$E_\gamma = E_{\gamma+1} = \cdots = E_\alpha = \cdots$, $\gamma \leqq \alpha < \Omega$. In generalizing some classical results of Baire the author proves that (*) is stationary if it is decreasing and if, in addition, each E_α is F_σ. The problem in the general case is unsolved.

3. This useful pamphlet consists of an Introduction followed by three chapters. In Chapter I the author gives a rapid but clear and rigorous survey of properties of convex and subharmonic functions, in some places giving new proofs or supplying missing details in older proofs existing in the literature. Chapter II contains the author's own investigation concerning the behavior of a subharmonic function in a neighborhood of an isolated singular point. The last Chapter, III, contains applications to the theory of harmonic functions and of solutions of the equation $\Delta u = \phi(P)$. The bibliographical references are numerous.

4. The operator $Af(x)$ defined over the space $L_2(-\infty, \infty)$ is said to be an integral operator if it can be represented in the form $\int_{-\infty}^{\infty} a(x, y)f(y)dy$ where $\int_{-\infty}^{\infty} |a(x,y)|^2 dy < \infty$ for almost all x. The main purpose of the author is to find necessary and sufficient conditions which have to be satisfied by a spectrum of a self-adjoint Hermitian operator in order that it be the spectrum of an integral operator. By a very elegant argument it is shown that necessary and sufficient conditions in question consist merely in the requirement that $\lambda = 0$ should be a limit point of the spectrum. An important place in the proof is occupied by the following theorem: *Being given any self-adjoint Hermitian operator H, it is always possible to find a completely continuous operator X of E. Schmidt's type of arbitrarily small norm, so that the spectrum of $H+X$ would reduce to a pure point-spectrum.* This theorem was proved by H. Weyl in the case of a limited operator H.

J. D. TAMARKIN

Introduction Mathématique aux Théories Quantiques. By Gaston Julia. Paris, Gauthier-Villars, 1936. vi+220 pp.

This book is the sixteenth of the well known series, *Cahiers Scientifiques*, and is the first of a series which proposes to give the mathematical foundation of quantum mechanics. In this first volume the essential difficulties of quantum mechanics (some of which concern the fact that Hilbert space is not finite dimensional) are merely foreshadowed, the attention being directed in the main to vector analysis in a space of finite dimensions. However, the treatment is sophisticated and designed, as far as possible, to carry over to the infinite dimensional case. Thus the treatment of linear equations is given in which the concept of rank is developed without the introduction of determinants. The derivation of the Jordan normal form for matrices is particularly clear, as is also the chapter on metric spaces. We can heartily recommend the book as an easy introduction to the well-known works of von Neumann (*Quantum Mechanics*), Stone (*Hilbert Space*), and Wintner (*Infinite Matrices*). There are only two parts of the book which seem to the reviewer other than highly commendable. The first of these contains the statement that unitary and Hermitian operators "se correspondent biunivoquement par les relations" (Cayley) $U = (H+i)/(H-i)$; $H = i(U+1)/(U-1)$. This is misleading since unitary matrices with a characteristic root unity are not cared for (except by a somewhat artificial limiting process). The correspondence $U=e^{iH}$ seems much more fundamental. The other point is in connection with the discussion of the values taken by the "quadratic" form associated with a matrix. The author seems unaware of the paper *On the Field of Values of a Square Matrix* by the present reviewer (Proceedings of the National Academy of Sciences, vol. 18 (1932)).

F. D. MURNAGHAN

The Rational Quartic Curve in Space of Three and Four Dimensions. By H. G. Telling. Cambridge, University Press; New York, The Macmillan Company, 1936. viii+78 pp.

This volume, Number 34 in the well known series of Cambridge Tracts in Mathematics and Mathematical Physics, is an introduction to the study of rational curves. The reviewer agrees that the best curve to select as representative of this type is the norm curve in four dimensions with its projections in ordinary space and in the plane. A detailed study of these rational quartics yields a wealth of geometric properties and of related configurations. On the other hand the analytic work involved is based on the algebraic theory of binary forms, and is not especially complicated for the quartic when expressed in the customary symbolic notation.

The present tract is condensed from a Fellowship Essay by the same author, and much of the material is here given as exercises. There are two chapters, the first of 40 pages on the norm quartic curve, and the second of 33 pages on the rational quartic in three dimensions. These are followed by several pages of notes on involutions on the curve. Only a few references are given in the text, but a selected bibliography is included which contains references to the extensive literature of the subject.

This little volume is well written, with excellent choice and arrangement of material. The author has produced a scholarly essay on a subject which richly deserves a place in this important series.

J. I. TRACEY

Idealtheorie. By W. Krull. (Ergebnisse der Mathematik, und ihrer Grenzgebiete, vol 4, No. 3.) Berlin, Julius Springer, 1935. vii+152 pp.

Krull's new book contains a very timely survey of the maze of material accumulated in recent years in the field of abstract ideal theory. Let it also be said to begin with that Krull's own fundamental contributions to the subject give him preeminent qualifications for the task.

The first concept of general ideal theory must be accredited to Dedekind. In his theory of the rings of all integers in fields of algebraic numbers one has the fundamental theorem that every ideal is a unique product of prime ideals. Dedekind also gives, however, consideration to rings in which this fundamental theorem is not true and where it has to be replaced by other decomposition theorems. Another introduction of general ideal theory came through Kronecker's theory of polynomial moduli. This theory was developed particularly by Lasker and Macauley, who showed its close relation with algebraic geometry. The impetus to the modern development came mainly through the work