F(x)$ is real, bounded, and never decreasing; (ii) $F(x)$ is of bounded variation in $(-\infty, \infty)$; (iii) $F(x)$ satisfies (ii) and is in addition absolutely continuous. The conditions are partly analogous to those of Hausdorff for a finite interval and are expressed in terms of properties of the Fourier transform of $\mu(\epsilon t)f(t)$, where $\mu(t)$ belongs to a suitably chosen class of kernels. The results extend to functions of several variables, and have applications to the theory of random processes. (Received February 17, 1939.)

171. J. L. Doob: *The law of large numbers for continuous stochastic processes.*

There are certain measurability difficulties which arise when the law of large numbers is to be applied to a one-parameter family of chance variables. A method of approach is given which overcomes these difficulties, and various forms of the law of large numbers are then proved. The following is an example of the results: Let $\{x_t\}$, $(t\geqq 0)$, be the variables of a centered temporally homogeneous differential process (so that if $0\leqq t_1<t_2<\cdots<t_n$, $x_{t_2}-x_{t_1},\cdots,x_{t_n}-x_{t_{n-1}}$ are mutually independent), and suppose that the expectation of x_1-x_0 exists: $E(x_1-x_0)=e$. Then $\lim_{t\to\infty}(x_t-x_0)/t=e$, with probability 1. Certain preliminary results on sums of independent chance variables are needed, of which the following is an example: Let $x_1, x_2, \cdots$ be independent chance variables, and let $\sum_1^\infty x_n=x$ with probability 1. Then if Ex exists, it follows that Ex_n exists for all n, that $Ex=\sum_1^\infty Ex_n$, and that $E(\text{L.U.B. } |x_1+\cdots+x_n|)$ exists. (Received March 16, 1939.)

172. F. G. Dressel: *A class of solutions for the heat equation*. Preliminary report.

The paper shows how to construct for each integer n a solution of the heat equation, whose nth derivative has the properties of the fundamental solution of this equation. The remainder of the paper is concerned with the properties and applications of this set of solutions. (Received March 7, 1939.)

173. D. M. Dribin: *Class field theory of solvable algebraic number fields*.

In the present paper a definition of class field is given which is applicable to nonabelian fields K/k, where k is an algebraic number field. This definition involves a set of prime ideals Π in k, rather than an ideal group. As simple consequences of this definition the analogues of the converse theorem and the uniqueness theorem of the abelian class field theory follow. It is shown that there exist solvable fields K/k which are class fields in this new sense provided that Π be "immersed" in an ideal group H in k. An analogue of the Artin law of reciprocity is also derived. (Received March 10, 1939.)

174. L. A. Dye: *A Cremona transformation associated with the rational normal curve of order n in S_n*.

A projectivity is established between a pencil of quadric primals H and sets of $n-1$ points of the rational normal curve C_n in S_n. The residual intersection of C_n with the prime determined by a general point P and the $n-1$ points of C_n associated with the quadric H passing through P is joined to P by a straight line meeting H again in P'. The points P, P' are conjugate in a transformation of order $6n+3$. The normal curve C_n and three $(n-2)$-dimensional loci of orders 4, 5, and $12n+2$ constitute the fundamental elements of the transformation. The last of these is the locus of parasitic lines. (Received March 1, 1939.)

175. R. A. Favila: *A surface with doubly stratifiable directrices*.

Let S_x be an integral surface of a Fubini system of equations of the form: $x_{uu}=px+\theta_u x_u+\beta x_v$, $x_{vv}=qx+\gamma x_u+\theta_v x_v$, where $\theta=\log\beta\gamma$ and p, q, β, γ are scalar functions of the asymptotic parameters u and v. Consider a pair of Green reciprocal congruences with respect to S_x, and let us denote them by I_1 and I_2. If l_1 and l_2 are any pair of corresponding lines of I_1 and I_2, respectively, then l_1 and l_2 may be taken as the lines joining the pairs of points (Px, Py) and (P_ρ, P_σ), respectively, where $y=x_{uv}$

$-ax_u-bx_v$, $\rho=x_u-bx$, $\sigma=x_v-ax$, and a, b are functions of u and v. Is it possible to determine the surface as well as I_1 and I_2 so that I_1 and I_2 will form a doubly stratifiable couple? The problem has a solution which depends on the integration of an equation of the form $\partial^2\Phi/\partial u\partial v=e^{\Phi}$. The surface is found to have asymptotics belonging to linear complexes, while I_1 and I_2 are directrices of Wilczynski with indeterminate directrix curves. (Received March 15, 1939.)

176. J. M. Feld: *Polygons as fundamental elements in the geometry of plane cubic curves.*

Geometry on a cubic regarded as a locus of a system of polygons was investigated by H. Oppenheimer (Monatshefte für Mathematik and Physik, 1909, p. 141). By defining "collinearity" of polygons, complete sets of polygons, and so on, Oppenheimer showed that theorems known to be true of points on a cubic hold true when certain kinds of polygons are substituted for points. In this paper a new geometry of polygons on a cubic analogous to the geometry of points on a cubic is discussed; the polygons in question are, however, of a type different from those of Oppenheimer. In addition it is shown how any configuration inscribed in a cubic can be used as a basis for the generation of an infinitude of other configurations. (Received March 8, 1939.)

177. A. S. Galbraith and S. E. Warschawski: *On the convergence of Sturm-Liouville series.*

This paper extends in a certain direction some results of M. Krein about the convergence of the derived series of expansions in terms of characteristic solutions of self-adjoint linear differential equations. Let $\{\phi_i(x)\}$ be the characteristic solutions, for values $\{\lambda_i\}$ of the parameter λ_1, of the Sturm-Liouville problem $(py')'+(\lambda r+q)y=0$, $h_1y(a)=h_2y'(a)$, $H_1y(b)=H_2y'(b)$, where p, $r>0$, and p', q, r are continuous. Let n be a positive integer, and if $n\geqq 2$, let $p^{(n-1)}$, $q^{(n-2)}$, $r^{(n-2)}$ be continuous. Then a necessary and sufficient condition that, for an integrable function $f(x)$ with the "Fourier coefficients" $a_i=\int_a^b fr\phi_i dx$, $f^{(n-1)}(x)$ exist and be absolutely continuous in (a, b), and that $\int_a^b[f^{(n)}]^2dx$ exist, is (i) that $\sum_{i=0}^{\infty}a_i^2\lambda_i^n$ converge, (ii) that the functions $G_s(x)$ defined by $G_0\equiv f_j-rG_s=(pG'_{s-1})'+qG_{s-1}$, $(s=1, 2, \cdots, [n/2]-1)$, satisfy the above boundary conditions, and, if n is odd, that $G_{[n/2]}$ vanish at an end point if the boundary conditions require the $\{\phi_i\}$ to do so. As a corollary, it follows that $f^{(m)}(x)\equiv\sum_{i=0}^{\infty}a_i\phi_i^{(m)}(x)$, $(m=0, 1, \cdots, n-1)$, and $\lim_{n\to\infty}\int_a^b[f^{(n)}-\sum_{i=0}^{n}a_i\phi^{(n)}]^2dx=0$. (Received March 11, 1939.)

178. H. H. Goldstine: *Minimum problems in the functional calculus.*

Necessary conditions and sufficient conditions that a functional defined on a region of a Banach space shall be a minimum are determined in terms of the first and second variations of the given functional. Then the more general problem of minimizing a given functional subject to a general side condition is discussed. Under certain hypotheses, necessary conditions for this problem, including a multiplier rule, are obtained; and it is shown that when these conditions are strengthened in the usual manner, sufficient conditions are obtained. The general problem includes both single and multiple integral problems of the calculus of variations as special cases. (Received March 15, 1939.)

179. G. D. Gore: *Transformations for two classes of surfaces.*

This paper develops three transformations for a surface bearing a family of curves in autoconjugacy of type ν, and for a surface bearing a system of curves in conjugacy of type ν. The first of these is an analogue of the transformation of Levy for conjugate nets. The second is somewhat analogous to the classical transformation of Laplace for conjugate nets, but has certain characteristics of the transformation of Levy. The third is an analogue of the fundamental transformation F for conjugate nets due to Eisenhart and Jonas. To determine the degree of prevalence of transformations for the above surfaces, a theorem of permutability is established. Earlier transformations for surfaces of these types, due mainly to B. Segre, have involved the linear osculants of order $\nu-1$ and higher to the curves in question, and have involved analytically the derivatives of correspondingly high orders in the point coordinates. The first two of the above transformations involve only the lines tangent to the curves in question, and are therefore accomplished analytically in terms of first derivatives of the point coordinates. (Received March 16, 1939.)

180. T. N. E. Greville: *Invariance of the admissibility of variates under certain general types of transformations.*

This paper develops a set of transformations on variates or "collectives" in terms of which it is possible to formulate completely a large class of problems for which the transformations given by von Mises and other authors are not adequate. These problems arise in the application of the classical theory of probability to the study of frequency distributions of statistical variates. The consistency of the assumptions of the classical theory is investigated by means of the invariance under the transformations of certain properties of the sequences. In an earlier paper the same method was applied to the simple type of collective in which the label space consists of exactly two elements (this Bulletin, abstract 39-7-202). (Received March 8, 1939.)

181. Marshall Hall: *The position of the radical in an algebra.*

The principal result in this paper is that any linear associative algebra is uniquely decomposable as the direct sum of a semi-simple algebra and an algebra bound to its radical. An algebra A is said to be bound to its radical R if when x belongs to A and $xR=Rx=0$ then x belongs to R. Whereas the semi-simple algebra bears no relation to the radical, the bound algebra is largely determined by the radical. In particular, the order of a bound algebra is limited by the order of its radical. For example, if R is of order one, the order of A is at most three and there are only five such algebras in all. (Received March 8, 1939.)

182. O. G. Harrold: *Invariance of the dimensionality of a compact metric space under certain continuous transformations.*

The continuous transformation T carrying A into B is called $(k, 1)$ provided that each point x in B has at most k inverse points. Let T be a $(k, 1)$ transformation defined on the compact metric space A, C the set of all points in B with fewer than k inverses. It is proved in this note that dim. $B=$ dim. A if, and only if, dim. $\overline{C}\leqq$ dim. A. In particular, if T is exactly $(k, 1)$, dim. $A=$ dim. B. (Received March 17, 1939.)

183. Philip Hartman: *On Dirichlet series involving random coefficients.*

This paper deals with the abscissa of convergence C, of uniform convergence U, of absolute convergence A of "almost all" of the Dirichlet series $\sum \pm a_n/n^s$. It is proved that $1/2 \geqq A - C \geqq U - C \geqq 0$, and that either $1/2 = U - C$ or $A - C = 0$ is possible. Of considerably more interest is the difference $A - U$; it is shown that $A - U = 1/2$ is also possible. The methods employed may be used for a more general class of Dirichlet series. (Received March 7, 1939.)

184. A. E. Heins: *The solution of the discrete equation of heat conduction.*

A solution is derived for the partial difference equation $f(t, y+2c) - 2f(t+h, y+c) + f(t, y) = 0$, where $f(0, y)$, $f(t, 0)$, and $f(t, a)$ are prescribed. The limiting value of the solution is also considered. (Received March 7, 1939.)

185. M. H. Heins: *Extremal problems for functions analytic and single-valued in a doubly connected region.*

It is well known that certain fundamental function-theoretic inequalities such as Julia's principle of the harmonic majorant, the two constant theorem, Lindelöf's principle, the principle of hyperbolic measure, and so on, are "best possible" when the domain of definition G_z for the functions $w(z)$ involved is simply connected. When one considers, however, functions which are analytic and *single-valued* in a multiply connected region, in general these inequalities are no longer the "best possible," and it is a question of interest to determine effectively the exact bounds in the above inequalities and the extremal functions associated with these bounds. By application of the Poincaré uniformization theorem and the Pick-Nevanlinna theory of interpolation, the exact bounds and their associated extremal functions can be determined effectively when the domain of definition G_z is doubly connected. (Received March 17, 1939.)

186. Olaf Helmer: *A theorem of the Picard type.*

Let ord f and exp f denote the order and exponent of convergence of the integral function $f(z)$. The function $f(z)$ will be called exceptional if $\exp f < \operatorname{ord} f$. Let $F(z)$ and $G(z)$ be polynomials, and $A(z)$ and $B(z)$ integral functions with max (ord A, ord B) $<$ max (deg F, deg G), then ord $(Ae^F + Be^G) =$ max (deg F, deg G). On the basis of this lemma it is possible to prove the following theorem: Let $f(z)$ and $g(z)$ be integral functions of distinct finite orders; then there is at most one integral function $A(z)$ with ord $A <$ max (ord f, ord g) for which the function $f(z) + A(z)g(z)$ is exceptional. This is not generally true when $f(z)$ and $g(z)$ are of the same order. We have however the following theorem: If $f(z)$ and $g(z)$ are integral functions of the same finite order ρ, of which at least one is not exceptional, then there are at most two functions $A(z)$ with ord $A < \rho$ for which the function $f(z) + A(z)g(z)$ is exceptional. (Received March 16, 1939.)

187. M. R. Hestenes and H. H. Goldstine: *The condition of Mayer for the problem of Bolza in the calculus of variations.*

Necessary and sufficient conditions that the second variation in the problem of Bolza be positive definite, when the strengthened Clebsch condition holds, have been given by Bliss, Reid, Hestenes, and others. Aside from the condition of Mayer arising from the study of a boundary value problem, no necessary and sufficient conditions for non-negativeness of the second variation have been given. In the present paper, conditions of this type are established, which when suitably strengthened are necessary and sufficient for the second variation to be positive definite. Conditions of Mayer are given for the fixed end point and the one variable end point problems of Bolza, as well as for the separated end point problem and the generalized problem of Bolza recently studied by Hestenes (abstract 45-1-22). (Received March 16, 1939.)

188. Einar Hille: *Contributions to the theory of Hermitian series.*

A study is made of Hermitian series in the complex plane. Convergent asymptotic expansions are derived for the Hermitian functions $h_n(z) = \exp(-z^2/2)H_n(z)$, leading to a simple determination of the strip of convergence. The differential operator $\delta_z = z^2 - d^2/dz^2$ has $h_n(z)$ as characteristic function for the characteristic value $2n+1$. Functions of δ_z are investigated; in particular, entire functions which preserve holomorphism or the property of being an entire function. This leads to a study of singularities of functions defined by Hermitian series and to a gap theorem which is the best of its kind. There exist Hermitian series with a strip of convergence of finite width which represent entire functions. Finally with the given series are associated certain Dirichlet series with exponents $\pm i(2n+1)^{1/2}$ having no other singularities than those of the Hermitian series. (Received March 31, 1939.)

189. P. G. Hoel: *A significance test for minimum rank in factor analysis.*

By treating n variables which are subject to errors of measurement as sums of true and error variables, the true correlation determinant is expressed as a function of n parameters. When all parameters are equal, this determinant reduces to the characteristic polynomial of the original correlation matrix. In factor analysis certain assumptions are made concerning the rank of the true correlation matrix. An inequality is obtained between certain values of the parameters and certain of the characteristic roots which serves as the basis for testing whether or not these assumptions concerning rank are justified. (Received March 3, 1939.)

190. E. V. Huntington: *Stirling's formula with remainder.*

This note gives the Stirling expansion of $n!$, both in the direct and logarithmic forms, through the term involving $1/n^9$, together with a remainder term by which the error involved in stopping the series at any term up to the ninth may be estimated. (Received March 28, 1939.)

191. M. H. Ingraham: *An algorithm for the solution of the unilateral matrix equation.*

An algorithm analogous to that for finding the rational roots of an equation with rational coefficients is given for finding all matrices X with rational elements that satisfy an equation $\sum A_i X^i$ where the A_i are matrices with rational elements. (Received March 16, 1939.)

192. Dunham Jackson: *Orthogonal polynomials on curves of the second degree.*

A theory of polynomials in two real variables orthogonal on the circumference of a circle with respect to an arbitrary weight function is at the same time a theory of orthogonal polynomials with an arbitrary weight function on an ellipse. But in connection with properties relating to a specific weight function, the transition from circle to ellipse gives rise to problems that are not trivial. By means of a theorem of Peebles on the boundedness of orthogonal trigonometric sums (for the case of polynomials in one real variable see Proceedings of the National Academy of Sciences, vol. 25 (1939), pp. 97–104) it is shown that the ordinary convergence theory of Fourier series carries over to series of orthogonal polynomials on an ellipse with unit weight function and arc length as variable of integration. A corresponding discussion is given for an elliptic arc and for arcs on a hyperbola. (Received March 16, 1939.)

193. Marie M. Johnson: *An extension of a covariant differentiation process.*

In this paper a covariant differentiation process is developed for functions which contain coordinate variables and derivatives of these variables with respect to a parameter. A special case is the process given by H. V. Craig (see this Bulletin, vol. 37 (1931), pp. 731–734). (Received March 16, 1939.)

194. Mark Kac and E. R. van Kampen: *Circular equidistribution and statistical independence.*

In order to obtain a satisfactory basis for the treatment of Buffon's needle problem, the circular equidistribution of an angular variable and a related type of discontinuous distributions are obtained as the only possible solutions of a problem concerning statistical independence. The problem depends on the determination of those transformations $\bar{\phi}_1=\phi_1+f(\phi_2)$, $\bar{\phi}_2=\phi_1$ of a (ϕ_1, ϕ_2)-torus into itself which conserve a given product measure on the torus. (Received January 16, 1939.)

195. Edward Kasner and J. J. DeCicco: *A characterization of the Moebius group of circular transformations.*

The Moebius group of circular transformations is ordinarily defined as the group of point transformations of the plane for which the entire family of ∞^3 circles is invariant. In this paper, the authors wish to find out if it is not sufficient for this definition to require that only some circles shall become circles. If a simple family (∞^2) of circles is understood to be any doubly infinite family possessing the property that there is one and only one circle of the family containing a given lineal element of the plane, then the main result may be stated as follows: If three simple families ($3\infty^2$) of circles become circles under a point transformation, then the same is true for all circles, and the point transformation is, therefore, a circular transformation of the Moebius group. For any other point transformation, there are at most two simple families ($2\infty^2$) of circles which become circles. The above result gives a minimal characterization of the Moebius group. (Received February 20, 1939.)

196. J. L. Kelley: *A note on the arc-wise connectivity theorem.*

Suppose M is the image of the unit interval I under a continuous transformation T. A simple proof for the existence of an arc between two chosen points x, y of M is obtained by finding a perfect set in I which maps under T into an arc xy. This proof depends only on the properties of compact sets, on the transformation T, and on the fact that the monotone transform of an arc is an arc. (Received March 9, 1939.)

197. R. B. Kershner: *Ergodic curves.*

Let M be an arbitrary bounded plane point set, and let $\epsilon>0$ be fixed. Then a continuous rectifiable curve C is called an ϵ-ergodic curve for M if it is a curve of minimum length having the property that every point of M is at a distance less than or equal to ϵ from C. This paper is devoted to the investigation of such curves (known to exist for arbitrary M and $\epsilon>0$). The main results are that C is simple and has a well defined tangent up to a countable number of corners. (Received March 16, 1939.)

198. Fulton Koehler: *Orthogonal polynomials on certain algebraic curves.*

An investigation is made of systems of orthogonal polynomials in two real variables which correspond to certain contours in the plane of the variables and certain weight functions. The contours considered are specifically a square (see also abstract 43-11-402), a pair of concentric circles, and a pair of intersecting line segments. By relating these systems of polynomials to systems of polynomials in one real variable, sufficient conditions are derived for the boundedness of the normalized polynomials and then for the convergence of series of the polynomials to an arbitrary function. Both uniform convergence and point by point convergence are considered. (Received March 16, 1939.)

199. C. F. Kossack: *The existence of collectives in abstract space.*

This paper considers the existence of admissible sequences or *Kollektivs* in abstract space. Such sequences arise in the study of the foundations of the theory of probability. Beginning with the admissibility conditions given by A. H. Copeland as an interpretation of the von Mises requirements, existence is established by showing that almost every (in the sense of Lebesgue) sequence in abstract space satisfies these conditions. This is first shown with respect to a denumerable field of sets in the space and then for an extended field which is, in general, nondenumerable. The Fubini theorem is then used to prove the existence of sequences satisfying the independence condition for almost every selection of a nondenumerable set. Finally the concept of admissibility is generalized so that the number of conditions has the power of the continuum, and it is shown that a nondenumerable set of sequences satisfies almost all of these generalized conditions. Many of the results previously established by Copeland, Tornier, and Reichenbach are obtained at once from the theorems of this paper merely by specializing the space. (Received March 22, 1939.)

200. C. G. Latimer: *The complete solution of certain Diophantine equations.*

Let f be a classic properly primitive ternary quadratic form, with integral coefficients, whose determinant D is not divisible by a square greater than 1. Let F be the adjoint of f, and let u, v, w be integers, not all zero, such that $f(u, v, w)=0$. There is a one-to-one correspondence between the solutions of $f=0$ and of $F=0$. In this paper the author obtains all solutions, in integers, of $f=0$ and of $F=0$ in terms of u, v, w and certain parameters. The fact that F is the norm of the general element of trace zero in a ring of integral elements in a generalized quaternion algebra is employed. (Received March 17, 1939.)

201. Yuan Lay: *On the imbedding of the skew part into an associative algebra.*

By an algebra is meant here a bilinear vector function. It is studied by splitting it up into symmetric and skew parts. The condition of associativity is translated into relations between these two parts from which, of course, the Jacobi relation for the skew part follows. Two problems arise: what conditions in addition to the Jacobi condition should be imposed on a skew function in order that an associative algebra may exist of which the function is a skew part; and if these conditions are satisfied, how to find the symmetric part. These problems are discussed for certain types of algebras, in particular, for centralized algebras, that is, algebras in which the skew part vanishes only when the arguments are linearly dependent modulo the centrum. A general theorem is proved covering the number of dimensions of the centrum in a centralized algebra, and the problem is solved completely for the cases of three- and four-dimensional space. Some general theorems are given also for certain other cases. (Received March 17, 1939.)

202. A. T. Lonseth: *The problem of Plateau in hyperbolic space.*

Let the hyperbolic geometry of three dimensions be the set $z>0$ of ordinary euclidean space (x, y, z), with Poincaré's metric $ds^2=(dx^2+dy^2+dz^2)/z^2$. Consider surfaces S represented parametrically by equations of the type $x=x(u, v)$, $y=y(u, v)$, $z=z(u, v)$, the coordinate functions being continuous in $u^2+v^2\leqq 1$. The area integral is defined, as usual, in terms of the metric, for surfaces whose coordinate functions have continuous first derivatives in $u^2+v^2<1$. It is shown that a Jordan curve, if it bounds a surface of finite (hyperbolic) area, must bound a hyperbolic minimal surface, that is, a surface whose coordinate functions have continuous second derivatives in $u^2+v^2<1$, and satisfy therein the Euler equations which arise from minimizing the area integral. (Received March 18, 1939.)

203. A. N. Lowan: *On some problems in the diffraction of heat.*

The boundary plane $x=0$ of the semi-infinite solid $x>0$, initially at 0°C, is impervious to heat except for an aperture of given shape. If the semi-infinite solid is subjected to an influx of heat at the rate $Q(P, t)$ per unit area, what is the subsequent temperature distribution in the solid? This problem is the thermal equivalent of the problem of diffraction of light. Rigorous solutions are obtained for the following cases: (A) The aperture is an infinite slit between two parallel lines. (B) The aperture is a circle. (C) The aperture is of finite extent and of any arbitrary shape. (D) The aperture consists of the half-plane $x=0$, $y>0$. The Laplace transforms of the temperature function are obtained with the aid of known Fourier or Bessel-Fourier integral representations of functions of one or two variables. The transition from the Laplace transforms to the temperature function is accomplished by means of standard methods in the operational calculus. (Received March 23, 1939.)

204. Saunders MacLane: *The universality of formal power series fields.*

Recently Gleyzal (Proceedings of the National Academy of Sciences, vol. 23 (1937), pp. 581–587) has constructed certain transfinite real numbers and has conjectured that certain corresponding modular fields will be universal; that is, will contain subfields isomorphic to any given field of the same characteristic and the same cardinal number. It is observed first that the transfinite real numbers are simply the formal power series of Hahn. By using Krull's generalized valuations, the corresponding fields are proved maximal, hence algebraically closed, and thus Gleyzal's conjecture is established. More generally, it is shown that any formal power series field is universal if its coefficient field is algebraically closed and if its group Γ of exponents always contains γ/n when it contains γ. (Received February 24, 1939.)

205. P. T. Maker: *The ergodic theorem for a sequence of functions.*

Let Ω be a space with measure m, $(m(\Omega)<\infty)$, $T(P)$ a measure-preserving transformation of Ω, and $\{f_n(P)\}$ a dominated, convergent sequence of complex-valued functions in $L_1(\Omega)$. For any set of positive integers n_{ij}, $(i, j=1, 2, \cdots; i\leqq j)$, such that $\lim_{i,j\to\infty} n_{ij}=\infty$, $\lim_{j\to\infty} 1/j\sum_{i=0}^{j-1} f_{n_{ij}}(T^iP)$ exists and equals the time mean of $\lim_{n\to\infty} f_n(P)$ except for points P of measure zero independent of the sequence $\{n_{ij}\}$. In the case of the flow, if $f_\lambda(P)$, $(a\leqq\lambda\leqq b)$, is a continuous family of dominated functions of $L_1(\Omega)$, and $\lambda(t, T)$, $(0\leqq t\leqq T\leqq\infty)$, is continuous with range in (a, b), then $\lim_{T\to\infty} 1/T \cdot\int_0^T f_{\lambda(t,T)}(P_t)dt$ exists for almost all P. (Received March 10, 1939.)

206. H. W. March: *Infinite plane strip of orthotropic material under a concentrated load.*

The deflection of such a strip satisfies the differential equation $D_1\partial^4w/\partial x^4+2K\partial^4w/\partial x^2\partial y^2+D_2\partial^4w/\partial y^4=0$, except at the point of loading (M. T. Huber, Bauingenieur, 1923–1926). After the change of variable $\eta=\epsilon y$ where $\epsilon=(D_1/D_2)^{1/4}$, the differential equation becomes $\partial^4w/\partial x^4+2\kappa\partial^4w/\partial x^2\partial y^2+\partial^4w/\partial y^4=0$ where $\kappa=K/(D_1D_2)^{1/2}$. The transformed differential equation can be used to advantage in treating numerous problems associated with flat plates of orthotropic material. For a strip with simply supported edges, $x=0$ and $x=a$, the deflection is readily expressed in the form $w=\sum Y_n(\eta)\sin n\pi x/a$. When $\kappa<1$, as it is for plywood, the bending moments m_x and m_y are found to be linear combinations of two functions which can be expressed in closed form as the real and imaginary components, respectively, of a function of a complex variable. This form is essentially identical with that given for a function ϕ by A. Nadai (*Elastische Platten*) for a strip of isotropic material, except that a real variable is replaced by a complex variable. (Received March 10, 1939.)

207. W. C. McDaniel: *The deflection of an orthotropic plate.* Preliminary report.

The differential equation for the deflection of an orthotropic plate can be reduced by a linear transformation (H. W. March, abstract 45-5-206) to the form $\partial^4w/\partial x^4+2\kappa\partial^4w/\partial x^2\partial y^2+\partial^4w/\partial y^4=P(x, y)/D_1$. When $\kappa>1$ and $\kappa<1$, the general solutions may be expressed as the sum of arbitrary functions of each of two complex variables. When $\kappa=1$ the equation is the biharmonic equation, and the two complex variables of the other cases reduce to a single complex variable. Series solutions are found for the infinite and semi-infinite strips and the rectangle, with simply supported edges, and having either a concentrated load or a load distributed over a small rectangular area. The solutions for $\kappa=1$ and $\kappa<1$ are found from those for $\kappa>1$ by noting that a quantity $\sigma=(\kappa-1/2)^{1/2}$ becomes zero when $\kappa=1$ and imaginary when $\kappa<1$. The bending moments are given as linear combinations of two functions ϕ and ϕ', which can be expressed in closed form for the infinite and semi-infinite strips with concentrated load. When $\kappa<1$, ϕ and ϕ' become conjugate complex functions, and the bending moments may be expressed in terms of their real and imaginary parts. (Received March 13, 1939.)

208. E. J. McShane: *On multipliers for Lagrange problems.*

Let $y=y(x)$, $(x_1 \leqq x \leqq x_2)$, minimize $\int f(x, y, y')dx$ in the class of curves satisfying differential equations $\phi_\alpha(x, y, y')=0$, $(\alpha=1, \cdots, m)$, and satisfying certain end conditions. It is well known that it is possible to choose a constant λ_0 and functions $\lambda_\alpha(x)$ in such a way that for the function $F(x, y, y', \lambda) \equiv \lambda_0 f + \lambda_\alpha \phi_\alpha$ the Euler equations and transversality conditions hold. In the present paper it is shown (without reference to normality) that λ_0 and the $\lambda_\alpha(x)$ can be so chosen that the Euler equations, transversality conditions, Weierstrass condition, and Clebsch condition are all satisfied. (Received March 10, 1939.)

209. E. J. McShane: *The condition of Legendre for double integral problems of the calculus of variations.*

Let $x=x(u, v)$ (that is, $x^i=x^i(u, v)$, $i=1, \cdots, n$), minimize the integral $\iint f(u, v, x, p, q)dudv$, where p, q are the partial derivatives of x with respect to u, v respectively. The analogue of the Legendre condition $\xi^\alpha f_{p^\alpha p^\beta}\xi^\beta + 2\xi^\alpha f_{p^\alpha q^\beta}\eta^\beta + \eta^\alpha f_{q^\alpha q^\beta}\eta^\beta \geqq 0$ is known to hold if ξ and η are linearly dependent. Otherwise it may fail. It has been shown by Albert and by Reid that if $n=2$ the condition (1) $\xi^\alpha F_{p^\alpha p^\beta}\xi^\beta + 2\xi^\alpha F_{p^\alpha q^\beta}\eta^\beta + \eta^\alpha F_{q^\alpha q^\beta}\eta^\beta \geqq 0$ holds for all ξ and η, the function F being the sum of f and a multiple of the jacobian of x^1, x^2 with respect to u, v. Here it is shown that if $n=3$ inequality (1) holds, F being the sum of f and a linear combination of the three jacobians of the x^i with respect to u, v. (Received March 10, 1939.)

210. A. D. Michal and A. B. Mewborn: *General projective differential geometry of paths.*

In this paper the authors study the invariant properties of the differential equation of paths in a Hausdorff topological space with a Banach coordinate space B, without postulated inner product, under projective changes of connection (see A. D. Michal, Proceedings of the National Academy of Sciences, vol. 23, p. 547, equation 4). A new element of structure is introduced by defining a geometric object with components which we call the *gauge form* Γ^0. This, in turn, is used to define distinguished parameters, called *projective normal parameters*, and a projective connection Π with components in a composite Banach coordinate space B_1 of couples (x, x^0), where x is in B and x^0, called the *gauge variable*, is real. This connection Π is a linear connection under a certain restricted set of coordinate transformations in B_1. By a proper choice of the coordinate space B, several new infinite dimensional cases are obtained as well as important finite dimensional cases from a somewhat different viewpoint. (Received March 17, 1939.)

211. Harlan C. Miller: *A separation theorem.*

The following result has been obtained: If M is a compact continuous curve, A and B are two points of M, and G is an uncountable collection of mutually exclusive point sets such that each set of G consists of two points and irreducibly separates A from B in M, then there exists an uncountable collection H of mutually exclusive sets such that each set of H contains two points and irreducibly separates A from B in M and no set of H separates two points of another set of H from each other in M, and M contains two arcs XPY and XQY having only X and Y in common such that one point of each set of H belongs to XPY and the other to XQY, and if K denotes the sum of all the sets of H, then $K\cdot XPY$ and $K\cdot XQY$ are closed. (Received March 20, 1939.)

212. Harlan C. Miller: *On the characterization of a certain type of continuous curve.*

The following results have been obtained: If every two points of a locally compact continuum M are irreducibly separated from each other in M by some pair of points, then for every two points A and B of M there exist uncountably many mutually exclusive pairs of points each irreducibly separating A from B in M. A necessary and sufficient condition that every two points of a locally compact continuum M be irreducibly separated from each other in M by some pair of points is that M be a continuous curve which contains no two simple closed curves having more than one point in common and which also has the property that if A and B are two of its points which are separated from each other in M by some point, the set of all points which separate A from B in M is totally disconnected. (Received March 20, 1939.)

213. M. G. Moore: *On fundamental systems of solutions for linear difference equations.*

Consider the difference equation $\sum_{\mu=1} c_\mu f(x+\alpha_\mu)=0$, where $N\geqq 2$ and α_μ and c_μ are arbitrary complex constants such that $c_\mu\neq 0$, $\alpha_\mu\neq\alpha_\nu$ for $\mu\neq\nu$. If σ is a zero of the generating function $h(t)=\sum_{\mu=1}^{N} c_\mu \exp(\alpha_\mu t)$, the difference equation is satisfied by $\exp(\sigma x)$, and, if ω is the multiplicity of σ, is also satisfied by $x^q \exp(\sigma x)$, $(q=1, 2, \cdots, \omega-1)$. These solutions are called fundamental solutions. On certain sets of curves in the complex plane, it is shown that all Lebesgue integrable solutions of the difference equation may be approximated as closely as desired by linear combinations of fundamental solutions. All solutions analytic in the neighborhoods of the points α_μ may be approximated uniformly in certain regions of the plane. The principal tool used is the property of biorthogonality discovered by the author (see abstract 44-1-41). (Received March 17, 1939.)

214. C. B. Morrey: *Multiple Denjoy-Perron integrals.*

In this paper, the special and general Denjoy-Perron integrals are generalized to n dimensions. The Perron form of the definition is used first as this exhibits most clearly the naturalness of the generalization. All the usual theorems for the Denjoy integrals are proved and it is shown that the multiple integrals reduce to the ordinary Denjoy integrals in one dimension. Fubini's theorem is proved, and it is shown that if $f(x)$ and $\phi(y)$ are both integrable D (or D^*) in m and n variables, respectively, then $f(x)\cdot g(y)$ is integrable D (or D^*) in $m+n$ variables. The integral includes the Lebesgue integral. Equivalent descriptive and constructive definitions are developed. Neither of the above integrals is equivalent to any previously defined multiple integral. An example is given of a continuous function $f(x, y)$ which possesses a partial derivative $\partial f/\partial x$ at each point which, however, is not two dimensionally integrable. (Received March 17, 1939.)

215. Marston Morse and C. B. Tompkins: *The existence of minimal surfaces of general critical type.* II.

The authors begin with a proof that the area of a harmonic surface of the type of a disc bounded by a simple rectifiable curve varies continuously with the curve, convergence in the space of the curves being convergence in variation. They then show how the hypotheses on the curve g, in the paper I with the same title in the Annals of Mathematics, can be weakened by the method of approximation, using the results of I. In the special case of the theorem that two disconnected minimizing critical sets imply a non-minimizing minimal surface it is sufficient to assume that g is rectifiable and that the ratio of chord to corresponding minimum arc length on g is bounded from zero. More generally Theorem 7.4 of I can be established if g is rectifiable and can be approximated by curves g_n admissible in I, and such that the ratio of the lengths of corresponding chords on g_n and on g tends uniformly to 1 as n becomes infinite and the length of the chord on g tends to zero. These conditions are considerably weaker than the condition that the curve be of class C'. (Received March 11, 1939.)

216. H. T. Muhly: *Valuations and infinitely near algebraic loci.* Preliminary report.

In the theory of continuous systems of surfaces in S_3, base conditions imposed by infinitely near base curves play an important role. These conditions can be described by the valuation ideals (v-ideals) in the ring $K(z)[x, y]=\Delta[x, y]$, belonging to zero dimensional valuations B of $\Sigma=\Delta(x, y)$. The author investigates these valuations of Σ for an arbitrary coefficient field Δ. Use is made of the results obtained by O. Zariski in the case where Δ is algebraically closed. It is proved that on extending Δ to its algebraic closure $\bar{\Delta}$, any $B(\Sigma)$ decomposes into a set $\bar{B}_1, \cdots, \bar{B}_n$ of valuations of $\bar{\Delta}(x, y)$ which are conjugate over Δ. If $\{Q_\alpha\}$ is the B-sequence of simple v-ideals in $\Delta[x, y]$ and $\{\mathfrak{Q}_{\alpha j}\}$ is the B_j-sequence in $\bar{\Delta}[x, y]=\mathfrak{D}$, the main lemma states that $\mathfrak{D}Q_\alpha=\mathfrak{Q}_{\alpha j_1}\cdot\mathfrak{Q}_{\alpha j_2}\cdot \cdots \cdot\mathfrak{Q}_{\alpha j_e}$ where $\mathfrak{Q}_{\alpha j_1}, \cdots, \mathfrak{Q}_{\alpha j_e}$ are the distinct ideals among $\mathfrak{Q}_{\alpha 1}, \cdots, \mathfrak{Q}_{\alpha n}$. This lemma enables one to relate the properties of B to those of $\bar{B}_j$. The lemma yields first the unique factorization theorem for products of simple ideals (complete ideals) in $\Delta[x, y]$, and second that the Q_β, $(\beta<\alpha)$, are uniquely determined by Q_α, that is, the Q_β are independent of the valuation to which Q_α belongs. These results serve as a starting point for a theory of infinitely near curves in S_3 or, more generally, infinitely near V_{n-2} in S_n. (Received March 8, 1939.)

217. I. M. Niven: *On a certain partition function.*

Consider the function $F(x)=f(x)f(x^6)f^{-1}(x^2)f^{-1}(x^3)$, where $f(x)=\prod_{j=1}(1-x^j)^{-1}$. It is a power series in x, convergent within the unit circle, the coefficient a_m of x^m being the number of partitions of an integer m, the summands being of the form $6n\pm 1$. Schur (Sitzungsberichte der Akademie der Wissenschaften, 1926, pp. 488–495) has show that the number a_m of partitions of m with summands of the form $6n\pm 1$ equals the number of partitions of m such that the difference between any two summands is at least three, and at least six in case both summands are divisible by three. By using the Rademacher modification of the Hardy-Ramanujan method, and Kloosterman sums of roots of unity, the values of the a_m are obtained in the form of convergent series involving Bessel functions. (Received February 24, 1939.)

218. Rufus Oldenburger: *Completely reducible forms.*

For any field F the minimal number of a completely reducible form (that is, a form which splits into linear factors) of degree n is not greater than 2^{n-1}. From this, the theory of nonsingular cubic forms introduced in a paper by the author in the Transactions of this Society, and minimal representations of such forms, the author solves the problem of complete reducibility of cubic forms for any field F with characteristic different from 2, 3. (Received March 9, 1939.)

219. G. H. Peebles: *The boundedness of certain systems of orthogonal functions.*

The purpose of this paper is to reformulate and extend certain results on the boundedness of orthonormal polynomials obtained by the writer in another paper (Proceedings of the National Academy of Sciences, vol. 25 (1939), pp. 97–104). The method, which does not depend on asymptotic formulas, is sufficiently simple and general to be of interest even in the case of Jacobi polynomials and other orthogonal polynomials for which the results are known. As applied to orthogonal trigonometric sums, it gives new results, which are then susceptible of further application in the corresponding convergence theory. (Received March 16, 1939.)

220. R. S. Phillips: *Integration in a convex linear topological space.*

This paper is essentially an extension of Birkhoff's generalization of the Lebesgue integral to a convex linear topological space. The chief difference lies in the replacement of unconditionally convergent sequences of sets by unconditionally summable sequences of sets for which it is required that the terms remain within a given neighborhood for sufficiently large terms and this independent of order. The integral is an absolutely continuous and completely additive set function depending linearly on the argument. Convergence theorems, the integrability of the product of an integrable function and a bounded summable function, and differentiation are discussed. Finally it is shown that the recently defined integrals of Birkhoff, Dunford, Gelfand, and Pettis are special instances of this integral. (Received March 16, 1939.)

221. R. S. Phillips: *On additive set functions.*

This paper is concerned with additive functions on a sigma-family of sets to real numbers or to a Banach space. The first section establishes for both of these cases a decomposition theorem in which the function is expressed as a denumerable sum. To each of the component functions corresponds a different cardinal number such that it is non-trivially defined on a set of this power and vanishes for all sets of lower power. It is shown that each additive family of sets which constitutes a separable space when metrized by a measure function is in a sense equivalent to the Lebesgue measurable sets on (0, 1). Then a non-separable set space is defined in a nondenumerable torus space. These results are applied to integration in vector spaces. An example is given of a function Pettis integrable but not Birkhoff integrable and another which is Dunford integrable but not Pettis integrable. Finally differentiation of functions defined to a Banach space is considered. (Received March 16, 1939.)

222. W. T. Puckett: *The images of 2-dimensional surfaces under 0-regular transformations.*

The transformation $T(A)=B$ is said to be 0-regular (see A. D. Wallace, abstract 44-3-161) if for every sequence of points $\{y_i\}$ converging to y in B the sequence $\{T^{-1}(y_i)\}$ converges 0-regularly to $T^{-1}(y)$. If A is a 2-dimensional pseudo-manifold and T is monotone 0-regular, either T is a homeomorphism or B is an arc or a simple closed curve. Moreover, if T is any 0-regular transformation, either B is a pseudo-manifold, an arc, or a simple closed curve. (Received March 9, 1939.)

223. Tibor Radó: *On cyclic elements.*

The fundamental concept of the cyclic element theory, as developed by Kuratowski and G. T. Whyburn (Fundamenta Mathematicae, vol. 16, pp. 305–331) is the concept of pairs of conjugate points of a Peano space S. Two points a, b of S are *conjugate* if for every choice of a point p, different from a and b, the points a and b are located in the same component of $S-p$. Write $a\sim b$ to express this fact. The binary relation $a\sim b$ is clearly determinative, symmetric, and reflexive, but generally it is *not* transitive. That is, $a\sim b\sim c$ does not imply, generally, that $a\sim c$. The present paper is based on the remark, apparently unnoticed so far, that if there exist two distinct points b_1, b_2 such that $a\sim b_1\sim c$, $a\sim b_2\sim c$, then it *does* follow that $a\sim c$. The purpose of the paper is to present a treatment of the cyclic element theory based on this *weak transitivity* property of the binary relation $a\sim b$. (Received March 6, 1939.)

224. Tibor Radó and P. V. Reichelderfer: *Some properties of continuous transformations in the plane.*

Given, on a fundamental rectangle in the w-plane, a continuous transformation carrying this rectangle into a flat surface in the z-plane, Radó (Duke Mathematical Journal, vol. 4 (1938), pp. 189–221) defines a multiplicity for each point of the z-plane, the kernel or essential multiplicity for the surface, this multiplicity being defined in terms of transformations close to the given one. In the present paper an intrinsic characterization of this multiplicity is developed. Given any point in the z-plane, this multiplicity is defined with reference to the image under the given transformation of every rectangle in the fundamental rectangle; thus a function of rectangles arises. Conditions for the existence of a completely additive extension of this function of rectangles, together with several closely related functions of rectangles, to a class including all Borel sets in the fundamental rectangle are derived, and the extent to which such extensions are unique is discussed. (Received March 10, 1939.)

225. Tibor Radó and J. W. T. Youngs: *On upper semicontinuous collections.*

During the course of the years 1924 to 1926, R. L. Moore, P. Alexandroff, and B. de Kerékjártó considered certain collections of sets which were called upper semicontinuous by Moore. Apart from the fundamental ideas, however, their work does not overlap: the space, the terminology, and the application are different in each case. The obvious importance of their work, together with the wide range of the applications, makes it interesting to observe that their definitions, though worded in entirely different fashions, become equivalent if applied in a certain conveniently chosen abstract space. This equivalence is rather obvious, but the purpose of this paper is to establish it in a way which is not suggested immediately by their work. Indeed, it will be shown that the definitions proposed represent in a sense the unique answer to a certain simple and fundamental question, and hence are necessarily equivalent. (Received March 9, 1939.)

226. E. D. Rainville: *Linear operational equations and the Laplace transformation.*

Some of the results of a previous paper (abstract 44-11-455) are applied to the solution of operational equations in σ (essentially the Laplace integral transformation) with coefficients which are linear differential operators. The methods employed are considered in the light of a more general theory of certain operators of any class and of finite order. Some of the results are carried over to the less restricted operators. Others, particularly because of the existence of a fundamental system of two linear differential operators invariant under σ, are peculiar to the operator σ. (Received March 14, 1939.)

227. E. D. Rainville: *Linear partial differential operators and the Laplace transformation.*

In this paper some of the results of a previous paper (abstract 44-11-455) are extended. The study of iterated σ transformations is brought to the stage where linear partial operational equations in σ may be solved. The methods used center around an expansion theorem for linear partial differential operators in direct analogy to one of the methods used in the case of one independent variable. (Received March 14, 1939.)

228. E. D. Rainville: *Relations among certain operators of class three.*

The paper considers linear operators which are defined as acting on ordinary linear differential operators. Relations are obtained between certain of these operators including, in particular, the σ and ψ of a previous paper (abstract 44-11-455) and the classical adjoint operator. (Received March 14, 1939.)

229. W. C. Randels: *On the absolute summability of Fourier series. II.*

Bosanquet (Proceedings of the London Mathematical Society, vol. 41, p. 517) gives conditions for absolute summability $C(\alpha)$ which imply that $C(\alpha)$ is a local property for $\alpha>1$, but the question for $\alpha\leqq 1$ is not answered. In this paper it is shown by an example that it is not a local property for $\alpha=1$. (Received March 3, 1939.)

230. W. T. Reid: *A note on the Du Bois-Reymond equations in the calculus of variations.*

McShane (Mathematische Annalen, vol. 109 (1934), pp. 746–755) and Tonelli (Annali della R. Scuola Normale Superiore di Pisa, vol. 3 (1934), pp. 213–237; pp. 214–219, in particular) have given conditions on the integrand function of a simple integral problem of the calculus of variations which insure that every absolutely continuous minimizing arc satisfies the Du Bois-Reymond form of the Euler equations. The principal purpose of the present note is to give a very simple proof of these results of McShane and Tonelli. Basically, the proof here given is intimately related to the proof of the fundamental lemma as given by Bliss (*Calculus of Variations*, Open Court, 1925, pp. 20–21). (Received March 17, 1939.)

231. J. H. Roberts: *A note on topological mappings.*

Examples have been given of acyclic curves having one of the following properties: (1) If f is a topological mapping of M into a subset of M, then $f(M)=M$. (2) If f is a topological mapping of M such that $f(M)=M$, then for each $p \varepsilon M, f(p)=p$. In the present note an acyclic continuous curve M is defined which has *both* of these properties; that is, the only topological mapping of M into a subset (proper or improper) of M is the identity. (Received March 8, 1939.)

232. M. S. Robertson: *The variation of the sign of v for an analytic function u+iv.*

Let $f(z)=u+iv\neq 0$ be regular within the unit circle. If z traces out the circle $|z|=r<1$ once, starting at any point on the circle at which $f(z)$ does not vanish, during the circuit either v is of constant sign or it changes sign an even number of times. Let p be a fixed nonnegative integer, and suppose that, on each circle $|z|=r<1$, v does not change sign more than $2p$ times. The author obtains a representation of the form $f(z)=\phi^p(z)\cdot F(z)$, $\phi(0)=0$, $\phi'(0)=1$, where $\phi(z)$ is star-like with respect to the unit circle, and where, for $|z|<1$, $F(z)$ is the limit of functions $F_n(z)$ which are regular for $0<|z|\leqq 1$ and which have the property that on $|z|=1$ the real part of $F_n(z)$ is nonnegative. At the origin each $F_n(z)$, when not regular there, has a pole of order not exceeding p. From this representation a study is made of the rate of growth of $f(z)$ as $|z|\rightarrow 1$. Inequalities are obtained for the coefficients of the power series of $f(z)$. (Received March 9, 1939.)

233. R. M. Robinson: *On numerical bounds in Schottky's theorem.*

If the function $f(z)$ is regular and different from ± 1 for $|z|<1$, and if $f(0)=a$, then, according to Schottky's theorem, $|f(z)|$ is bounded for $|z|\leqq r$ by a number depending only on a and r. Let $K(a, r)$ be the smallest such number. It is shown that $8K(a, r)-10<(8|a|+10)^{(1+r)/(1-r)}$, and that $8K(a, r)+10>(8|a|-10)^{(1+r)/(1-r)}$ provided $8|a|-10>0$. Therefore one has obtained (for the first time) numerical bounds for $K(a, r)$, which are close enough so that the asymptotic formula $8K(a, r)\sim(8|a|)^{(1+r)/(1-r)}$, for $a\rightarrow\infty$ and fixed r, will follow from them. The proof uses the geometrical properties of the modular function, defined as the function which maps a circular-arc triangle, with zero angles, on a half-plane. (Received March 10, 1939.)

234. O. F. G. Schilling: *Units in p-adic algebras.*

Let A be a simple algebra over the field of p-adic numbers. The author investigates the structure of unit groups in orders of A. It turns out that every unit group has a finite base so that each unit can be represented as an infinite convergent product. Convergence is to be understood in terms of the unique pseudo-valuation of the given algebra. (Received March 2, 1939.)

235. H. M. Schwartz: *A study of a certain class of continued fractions.*

This paper is an extension of the author's abstract 44-5-262. The continued fraction $k_1/(z-c_1)+k_2/(z-c_2)+\cdots$ (k_i and c_i complex, $k_i\neq 0$, z a complex variable) is studied under certain restrictions on the coefficients which lead to a number of cases in which the numerators and denominators of the approximants, when multiplied by suitable factors, converge uniformly in certain regions of the z-plane. The nature of the limit functions is discussed. As an application, the continued fractions representing the quotient $J_\nu(z)/J_{\nu-1}(z)$ ($J\nu(z)$—Bessel's function) and the related system of Lommel polynomials are studied. The latter are shown to form an orthogonal set for all values of the parameter ν, real or complex, under an extended definition of orthogonality. Application is also made to a certain special case in the moments problem of Stieltjes and Hamburger. (Received March 9, 1939.)

236. G. E. Schweigert: *A note on the limit of orbits.*

This note sets forth a proposition based on a paper published recently (Duke Mathematical Journal, vol. 4 (1938), pp. 719–724) by D. W. Hall and the author. Let $T(M)=M$ be a pointwise periodic transformation of a compact metric space onto itself. Let L be the limit set for a convergent sequence of orbits each of which consists of a point x together with its images. Under these conditions, it is found that L contains only a finite number of components which permute under the transformation. An example is given to show the effect of this theorem in the case of a well known dendrite. (Received March 15, 1939.)

237. W. T. Scott and H. S. Wall: *Power series in which each exponent is at least twice the preceding.*

In the corresponding continued fraction $1+a_1x^{\alpha_1}/1+a_2x^{\alpha_2}/1+a_3x^{\alpha_3}/1+\cdots$ for the power series $1+\sum_{i=0}^{\infty}c_{i+1}x^{\lambda_i}$, $(c_i\neq 0,\ \lambda_i\geqq 2\lambda_{i-1},\ i=0, 1, 2, 3, \cdots)$, the a_n are independent of the λ_n and the α_n are independent of the c_n, and simple formulas exist for the a_n, α_n in terms of the power series and for the c_n, λ_n in terms of the continued fraction. For example, $\lambda_n=\alpha_1+\alpha_3+\alpha_5+\cdots+\alpha_k$, $(k=2^{n+1}-1,\ n=0, 1, 2, 3, \cdots)$. The corresponding continued fractions for the derivative, and for the reciprocal, of the power series are easily obtained from that for the power series itself. If $1+\sum_{i=0}^{\infty}d_{i+1}x^{\mu_i}$ is another series of the same type, then the corresponding continued fraction for the "composite" series $1+\sum_{i=0}^{\infty}c_{i+1}d_{i+1}x^{\lambda_i+\mu_i}$ is obtained from the separate corresponding continued fractions by the same sort of composition except for possible changes of signs in the partial numerators. The convergents of the continued fraction are Padé approximants for the series and fill all the "fields" (i, j) of the Padé table for which $j\geqq i$. (Received March 13, 1939.)

238. D. M. Seward: *Note on an integral equation.*

The author proves the following theorem. Let k be a positive constant. Let $F(x)$ $x=x_1+ix_2$, be analytic and bounded in the domain D: $|x_2|<k(1-x_1)$, $|x_2|<k(1+x_1)$ Let $K_1(x, y)$ be analytic in the domain D_1: x in D, y in D, $y_1<x_1-|x_2-y_2|/k$. Let $K_2(x, y)$ be analytic in the domain D_2: x in D, y in D, $y_1>x_1+|x_2-y_2|/k$. Let both K_1 and K_2 satisfy $|K_i(x, y)|<M|x-y|^{-\alpha}$ where $\alpha<1$ and M are constants. Let λ be a non-characteristic value for the equation $f(x)=F(x)+\lambda\int_{-1}^{x}K_1(x, y)f(y)dy+\lambda\int_{x}^{1}K_2(x, y)f(y)dy$, x and y real. Then this equation has a unique solution $f(x)$, analytic and bounded in D. (Received March 10, 1939.)

239. J. A. Shohat: *On the Lagrange interpolation formula based on the zeros of orthogonal Tchebycheff polynomials.*

The author considers the Lagrange interpolation formula based on the zeros of orthogonal Tchebycheff polynomials (OP) corresponding to any given distribution function. A horizontal step-function is introduced which has the first $2n$ moments of the given distribution. The interpolation polynomial is now represented as a finite expansion in series of OP corresponding to the above step-function. Various properties of this polynomial are given, and applications made to polynomials in general. (Received March 9, 1939.)

240. J. A. Shohat and Vivian E. Spencer: *Mechanical quadratures coefficients as functions of the moments.*

Extending a method of Markoff for the zeros of orthogonal polynomials (OP) theorems have been developed concerning the behavior of the coefficients of the associated mechanical quadratures formula as functions of the moments. Applications are made to various special classes of OP. (Received March 9, 1939.)

241. P. M. Swingle: *A finitely containing connected set.*

In this paper, using the hypothesis of the continuum and a modification of the method used by E. W. Miller (Fundamenta Mathematicae, vol. 29, pp. 123–133), it is shown that there exists a connected set which, for every positive integer n greater than one, is the sum of n mutually exclusive biconnected subsets but does not contain infinitely many mutually exclusive connected subsets. This solves a number of problems previously proposed (American Journal of Mathematics, vol. 53 (1931), pp. 394–395). (Received March 18, 1939.)

242. Otto Szász: *On the order of the partial sums of Fourier power series.*

It is shown by a simple construction that corresponding to any sequence $d_n \to 0$, there exist Fourier power series $\sum c_n e^{inx} \sim F(e^{ix})$ such that $\limsup_{n\to\infty} \sum_0^n c_v / n d_n = +\infty$. Moreover, a slight change in the coefficients provides a function $F(z)$ having the same property and such that $F(z)/(1-z)$ is a generalized Fourier power series for $|z|=1$. (Received March 9, 1939.)

243. R. M. Thrall: *Trilinear forms and determinental manifolds.* Preliminary report.

Let $(x_{\alpha\beta})$ be a p by q matrix whose elements are linear forms in $x_1, \cdots, x_n$, say $x_{\alpha\beta} = a_{\alpha\beta\gamma} x_\gamma$, and denote by V_r the manifold defined by the vanishing of the $(r+1)$-rowed minors of $(x_{\alpha\beta})$. Then the projective invariants of V will be invariants of the trilinear form whose coefficients are $a_{\alpha\beta\gamma}$. The purpose of this paper is the application to trilinear form classification of the theory of the manifolds V as developed by T. G. Room, *The Geometry of Determinental Loci*, Cambridge University Press, 1938. In the general case (that is, coefficients arbitrary and excluding certain values of p, q, n), it is possible to give a complete set of invariants for the trilinear form in terms of its related manifolds V. The significance of special cases is also discussed. (Received March 14, 1939.)

244. H. E. Vaughan: *Locally peripherally compact spaces.*

It has been shown by Zippin (this Bulletin, vol. 40 (1934), p. 56) that any complete separable metric space having the property that every point has arbitrarily small neighborhoods with compact boundaries (local peripheral compactness) may be compactified by the addition of a denumerable set of points. The object of the present paper is to show that a space having the above properties with the exception of completeness may be imbedded in a complete locally peripherally compact space which differs from it by a zero-dimensional set; and, hence, may be imbedded in a compact metric space which differs from it by the sum of a zero-dimensional set and a denumerable set. If a determining system for the original space can be chosen which consists of neighborhoods with compact boundaries all of which are of dimension at most

$n-1$, the imbedding space will also be at most n-dimensional. (Received March 17, 1939.)

245. C. W. Vickery: *Deformations of frequency functions.*

If $P(x_1, \cdots, x_k)$ and $Q(x_1, \cdots, x_k)$ are frequency functions, $\beta(x_1, \cdots, x_k) = Q(x_1, \cdots, x_k)/P(x_1, \cdots, x_k)$ is a *deformation* of P; a *continuous deformation* provided β is a continuous function. The identity deformation is produced when $\beta \equiv 1$. The mean value of β with respect to P is 1. Every continuous deformation leaves at least one point with fixed probability. The second moment of β about the identity deformation is a measure of deformation. Under certain conditions there exists a transformation of variables equivalent to deformation β and β is equal to the reciprocal of the absolute value of the jacobian of the transformation. If $Q(x)$ and $\beta(x)$ are estimated from a sample, $\beta(x)$ may be supposed an algebraic function of degree n. As an estimate of $\beta(x)$ one may use $\beta'(x)$, that algebraic deformation of degree n which minimizes χ^2 for the sample distribution with respect to $P(x)\cdot\beta'(x)$. If $\beta'(x)$ is linear, its slope is a measure of linear bias. One may thus classify and measure the bias of samples and test the hypothesis that a sample S is drawn with a particular form of bias from a population π. (Received March 29, 1939.)

246. C. W. Vickery: *Random and biased sampling machines.*

A machine has been designed for drawing independent samples at random or, with a predetermined form of bias, from a population of given distribution. One form of this machine consists essentially of a Rutherford-Geiger counter provided with a source of α rays and attached through an amplifier to a device for drawing cards. As cards flow through the machine, each α particle actuates the drawing of a card. The ratio of the rate of detected emission s to the rate of flow of the cards r is adjusted to produce the desired size of sample. The rate s is varied by varying the distance of the radioactive substance from the detecting vessel, thus introducing various forms of bias. A varying r equivalent to a varying s produces a transformation equivalent to a deformation. Another form of the machine dispenses with cards and records the emissions on a chronograph tape. The constant speed of a tape with linear scale produces a sample from a rectangular distribution. The scale or speed of the tape may be varied to obtain samples from other distributions, or a transformation equivalent to a deformation. (Received March 29, 1939.)

247. R. K. Wakerling: *On the loci of $(k+1)$-secant k-spaces of a curve in r-space.*

In r-space the $(k+1)$-secant k-spaces of a general curve form a $(2k+1)$-dimensional variety V, $(2k+1<r)$. If the curve is rational, the properties of V, and of its sections by spaces of lower dimension, can be studied by means of a $(1, 1)$ correspondence which may be established between the k-spaces of V and the points of a $(k+1)$-space. This correspondence is constructed by employing for the space of representation a fixed, osculating $(k+1)$-space having $(k+1)$-point contact with the curve. It is found that there is a close connection between the correspondence in question and the Cremona $(r-k)$-*ic* transformation in an S_{r-k} of S_r. (Received March 14, 1939.)

248. A. D. Wallace: *On ergodic continua.*

Using the linear measure of continua as developed by Frink (American Journal of Mathematics, vol. 58 (1936), p. 514) the results of Martin on ergodic curves (American Journal of Mathematics, vol. 58 (1936), p. 727) are extended to continua in a compact metric space. It is shown that certain of Martin's theorems are consequences of theorems on lower semicontinuous functions. In particular, it is shown that the ergodic function is monotone nonincreasing and continuous on the right (Martin, Theorem 3). (Received March 8, 1939.)

249. A. D. Wallace: *Some cyclicly extensible and reducible properties.*

It is assumed that S is a Peano continuum and that P is a property of point sets. The following properties are considered. $\Delta_0(P)$: If S is the sum of two continua, their product has property P. $\Delta_1(P)$: The boundary of each component of the complement of a continuum has property P. $\Delta_2(P)$: Every irreducible separation of S between two points has property P. $\Delta_3(P)$: If X and Y are disjoint closed sets containing x and y, there exists a set Z having property P which separates S between x and y and is disjoint with $X+Y$. It is known that for any property P each of the above properties implies the following. In this paper the following theorems are proved: (1) For any property P, $\Delta_0(P)$, $\Delta_1(P)$, and $\Delta_2(P)$ are cyclicly reducible. (2) If P is such that X having P implies that XE has P for each true cyclic element E of S, then $\Delta_3(P)$ is cyclicly reducible. (3) If each point has property P, then $\Delta_2(P)$ and $\Delta_3(P)$ are cyclicly extensible. From these results several known theorems are obtained. (Received March 8, 1939.)

250. J. L. Walsh: *On interpolation by functions analytic and bounded in a given region.*

Let C be a Jordan curve interior to a second Jordan curve B. Let the points $\beta_{n1}, \beta_{n2}, \cdots, \beta_{nn}$ interior to C satisfy an asymptotic condition $\lim_{n\to\infty} |(z-\beta_{n1})(z-\beta_{n2}) \cdots (z-\beta_{nn})|^{1/n} = \Psi(z)$ uniformly on any closed limited set exterior to C. Let the function $f(z)$ be analytic on and within C but not throughout the interior of B. Let $f_n(z)$ be the function analytic interior to B interpolating to $f(z)$ in the points $\beta_{n1}, \cdots, \beta_{nn}$ the least upper bound of whose modulus interior to B is a minimum. Then the sequence $f_n(z)$ converges in a characteristic way to $f(z)$ on C and even in certain points interior to B exterior to C (overconvergence). Compare for similar results a recent paper by the writer, Proceedings of the National Academy of Sciences, vol. 24 (1938), pp. 477–486, especially Theorems 1 and 3. (Received March 10, 1939.)

251. L. F. Walton: *On the theory of ideal numbers.* Preliminary report.

This investigation is based on the ideal theory developed by B. L. van der Waerden and E. Artin for integrally closed domains of integrity satisfying the finite chain condition ("Teilerkettenbedingung") and whose prime ideals are not necessarily divisorless. In the domains of integrity which are the subject of the present study, the existence of a denumerably infinite set of higher prime ideals is postulated. By introducing a "quasi-containment" relation defined in terms of the "quasi-prime decomposition" of the above ideal theory, it has been possible to imbed these domains in rings whose elements exhibit many of the properties of the Prüfer-Neumann ideal numbers. In particular, one obtains a unique additive decomposition in terms of p-adic components. (Received March 16, 1939.)

252. J. A. Ward: *Theory of analytic functions in linear associative algebras.* Preliminary report.

By means of the regular representations of an algebra, generalizations of the Cauchy-Riemann differential equations are obtained in terms of which analytic functions are defined. The usual properties of analytic functions are easily seen to follow, including results of Scheffers, Hausdorff, Ketchum, and Ringleb. The theory for commutative algebras is particularly simple. If $y=y_1e_1+\cdots+y_ne_n$ is an analytic function of $x=x_1e_1+\cdots+x_ne_n$, the jacobian matrix $\partial(y_1,\cdots,y_n)/\partial(x_1,\cdots,x_n)$ is the matrix representation of the derivative of y with respect to x. (Received March 16, 1939.)

253. F. J. Weyl: *Exponential curves.*

A normal exponential curve shall be defined by setting up $x_i=e^{\lambda_i z}$ in projective complex n-space $\{x_0, x_1, \cdots, x_n\}$, where the parameter z varies over the finite z-plane and the λ_i are distinct but arbitrary complex numbers. To this curve the theory of meromorphic curves (Hermann and Joachim Weyl, *Meromorphic curves*, Annals of Mathematics, (2), vol. 39 (1938), no. 3) is applied. Since the order $T(r)$ of such a curve, except for a bounded term, is equal to $r/2\pi$ times the circumference of the smallest convex polygon containing all points λ_i in the complex number plane, the second and third main theorems of this theory lead to certain results concerning circumferences of smallest convex polygons containing a given set of points in the plane. Similar considerations are carried through for the projection $y_j=\sum_{i=0}^{n}a_{ji}e^{\lambda_i z}$, $(j=0, 1, \cdots, m; m\leqq n)$, of a normal curve into a space of lower dimensionality. (Received March 17, 1939.)

254. G. T. Whyburn: *On the existence of certain transformations.*

It is shown that in order for a compact locally connected continuum to be mappable onto the interval (0, 1) by a non-alternating interior transformation $f(x)$ so that $f(a)=0$, $f(b)=1$, it is necessary and sufficient that M be a cyclic chain $C(a, b)$ from a to b. It follows from this that for each y, $(0<y<1)$, the set $f^{-1}(y)$ separates M irreducibly between a and b into just two components. Also, if M is unicoherent, f will be monotone. Related questions, such as the mappability of graphs and dendrites into an arc by interior transformations and the mappability of locally connected curves into an arc by light interior non-alternating transformations, are also considered. (Received March 8, 1939.)

255. L. R. Wilcox: *A theorem on curves in a projective space.*

Let C be a curve in a projective space S_n of n dimensions, and let T: $S_2\subset S_3\subset\cdots\subset S_{n-1}$ be a sequence of subspaces of S_n, the dimension of S_k being k. This paper considers configurations generated in the P_i by intersecting them with the linear osculants of C. In particular, a sequence of curves C_k, $(k=0,\cdots,n-2)$, is inductively defined as follows: $C_0\equiv C$; C_k is the set of intersections with S_{n-k} of the tangent lines to C_{k-1}. It is shown under mild hypotheses that C_k is the set of intersections with S_{n-k} of the osculating k-spaces of C. Special attention is devoted to the case when T is the sequence of linear osculants of C at a fixed point. (Received March 17, 1939.)

256. Max Wyman: *Non-holonomic covariant vector fields.*

In discussing covariant vectors fields for a Hausdorff space H, with coordinates in a Banach space E, Michal (*Abstract covariant vector fields in a general absolute calculus*, American Journal of Mathematics, vol. 59 (1937), pp. 306–314) points out the advisability of postulating an inner product $[x, y]$ for the space E. In the non-holonomic case the author considers in addition to H and E another Banach space E_1, and takes $\bar{V}(\bar{x}) = M(x, V(x))$ to be the law of transformation of a non-holonomic contravariant vector field, where $M(x, y)$ is a solvable linear function of y on EE_1 to E_1, with $N(\bar{x}, y)$ as inverse. To introduce non-holonomic covariant vector fields he also postulates an inner product $[v, w]$ for E_1, and assumes $N(\bar{x}, y)$ has an adjoint $N^*_{(2)}(\bar{x}, w)$. The law of transformation of covariant vector fields is $\bar{W}(\bar{x}) = N^*_{(2)}(\bar{x}, w)$ and that of a covariant linear connection is $\bar{L}(\bar{x}, \bar{w}, \bar{\xi}) = N^*_{(2)} \ (\bar{x}\, L(x \ w, \xi)) + N^*_{(2)}(\bar{x}, w; \xi)$. Under these definitions it is found that the usual tensor theorems can be proven, and one can introduce a normal representation theory for the covariant case. These results are in the main generalizations of results obtained by Michal and Botsford (Annali di Matematica, vol. 12, pp. 13–32) for the n-dimensional case. (Received March 17, 1939.)

257. Max Wyman: *Postulates for the determination of a non-holonomic linear connection.*

The author considers in non-holonomic tensor analysis a Hausdorff space H with coordinates in a Banach space E, and in addition another Banach space E_1. In this paper H is taken to be a generalized Riemannian space as defined by Michal (Paris Comptes Rendus, vol. 205 (1937), no. 14, p. 552). The existence of two functions $R(x, v)$ on EE_1 to E_1, and $\gamma(x, w)$ with the following properties, (i) $R(x, v)$ has the same properties as the metric form $g(x, \xi)$ of E, and (ii) $\gamma(x, w) = 0$ has only a finite number of linearly independent solutions, is assumed. It is then possible to state three postulates by means of which one can calculate a unique linear connection $K(x, v, \xi)$ in terms of $R(x, v)$, $\gamma(x, w)$, and r arbitrary covariant vector fields. These results are generalizations of results obtained for the n-dimensional case by Michal and Botsford (Annali di Matematica, vol. 12, pp. 13–32). (Received March 7, 1939.)

258. Max Zorn: *Menger-Brouwer dimension in subanalytic spaces.* Preliminary report.

The author has introduced the subanalytic spaces in connection with the theory of discontinuous groups. In this paper the theory of dimension in these spaces is examined. It is conjectured that the essential theorems of Menger's theory hold even when the concept "arbitrarily small" is defined by means of an arbitrary, not necessarily finite, covering of the subanalytic space by open sets. (Received March 18, 1939.)