Inversive Geometry. By Frank Morley and F. V. Morley. New York, Ginn and Company, 1933. 272 pp., and 67 figures.

This book is to serve as an introduction to algebraic geometry, that term being defined as the geometry of circular inversion. The problems considered are chosen from those that lend themselves readily to that treatment, but the number and the breadth of those selected is astonishingly large.

The most important prerequisite for the reading of the book is dexterity in manipulation of algebraic processes; groups, linear dependence, polarization, invariants and covariants, vectors, are all defined and even sometimes illustrated, but so concisely as to be of little use to a reader not already somewhat familiar with them. The procedure is everywhere rapid and direct; most of the theorems are at once applicable to a large number of cases. The elementary operation is that of reflection of a plane figure about a line in the plane. By combinations of reflections, rotation and translation are obtained. The product of reflections about perpendicular axes is called reversion.

The chapter on algebra develops the ordinary algebra of the complex number, and emphasizes the geometric meaning of each step. Hermitian forms play an important part. The euclidean group is then interpreted in terms of the algebra just developed, and this is enlarged by adjoining circular inversion, including stereographic projection. A chapter on quadratic and bilinear forms prepares for the inversive group in the plane, featuring the necessary conditions for finite inversive groups, the treatment being rather similar to that met with in the theory of automorphic functions.

Thus far the material developed provides the necessary tools for the subject proper. The binary linear transformation is interpreted in terms of parabolic, hyperbolic, or elliptic geometry, according to the reality of the fixed points. The idea is then extended to the surface of a sphere and lines are replaced by arcs of normal circles. The rectangular hexagon and the case of six perpendicular lines are treated in some detail.

Now follows the idea of conformal mapping, with a consistent physical interpretation. A short treatment of the differential geometry of these processes closes the first Part. The second Part develops the technique for the map equations of a line and of a circle, applies them to regular polygons, and skilfully shows just where the constructive difficulty of the regular polygons lies.

The introduction of rigid motion in terms of stretches and turns serves to make the operations concrete and tangible. The geometry of the triangle now offers no new difficulty, and a large number of theorems are speedily developed. Invariants under homologies are applied to triangles, four points, and hexagons, with an interpretation in terms of barycentric coordinates.

Rational curves are defined algebraically in terms of a parameter, as is customary, but with a number of special features peculiar to the coordinates employed. Here the parameter is the complex number of modulus 1. Conics, the cardioid, and the deltoid come in for more detailed treatment. The characteristic properties of these latter curves lend themselves particularly to treatment by the methods here developed. A short chapter on Cremona transformations is, in the opinion of the reviewer, less successful. For the elementary circular cases it is entirely appropriate, but the application to the Geiser and Bertini involutions is decidedly sketchy.

The book closes with a well written chapter on the n-line, developing from the beginning the essentials of the Clifford-Morley chain.

The press work is excellent and the proof reading faultless. The authors have succeeded in presenting the power and the fascination of the uses of inversive geometry in a competent and dignified way.

VIRGIL SNYDER

Le Calcul Vectoriel. By Alex. Véronnet. Paris, Gauthier-Villars, 1933. xviii+252 pp.

Except for a few chapters on the fundamental operations, this book is concerned almost entirely with the analytical properties of vectors in n dimensions. With a set of n coordinates is associated a vectorial number having those coordinates as components. A function of the coordinates is then regarded as a function of the vectorial number and operations of differentiation are defined analogous to those for a single variable. By taking the unit vectors as variable, the operations of tensor analysis are given a natural and simple interpretation. The treatment is formal in the sense that there are no convergence or limit proofs. No special notation is used, it being the view of the author that the reader should know from its significance what type of scalar or vector each letter represents. There are no problems or applications to physics. This book, particularly the chapter on tensors, should prove valuable collateral reading for the student interested in the analytical or multiple-algebraic phase of vector analysis.

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