If $f(x)=x^r$, (4) requires that $a^r=m_r$. This can be satisfied if $m_r<a^r$ for some r; since m_r is a continuous function of r, and if r is large enough,

$$m_r \geqq b \int_a^{a+k} x^r\, dx > a^r .$$

In any case, however, (4) can be satisfied by

$$f(x)=cx^r+z;\quad z=2x-x^2/a,\quad x\leqq a;\quad z=a,\quad x\geqq a,$$

because a positive c can be found satisfying $ca^r+a=cm_r+g$, where $g<a$, noting that $z\leqq \alpha$, and $z<a$ inside $(a-k,a)$ where $\phi(x)\geqq b$.

This M satisfying (3) reduces to the arithmetic mean only if $m_1=a$.

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CONSECUTIVE QUADRATIC RESIDUES*

BY A. A. BENNETT

By an extension of the methods described in a paper to appear shortly in the TÔHOKU MATHEMATICAL JOURNAL, I have succeeded in proving that for each prime greater than 193 there is at least one sequence of five consecutive positive reduced quadratic residues. The proof entails the examination of many hundred linear forms which together include all primes. Since the method would prove excessively laborious for even the next case, that of six consecutive quadratic residues, the computational details seem hardly to warrant the space required for their complete publication. As a result of the actual construction of a complete table of quadratic residues for all primes less than 331, we obtain the brief table subjoined. Here p denotes in turn each prime number, r denotes for the given p the maximum number of positive reduced quadratic residues which appear

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consecutively, while n denotes correspondingly the maximum number of positive reduced quadratic non-residues appearing consecutively. For primes of the form $4k+3$, the distribution of quadratic residues among the integers $1, 2, 3, \cdots$ is the same as that of quadratic non-residues among the integers $p-1, p-2, p-3, \cdots$, so that for each prime of this form, r and n are necessarily equal.

Within the limits of this list the actual irregular distribution is fairly closely approximated by the smooth approximate empirical relation $r=n=\frac{1}{2}\sqrt{p+20\log_{10} p}$.

Table of the number of consecutive quadratic residues and of consecutive quadratic non-residues:

p=2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59
r=1	1	1	2	3	2	2	4	4	4	4	4	3	5	4	3	5
n=0	1	2	2	3	4	3	4	4	3	4	4	5	5	4	6	5

p=61	67	71	73	79	83	89	97	101	103	107	109	113	127	131	137	139
r= 5	6	6	4	6	7	4	4	7	7	6	5	5	7	8	6	5
n= 6	6	6	4	6	7	6	6	5	7	6	10	4	7	8	5	5

p=149	151	157	163	167	173	179	181	191	193	197	199	211	223	227	229	233
r= 4	7	6	6	6	6	6	6	6	4	7	6	7	7	7	5	6
n= 6	7	6	6	6	8	6	6	6	5	5	6	7	7	7	6	7

p=239	241	251	257	263	269	271	277	281	283	293	307	311	313	317
r= 6	6	7	6	7	8	7	10	7	9	9	7	10	5	5
n= 6	5	7	6	7	9	7	7	7	9	5	7	10	7	7

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