THE MAY MEETING IN NEW YORK

The two hundred forty-second regular meeting of the Society was held at Columbia University, on Saturday, May 2, 1925, extending through the usual morning and afternoon sessions. The attendance included the following sixty-three members.

Alexander, R. L. Anderson, C. R. Ballantine, J. P. Ballantine, Boyajian, Brant, R. W. Burgess, Carson, Alonzo Church, Dostal, Dresden, Eversull, Feldstein, Fenn, Fine, Fiske, Fite, D. A. Flanders, Fort, R. M. Foster, Philip Franklin, Frink, Gafafer, Garretson, Gehman, Gill, Glenn, W. C. Graustein, Gray, Gronwall, C. C. Grove, Hazlett, Hille, Himwich, Hoyt, Joffe, Kerékjártó, Kline, Lefschetz, Harry Levy, MacColl, S. P. Mead, Meder, Metz, Molina, Paxton, Pell, R. G. Putnam, Ritt, R. B. Robbins, Saurel, Schelkunoff, Seely, J. H. Taylor, J. M. Thomas, Tracey, Veblen, Wedderburn, Whited, Widder, Widmark, Wiener, Zobel.

There was no meeting of the Council or of the Trustees. At the morning session Professor Arnold Dresden, Assistant Secretary of the Society, gave a report on the business transacted by the Council at the April meeting in Chicago and by mail.

Vice-President Wedderburn presided, relieved by Ex-Presidents Fine and Veblen and Professor Alexander. At the beginning of the afternoon session addresses were presented, at the request of the program committee, by Mr. J. R. Carson and Dr. T. H. Gronwall on *The Heaviside operational calculus and its applications to electric circuit theory*. A number of engineers and physicists were present during this session by invitation, in addition to members of the Society.

Titles and abstracts of the papers presented at this meeting follow below. The papers of Dr. Castellani and Professor Hollcroft were read by title.

1. Professor Norbert Wiener: *The operational calculus.*

The author employs the method of separating the Fourier integral of a function into low and high frequency ranges to justify the asymptotic expansions derived by Oliver Heaviside by operational methods for the solution of the differential equations of the electric circuit.

2. Professor Norbert Wiener and Dr. Philip Franklin: *Analytic approximations to topological transformations.*

The leading theorem of this paper asserts the existence of an analytic transformation approximating to any degree of exactness any given one to one continuous transformation of the spherical surface into itself. The analytic transformation is obtained after an approximation with continuous partial derivatives is set up. This last construction depends on a dissection of the sphere and its transform into convex polygons which are nearly transforms of one another. All the vertices are triple, and we use affine transformations at these vertices. We then interpolate our transformation appropriately on the sides and in the interiors.

3. Dr. Philip Franklin: *Functions with assigned derivatives.*

In this paper it is shown that, given an infinite number of isolated points in the complex plane, with one branch cut drawn to infinity from each point, a function exists in the cut plane which is analytic at all points except those of the given set, and at these points possesses derivatives of all orders whose values may be arbitrarily assigned. The method of proof is an extension of that used by J. F. Ritt for a simpler, but related, problem. The function is also shown to exist when the further condition is imposed on it that, in a finite region not including any points on the branch cuts, it approximates uniformly to a given degree of exactness a given analytic function.

4. Professor Olive C. Hazlett: *On the types of division algebras.*

In this paper the author extends somewhat the results of a paper by Dickson in the TRANSACTIONS OF THIS SOCIETY for 1924 and one by Wedderburn in the TRANSACTIONS for 1921. It is proved that any quadratic division algebra of the type discussed by Cezioni in the RENDICONTI DI PALERMO for 1923 is a direct product of Dickson algebras.

5. Professor T. R. Hollcroft: *On conditions for self-dual curves.*

In accordance with the Lefschetz postulate of singularities for a given order $n \geqq 14$, self-dual plane curves exist only for a genus within and including the limits 0 and $2n-7$. This theorem results from Plücker's equations

and a theorem of Klein's: For an algebraic curve whose equation has real coefficients, a self-dual curve, and a self-dual curve only, may have all of its point and line singularities (1) real with real tangents and points, (2) imaginary. Six sets of equalities among the singularities of a curve are found all of which are necessary for the curve to be self-dual. Of these, three sets only are always sufficient. The remaining three are sufficient except for certain definite curve types, dual in pairs. For space self-dual curves, in general the plane projection curve is not self-dual, and conversely. Such curves whose plane projection curves are also self-dual exist only for $n \geqq 8$ and for a very limited genus. There are eight equalities among the characteristics of space curves, four of which are both necessary and sufficient for the curve to be self-dual, and four necessary, but sufficient only with the exception of certain curve types.

6. Professor O. E. Glenn: *The invariant system of two associated bilinear connexes.*

Systems of invariants of connexes were studied by Clebsch and Gordan, who published, jointly, a paper on the theory in MATHEMATISCHE ANNALEN, vol. 1. The subject of complete systems remained stationary until 1916, when the present writer improved the algorism for simultaneous connexes (TRANSACTIONS OF THIS SOCIETY, vol. 17), and determined a complete system for a conic p_x^2 and a connex $a_x \alpha_u$. The present paper gives the determination of a fundamental system of two bilinear connexes $a_x \alpha_u$, $p_x \theta_u$. Subject to some reduction methods yet to be tried, the system contains, approximately, 130 invariant formations.

7. Dr. Maria Castellani: *Algebraic surfaces with reducible bitangent and osculating hyperplanar sections.*

The object of this note is to study, first, the algebraic surfaces in hyperspace which are cut by any bitangent hyperplane in reducible curves and, second, algebraic surfaces which are cut by any osculating hyperplane in reducible curves. Among the results is the following theorem: The algebraic irreducible surfaces $S_r (r > 5)$, not cones, which any bitangent hyperplane cuts in reducible curves are the ruled surfaces and the surfaces with hyperplanar sections of genus $p = 1$.

8. Professor J. H. M. Wedderburn: *The elementary divisors of a real symmetric matrix.*

This paper gives a short proof of the well known theorem that a real symmetric matrix has real roots and simple elementary divisors. The proof is based on the reduced characteristic equation introduced by Frobenius and on the fact that $a^2 + b^2 = 0$ implies $a = b = 0$ when a and b are real symmetric matrices.

9. Professor J. H. M. Wedderburn: *The absolute value of the product of two matrices.*

This paper appears in the July number of this BULLETIN.

10. Dr. T. H. Gronwall: *The algebraic structure of the formulas in plane trigonometry.*

The set of formulas connecting the sides and angles of a plane triangle is investigated from the point of view of Hilbert's theory of algebraic forms (MATHEMATISCHE ANNALEN, vol. 36). A linear basis of the set of formulas is constructed, the syzygies of various orders are set up, and finally the group of birational transformations leaving the formulas invariant is considered.

11. Dr. T. H. Gronwall: *The behavior at infinity of the gamma and associated functions.*

Regions are constructed in which the functions $\Gamma(z)$, $P(z)$, and $Q(z)$ tend uniformly to a limit as $|z| \to \infty$ (the limit being either zero or infinity) and asymptotic formulas are given for the roots of the equations $\Gamma(z) = a$, $P(z) = a$, $Q(z) = a$.

12. Professor Norbert Wiener: *A canonical form for biunivocal continuous sense-preserving transformations of a sphere.*

The author proves a theorem suggested by Professor Alexander, to the effect that it is possible to interpolate a continuous sequence of continuous biunivocal transformations between two biunivocal continuous sense-preserving transformations of a sphere into itself. In the course of this argument, these transformations are reduced to a canonical form.

13. Dr. Harry Levy: *Ricci's canonical congruences.*

Ricci has associated with a given congruence of curves in a Riemann space n—1 others which he calls the congruences *canonical* with respect to the given one. In this paper we obtain a geometrical interpretation of the canonical congruences dependent upon the parallelism of Levi-Civita; we derive the conditions that congruences be mutually canonical, and from them we deduce several theorems of geometric interest.

14. Mr. Orrin Frink: *Operations of boolean algebras.*

In this paper the problem of finding all operations of boolean algebras which have all the formal properties of a given boolean operation is solved by means of a theory of transformation. The author also finds all boolean operations which are unique, i. e., which share all their formal properties with no one other operation. In terms of such a unique triadic operation taken as a single fundamental operation a postulate system for boolean algebras is given. All boolean operations with idemfacient and nilfacient elements are determined. A method is given of deriving from a non-invariant relation between n elements an invariant relation between $n+1$ elements, with similar properties.

15. Mr. Alonzo Church: *Alternatives to Zermelo's assumption.* Preliminary report.

If we reject the axiom of choice as a principle of logic applicable to all classes, the following possibilities for the second ordinal class appear: (A) There exists an assignment of a unique fundamental series to every ordinal of the second kind in the second ordinal class. (B) There exists no assignment of a unique fundamental series to every ordinal of the second kind in the second ordinal class; but, given any ordinal, α, of the second ordinal class, there exists an assignment of a unique fundamental series to every ordinal of the second kind less than α. (C) There is an ordinal, α, of the second ordinal class, such that there exists no assignment of a unique fundamental series to every ordinal of the second kind less than α. These three possibilities may be regarded as postulates, determining three distinct kinds of second ordinal classes, and the consequences of each may be investigated on this basis, with an open mind about the possibility of obtaining

a contradiction from one or more of them. Any one of these postulates is found to be an effective instrument in obtaining theorems. On grounds of elegance, the theory resulting from (B) seems preferable, although it is in contradiction to the axiom of choice.

16. Professor Oswald Veblen and Dr. J. M. Thomas: *Projective normal coordinates for the geometry of paths.*

This paper introduces into the geometry of paths a system of coordinates which have the following properties: (1) they are independent of the particular affine connection associated with the paths and are therefore projective in character; (2) in terms of them the equations of the paths through the origin are linear in a specially chosen projective parameter; (3) they undergo a linear fractional transformation when the general coordinate system suffers an arbitrary analytic transformation. A projective normal tensor Q^i_{jkl} is defined in terms of them, and the Weyl projective curvature tensor is found to be $Q^i_{jkl}-Q^i_{jlk}$. An outline of this paper appeared in the PROCEEDINGS OF THE NATIONAL ACADEMY for April, 1925.

17. Dr. J. M. Thomas: *Conformal correspondence of Riemann spaces.*

In this paper is pointed out the existence of a set of quantities which are formed from the Christoffel symbols of the second kind and have the same value at corresponding points of two Riemann spaces mapped conformally on each other so that corresponding points have the same coordinates in both spaces. These quantities are useful in obtaining conformal invariants. The conformal curvature tensor of Weyl is obtained by expressing integrability conditions of their law of transformation under a change of coordinate system.

18. Dr. J. M. Thomas: *Asymmetric displacement of a vector.*

The changes in the components of a vector ξ^i by displacement from a point x^i to a nearby point x^i+dx^i of an n-dimensional manifold are assumed to be $-H^i_{jk}\xi^j\,dx^k$, where the quantities H have the same law of transformation as the Christoffel symbols of the second kind for a Riemann space, but are otherwise absolutely general. When the H's are changed, it is found that the directions of all displaced

vectors are preserved if, and only if, the changes in the symmetric and skew symmetric parts of H have the forms $\delta^i_j\varphi_k+\delta^i_k\varphi_j$ and $\delta^i_j\varphi_k-\delta^i_k\varphi_j$ respectively, φ_j denoting an arbitrary vector. The most general change of affine connection which preserves geodesics (cf. Weyl, GÖTTINGER NACHRICHTEN, 1921, p. 99) is brought about, therefore, by such a change in the H's. Tensors independent of φ_j are found, and a process for forming tensors of the same nature but of higher rank is given. A necessary and sufficient condition for the H's to assume the special form considered by Friedmann and Schouten (MATHEMATISCHE ZEITSCHRIFT, vol. 21, p. 18) is the existence of a coordinate system in which the original and displaced vectors have proportional components. The corresponding H's can be made symmetric by a change preserving displaced directions.

19. Dr. J. H. Taylor: *On the inverse problem of the calculus of variations.*

In this paper application of existing theory is made to the problem of determining when a given system of differential equations are such that they define the extremals for a definite integral, whose integrand is expressed in parametric form and which is homogeneous of degree one in the first derivatives. It is shown that if the "paths" of the geometry of paths minimize a definite integral of the above type its integrand is necessarily the square root of a quadratic form, that is, the paths then admit of being the geodesics of a Riemann space.

20. Dr. J. P. Ballantine: *On a certain functional equation.*

In taking a mean between two numbers x_1 and x_2, one has an equation of the form $p_1f(x_1)+p_2f(x_2)=(p_1+p_2)f(x_3)$. The mean based on the function f is called the f-mean. The problem is to find all functions f whose f-mean has the property that if x_1 and x_2 are each multiplied by the same number a, then the mean, x_3, is also multiplied by that same number. Additive and multiplicative constants in f have no effect on the mean, and, disregarding these, it is shown that x^n and $\log x$ are the only such functions. If a instead of being multiplied by x_1, x_2, and x_3 is added to them, the only functions are x and k^x, where k is an arbitrary constant.

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