has particularly interesting properties due to the fact that any orientation can be defined in terms of three, which is equal to the number of dimensions of the hypersurface. Many special hypersurfaces are also of considerable interest; among these may be mentioned hyperspheres, ellipsoids, and minimal hypersurfaces. This is only a small indication of the many things treated in this part of the book.

V. Part five, Invariants (120 pages). A book on this subject that makes any claim to completeness must necessarily discuss this subject. The treatment is based on Lie's continuous groups and covers the subject quite adequately.

C. L. E. MOORE

The Logic of Discovery. By R. D. Carmichael. Chicago, The Open Court Publishing Co., 1930. 274 pp.

Thanks to the analyses of both mathematicians and philosophers the science of logic has enjoyed greater development in the last seventy-five years than in the entire twenty odd centuries of its previous history. Up to 1850 it was identified with the formal logic of Aristotle. Since then five generalizations beyond Aristotle have occurred, each one of which has produced a new and more fertile system of logic. These systems are (1) the logic of classes initiated by Boole and Schröder, (2) the logic of relations associated with the names Peirce, Royce, Russell and others, (3) the logic of propositional functions formulated by Russell, (4) the logic of systems or doctrines and system or doctrinal functions, recently developed into an even higher generalization by Sheffer, and (5) the calculus of propositions first formulated by Russell and Whitehead in the *Principia Mathematica*, recently reinterpreted by Wittgenstein, and modified and perhaps even transcended by Hilbert, C. I. Lewis, Weyl, and others.

Carmichael's *Logic of Discovery* is based upon the fourth of these generalizations, the logic of systems or doctrines. Nothing new concerning this logic appears in his book. Its originality consists in its specification of the changes in our conception of mathematics, of reasoning, and of scientific method in the physical and social sciences, which the established concepts of this logic entail.

The well known fact that logical inference in a doctrinal function depends solely on the form is brought into the foreground. The key to this is really the notion of the variable and the discovery of the formal properties of relations which came into the logic of systems from the logic of propositional functions and relations. Carmichael's use of an experiment carried on by Veblen is most convincing on this point. The equivalence of postulate sets for the same system or system function is also indicated. These considerations enable Carmichael to reveal the relativity of the ordinary linguistic statements of a system, and the formal and purely rational side of scientific procedure.

His most original contribution concerns our conception of scientific method. It has been known for a long time by all acquainted with the historical evidence that Bacon's naive theory of induction has very little to do with actual scientific procedure, particularly as it occurs in the most mature sciences. Recently it has become the custom to conceive of all scientific work in terms of the method of hypothesis. But both conceptions have a common presupposition: Scientific statements are considered in terms of a few isolated propositions. The logic of systems and the part which it plays in mathematical physics forces a modification of these conceptions. It reveals that bodies of propositions have formal properties and that they can be handled as a whole. Moreover, these formal properties of bodies of propositions prescribe rules which are quite independent of any specific subject matter whatever. In technical language, the properties of the system remain the same if variables are substituted for constants, thereby turning each proposition into a propositional function, and the system or doctrine as a whole into a system or doctrinal function.

Unconsciously science has been taking advantage of this fact. Thus much of scientific procedure consists in establishing identifications between the variables of some purely mathematical system and the physical constants of some empirical subject-matter. Thus instead of working by observation from isolated facts to general principles, or from a hypothesis, put up more or less arbitrarily, to the verification of its deductive consequences, science confronts the actual system of an entire empirical subject-matter with the possible doctrine functions of pure mathematics. Einstein's identification, in the general theory of relativity, of the potentials of the gravitational field with the g_{ik}'s of a tensor form in pure mathematics is a case in point.

This is but one example of the many ways indicated in Carmichael's book in which the modern logic of doctrinal or system functions forces us to modify our conceptions of important things. It is easily the most readable treatment of this aspect of modern logic that has been written.

F. S. C. NORTHROP

Lezioni di Geometria Analitica e Proiettiva; parte seconda. By Annibale Comessatti. Padova, Casa editrice A. Milani, 1931. xii+440pp.+57 figures. Price Lire 60.00.

This second volume of the comprehensive treatise on the basic concepts of analytic geometry is concerned entirely with projective geometry of one, two, and three dimensions, treated analytically and restricted to entities generated by linear forms: points, lines and planes.

While the results obtained in the preceding volume (reviewed in this Bulletin, vol. 36, p. 474) are occasionally cited, it would be possible to read the present one without any knowledge of it further than the general concepts of coordinates and of the elements of projective geometry, treated synthetically.

The general plan of this second volume is much more closely related to our American scheme of instruction than was that of the first. It does not treat a wide range of subjects, but develops the one concept of projective transformations systematically and exhaustively. Following each chapter is a generous list of exercises, and the reader is urged to supplement the algebraic discussion with a free use of synthetic confirmation.

After binary collineations in a purely projective field have been introduced, they are applied to various metrical cases, including pencils of circles and groups of motions. In the ternary field the ordinary problems of projection and section, homology, involutions are first presented; then follow the types of plane collineations, cyclic collineations, and numerous metrical particularizations. Duality and polarity are given an entire chapter, which includes the duality between a bundle and a plane field.

In space of three dimensions more details are given in the development of homogeneous point coordinates; then follow the canonical types of collineations