SHORTER NOTICES

Traité du Calcul des Probabilités et de ses Applications. Edited by Émile Borel. Paris, Gauthier-Villars, 1933.

Vol. I, Section IV: *Les Principes de la Statistique Mathématique.* By R. Risser and C. E. Traynard. xi+388 pp.

Vol. III, Section IV: *Théorie Mathématique de L'Assurance Invalidité et de L'Assurance Nuptialité, Définitions et Relations Fondamentales.* By Henri Galbrun. 156 pp.

Vol. III, Section V: *Théorie Mathématique de L'Assurance Invalidité et L'Assurance Nuptialité, Calcul des Primes et des Réserves.* By Henri Galbrun. 183 pp.

This is an excellent, up-to-date treatment of mathematical statistics, covering an extensive range of well-selected topics. The earlier problems of statistics are given correct perspective; but the main purpose of the book seems to be to put the reader into contact with important recent developments, so far as space permits. Means and moments for one or more variables are handled with the aid of characteristic functions. For classifying the Pearson Types use is made of the following two criteria:

$$s = \{6\beta_2 - \beta_1 - 1\} \div \{3\beta_1 - 2\beta_2 + 6\};$$
$$p = \{4s^2(s+1)\} \div \{\beta_1(s+2)^2 + 16(s+1)\}.$$

Following the Bruns-Charlier series in Hermite polynomials, there appears Romanovsky's generalization with the substitution for the normal function of other frequency functions. Probable errors of statistical constants are discussed briefly with references to Karl Pearson, R. A. Fisher, and Rietz's *Handbook of Mathematical Statistics*; but small samples are not given detailed treatment. For drawing balls from urns, the schemes of Bernoulli, Poisson, Lexis, Borel, and Pólya are described. For the resolution of a frequency curve into components, Karl Pearson's method is given; and successive approximations are considered. This completes Part I.

Part II, devoted to correlation and covariation, draws from the works of Tschuprow, Galton, K. Pearson, Fechner, Cheysson, Norton, March, Keynes, Yule, Darmois, Lévy, Cantelli, Lexis, Bachelier, Edgeworth, Kneser, Myller-Lebedeff, Gram, Thiele, Charlier, Bruns, Galbrun, Guldberg, MacMahon, Student, Tchebycheff, Markoff, Bortkiewicz, Anderson, Steffensen, Sheppard, and Frisch. As part of this wealth of material may be mentioned the subject of probable values or mathematical expectation as developed by Tschuprow, the correlation surface in n dimensions as established by Pearson, also his chi-square surface, the Yule treatment of multiple and partial correlation, Anderson's variate difference method of dealing with correlation or covariation, Steffensen's modification of mean-square contingency, and Guldberg's difference equations for choosing the form of frequency functions.

In Section IV of the third volume a decidedly general treatment, based upon instantaneous rates, is given for insurance supplementary to life insurance. Disability is taken as representative of the entry into a new class where new mortality rates must be used; marriage as representative of the entry into another class where new mortality rates are not needed. This marriage insurance seems to be intended primarily for children or minors who upon marriage are to receive the insurance as a species of dowry. In France, marriage insurance was authorized by law in May 1921. From $p(x, y)$, the probability that a man of age x will reach age y, usually written ${}_{y-x}p_x$, there is formed $q(x, y)dy = [-\partial p(x, y)/\partial y]dy$, the probability that a man of age x will die between ages y and $y+dy$. Likewise $w(x, y)dy$ is written as the probability that a man of age x will enter a new class—of the disabled or the married—between ages y and $y+dy$; and $p^b(\beta, \beta, y)$ is the probability that a man now of age β, who entered the new class b at age β, will survive to age y. With the original class designated by the superscript a, a typical equation is

$$p(x, x, y) = p^a(x, y) + \int_x^y w(x, \beta)p^b(\beta, \beta, y)d\beta.$$

This is the probability that a man now of age x and in class a will survive to age y in class a *or* in class b. The annual rate $W(x, x+1)$ of withdrawal from class a, by disability or by marriage, is the integral of $W(x, \alpha)d\alpha$ from x to $x+1$. After a great many pertinent distinctions have been made and numerous probabilities defined, suitable approximations for the required aggregates are made with the aid of the Newton interpolation formula. Then follows a critical note on the use of the product formula for compound probability. Finally, there are tables giving disability and marriage rates.

Section V is a continuation, in the same excellent style, of Section IV. From the complex probabilities evolved in Section IV, net premiums for a large variety of forms of insurance and annuities are deduced, and the corresponding reserves or policy values. The proper loading to be added to the net premium to form the office premium is treated at some length with the aid of the Tchebycheff inequality. Thus, for a large company, the percentage loading for safety is very small. The last chapter is devoted to a somewhat detailed discussion of the validity of the Tchebycheff inequality as utilized in the preceding chapters.

E. L. Dodd

Höhere Mathematik. Teil IV: *Uebungsaufgaben mit Lösungen.* Heft II. By R. Rothe. Leipzig, B. G. Teubner, 1933. 53 pp.

This pamphlet gives interesting and instructive exercises (followed immediately by their solutions) on calculus topics included under the headings: (a) functions of two variables, their geometric representation, partial derivatives, and maxima and minima; (b) the differential geometry of plane curves, including properties of tangents and normals, order of contact, curvature, polar coordinate forms, asymptotes, and singular points. Further there are questions relating to complex numbers and the simpler functions of a complex variable.

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