SHORTER NOTICES

Einleitung in die Mengenlehre. By A. Fraenkel. 3d ed. Berlin, Springer, 1928. 14+424 pp.

Mengenlehre. By E. Kamke. (Sammlung Göschen, No. 999.) Berlin and Leipzig, de Gruyter, 1928. 159 pp.

Books on the theory of aggregates all seem to present much the same approach to the subject. They begin with Cantor's definition of an aggregate and then build up a theory of cardinal and ordinal numbers. It seems to the reviewer that one of the main features of the aggregate theory is its extremely careful avoidance of any reasoning which is not clear cut and rigorous. For a student entering for the first time on rigorous thinking, the aggregate theory seems almost uselessly careful. But should a student have his first dip into rigor in this field? As a matter of fact most students get an introduction to rigor in a course in real variable. Then why, in a course on aggregate theory; and why, in the texts on the subject, is it not possible to put down a system of axioms which is considered sufficient for the purpose in hand and to proceed by indisputable steps to the erection of the structure of the theory? The two books about to be reviewed do not do this. The first, by Fraenkel, discusses the axioms very carefully, but not until the very end of the book! The second even proves the well ordering theorem without any mention of Zermelo's Axiom.

The treatise by Fraenkel on the theory of aggregates is now one of the finest. The book has been made much clearer by the definite statements of theorems in italics, which was not done in the previous editions.

This book is divided into five chapters: I. Foundations and Cardinal Numbers, II. Operations with Cardinal Numbers, III. Order Types and Ordinal Numbers, IV. Attacks on the Foundations and their Consequences, V. The Axiomatic Structure of the Theory of Aggregates and the Axiomatic Method.

The first three chapters cover the usual ground quite clearly except that what is assumed is sometimes a bit in doubt because no axioms are presented. The book contains problems which make it very suitable as a text for graduate students.

The fourth chapter has been enlarged to twice its former size and presents a most interesting discussion of Brouwer's new work and of Russell's work.

The fifth chapter discusses a system of axioms and Hilbert's theory of logic and metamathematics.

Not as much attention is paid in the book to the application of the theory to function theory or point sets as might be in a text of its size; it is a text on the theoretical side of the subject only.

In matters of printing and arrangement both author and publisher are to be complimented.

Kamke's little book is a very convenient summary of the subject of abstract aggregate theory. Its four chapters are concerned with, first, the generalities of the subject; second, cardinal numbers; third, ordered sets and ordered types; fourth, well ordered sets and their ordinal numbers.

Applications of the theory to point sets and function theory are sometimes given as examples to illustrate the matters discussed, but no detailed discussion of point set theory or of function theory is given. This is, of course, not to be expected in so small a book.

There is a short but very interesting discussion of the paradoxes.

It is to be hoped these other topics relating to the subject of this book will be dealt with in other volumes of the series.

T. C. BENTON

Höhere Mathematik für Mathematiker, Physiker, und Ingenieure. By R. Rothe. Part 2. Leipzig and Berlin, B. G. Teubner, 1929. viii+201 pp.

This little volume is the second of a series of three volumes intended to give the principal elementary facts of higher mathematics, for the benefit of mathematicians, physicists, and engineers. The present volume covers the integral calculus, infinite series and a brief treatment of vector analysis. The integration sections discuss formal indefinite integration processes, the definite integral, approximate integration by computation and mechanical means, and the definite integral as a function of a parameter. The infinite series section takes up the principal theory of convergence, a treatment of power series, expansions of special functions, and some applications. The last chapter is devoted to determinants, and the algebra of vectors, with applications to geometry and mechanics, mostly the former. On the whole one gets the impression that the physicist and engineer has had but scant attention in the brief mention of an occasional application to physics and mechanics. For instance, in integration second moments are not mentioned at all, and first moments only incidentally as mean value integrals.

The book has a number of interesting points and formulas. By way of example, there is developed an integration by parts formula of the nth order (suggestive of existence proofs in differential and integral equations); there are derivations of Wallis' product, Stirling's formula, and Euler's constant; determinants are defined by means of an expansion formula in terms of minors, which with the aid of mathematical induction can be (but is not by the author) made the basis for derivation of the principal properties of determinants.

The book makes an attempt and succeeds quite well in maintaining a high standard of rigor. There are occasional slips. For instance, in discussing the method of determining constants in dividing a rational fraction into partial fractions, the impression is given that the method of substitution for the variable is applicable only when the factors of the denominator of the given fractions are distinct; in stating Darboux's theorem on upper and lower integrals one is led to believe that the proof is an immediate consequence of earlier theorems, which it is not; the vector product of two vectors is expressed as a determinant, involving vector quantities, though determinants have been defined only for numbers.

On the whole, the book contains much that is worth while for the embryo mathematician, and presents the material in a very acceptable way.

T. H. HILDEBRANDT