one of their characteristic features. In other ways also much valuable material has here been brought together for which persons wishing to learn or teach the subject will feel grateful to the authors.

MAXIME BÔCHER.

Ueber die analytische Fortsetzung des Potentials ins Innere der anziehenden Massen. Preisschriften gekrönt und herausgegeben von der Fürstlich Jablonowskischen Gesellschaft zu Leipzig. XLIV. By GUSTAV HERGLOTZ. Leibzig, Teubner, 1914. 52 pp.

THE problem for which this memoir is a solution was stated as follows: A given ellipse is transformed by the method of reciprocal radii into a certain oval. Consider a plane surface of homogeneous material (flat plate) bounded by such an oval. In the *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften,* 1909, C. Neumann proved on the basis of the theory of the logarithmic potential that, so far as its action on exterior points is concerned, the surface in question can be replaced by a material line bounded by two mass points. Since it might be of interest to extend this result to space of three dimensions, the Society proposed the following question: In the theory of the newtonian potential what is the analogue of Neumann's theorem? That is, what can be said about the newtonian potential of a homogeneous solid bounded by the ovaloid which is obtained from an ellipsoid by the method of reciprocal radii?

The point of view adopted by the author is exhibited first in a generalization of the result obtained by Neumann for the case of the logarithmic potential. If the potential of a body for outside points can be represented as the potential of a suitable mass distribution within the body, then certainly it can be continued analytically inside the body up to the mass distribution. Conversely, if the potential can be continued analytically to points inside the attracting body and if the singular points of the potential which are encountered be enclosed by any surface (curve in the plane problem) F, then the potential, because it is regular outside F, can be represented as the potential of a mass distribution on F. In general the mass consists of a single and a double distribution, the density of which depends on the surface chosen.

The exact meaning of the analytic continuation of the potential is apparent only by following the transformation by which the potential is expressed as a line integral of a function of a complex variable. The logarithmic potential of a flat plate of unit density upon an exterior particle at (ξ, η) is defined by

$$V(\xi, \eta) = \int\int \log \frac{1}{R} dxdy, \quad \{R^2 = (x - \xi)^2 + (y - \eta)^2\}.$$

The components of attraction are

$$X = \frac{\partial V}{\partial \xi} = \int\int \frac{x - \xi}{R^2} dxdy,$$

$$Y = \frac{\partial V}{\partial \eta} = \int\int \frac{y - \eta}{R^2} dxdy.$$

If we set $\zeta = \xi + i\eta$, $u = x + iy$, $v = x - iy$, and define Ω by

$$\Omega(\zeta) = X - iY,$$

then

$$\Omega(\zeta) = \int\int \frac{dxdy}{u - \zeta},$$

and

$$V(\xi, \eta) = \Re \int \Omega(\zeta) d\zeta,$$

where $\Re$ denotes the real part of the integral. If

$$G(x, y; \zeta) = \frac{v}{u - \zeta},$$

then, by Green's theorem,

$$\int\int \left(\frac{\partial G}{\partial x} + i\frac{\partial G}{\partial y}\right) dxdy = \int_C G(dy - idx),$$

where the integral in the second member is taken around the curve which forms the boundary of the plate. Since the first member is equal to 2Ω, this equation becomes

$$\Omega(\zeta) = \frac{1}{2i} \int_C \frac{v}{u - \zeta} du.$$

The denominator is single-valued and does not vanish throughout the integration. The singularities must enter through v, which is determined as a function of u from the equation of the curve C. If this curve is algebraic the possible singularities are branch-points and poles, which occur at the foci of the curve. As defined by Plücker the ordinary foci of an algebraic curve are the real points on the tangents to the curve which pass through the circular points at infinity. If the curve includes the circular points then the tangents at these points furnish extraordinary foci. The function v has a branch-point at an ordinary focus and a pole at an extraordinary focus. The analytic continuation of the potential means the process of allowing the curve C to shrink without passing over a singular point. If the singular points are joined by a curve L then L represents the limiting form which C may take.

The author shows that it is possible to determine (1) the density of a simple distribution of matter along L, (2) the moment of a double distribution along L, and (3) the masses of particles situated at the extraordinary foci so that the potential shall be the same as the potential of the plate on an exterior point.

This result is a generalization of Neumann's statement. Some particularly simple theorems are deduced for certain curves consisting of two ovals. If one oval is allowed to shrink to a point or the two ovals allowed to coalesce into a loop one obtains the general inversion curve of the ellipse or hyperbola and every theorem goes directly into those discovered by Neumann.

The general treatment of the newtonian potential for algebraic surfaces is promised by the author in a later dissertation. In the present paper the work is confined to solids bounded by surfaces of revolution of the form

$$(x^2 + y^2 + z^2)^2 + 4C(x^2 + y^2) - 4Bz^2 + 4D = 0,$$
$$(D - B^2)(D - C^2) > 0.$$

These surfaces fall into six different types, and, as limiting cases, they include: for $D = 0$ the surface obtained by inversion of a surface of the second order, which is the proposed prize problem; for $D = B^2$ the torus considered by Bruns.

For homogeneous solids bounded by these surfaces of revolution the following conclusion is given: the potential can be represented as the potential due to a mass arranged as (1) a simple distribution along a segment of the Z-axis, (2) a simple distribution over a circular plate in the XY-plane ($x^2 + y^2 \leqq a^2$), and (3) one or more mass-points at the ends of certain segments or a radial double distribution on the circular plate.

The density of the mass distribution in the XY-plane and on the Z-axis is explicitly determined for the various feasible cases.

W. R. LONGLEY.

Das Schachspiel, und seine strategischen Prinzipien. VON M. LANGE. Zweite Auflage. No. 281, Aus Natur und Geisteswelt. Leipzig, Teubner, 1914. 108 pp. Mark 1.25. With portraits of E. Lasker and Paul Morphy.

THIS little volume, with the portrait of a mathematician as frontispiece, is included in the announcement of the series in which it appears among the mathematical works. While the strictly mathematical treatment is, of necessity, slight yet the attempt is seriously made to present an introduction to chess based upon somewhat fundamental, and partly mathematical, principles. The work marks a distinct advance, in a pedagogical way, in the literature of chess.

LOUIS C. KARPINSKI.

A Course in Descriptive Geometry and Photogrammetry for the Mathematical Laboratory. By E. LINDSAY INCE. Edinburgh Mathematical Tracts, No. 1. London, E. Bell and Sons, 1915. viii+79 pages, 42 figures.

THIS little book makes no claim of being a treatise, but endeavors to present the important features of descriptive geometry in such a manner that one may be instructed rapidly in the general processes employed. A short introduction sketches the whole problem as treated by the methods of orthogonal double projection, perspective and plane projection. Only about twenty pages are devoted to the treatment of lines and planes, yet in this short space many of the standard problems are well discussed. In the chapter on the applications to curves and surfaces no general statements are found, no attempts being made to have the processes apply to other surfaces than cones, cylinders, and spheres. The mathe-