THE APRIL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and forty-eighth regular meeting of the Society was held in New York City on Saturday, April 30, 1910, extending through the usual morning and afternoon sessions. The following forty-one members were present:

Mr. E. S. Allen, Mr. F. W. Beal, Professor W. J. Berry, Dr. E. G. Bill, Professor G. D. Birkhoff, Professor E. W. Brown, Mr. R. D. Carmichael, Miss Emily Coddington, Professor F. N. Cole, Professor J. L. Coolidge, Professor L. P. Eisenhart, Professor H. B. Fine, Professor T. S. Fiske, Professor C. C. Grove, Dr. G. F. Gundelfinger, Dr. L. C. Karpinski, Professor Edward Kasner, Mr. W. C. Krathwohl, Professor W. R. Longley, Professor J. H. Maclagan-Wedderburn, Dr. H. F. MacNeish, Dr. Emilie N. Martin, Mr. A. R. Maxson, Mr. H. H. Mitchell, Professor C. L. E. Moore, Professor Frank Morley, Professor G. D. Olds, Professor W. F. Osgood, Dr. H. B. Phillips, Mr. H. W. Reddick, Professor R. G. D. Richardson, Mr. L. P. Siceloff, Professor D. E. Smith, Professor P. F. Smith, Dr. W. M. Strong, Professor J. H. Tanner, Professor C. B. Upton, Professor Oswald Veblen, Mr. H. E. Webb, Miss M. E. Wells, Professor H. S. White.

Ex-President W. F. Osgood occupied the chair at the morning session, Ex-President T. S. Fiske and Professor Frank Morley at the afternoon session. The Council announced the election of the following persons to membership in the Society: Mr. F. W. Beal, Princeton University; Professor W. J. Berry, Brooklyn Polytechnic Institute; Mr. J. K. Lamond, Yale University; Mr. R. M. Mathews, University High School, Chicago, Ill.; Professor F. E. Miller, Otterbein University; Mr. J. E. Rowe, Johns Hopkins University; Mr. W. H. Terrell, Clyde, N. C.; Mr. George Wentworth, Exeter, N. H.; Mr. W. A. Wilson, Yale University. Eight applications for membership in the Society were received.

Professor Bôcher was elected by the Council to succeed Professor Osgood as a member of the Editorial Board of the *Transactions*. Professor Dickson was appointed to fill the unexpired term of Professor E. B. Van Vleck, who retires from the Editorial Board in July.

It was decided to hold the April, 1911, meeting of the Society at Chicago, Ill. The publication of the lectures delivered at the Princeton Colloquium by Professors G. A. Bliss and Edward Kasner was placed in charge of the Committee of Publication. The New Haven Colloquium Lectures have recently been issued by the Yale University Press.

The following papers were read at this meeting:

(1) Dr. H. B. PHILLIPS: "Application of Gibbs's indeterminate product to the algebra of linear systems."

(2) Dr. H. B. PHILLIPS: "Concerning a class of surfaces associated with polygons on a quadric surface."

(3) Professor VIRGIL SNYDER: "Conjugate line congruences contained in a bundle of quadric surfaces."

(4) Professor W. B. CARVER: "Ideals of a quadratic number field in canonic form."

(5) Professor G. A. MILLER: "On a method due to Galois."

(6) Dr. E. H. TAYLOR: "On the transformation of the boundary in conformal mapping."

(7) Professor W. B. FITE: "Concerning the invariant points of commutative collineations."

(8) Professor R. G. D. RICHARDSON: "On the saddle point in the theory of maxima and minima and in the calculus of variations."

(9) Mr. H. H. MITCHELL: "Note concerning the subgroups of the linear fractional group $LF(2, p^n)$."

(10) Mr. H. H. MITCHELL: "The subgroups of the linear group $LF(3, p^n)$."

(11) Professor C. L. E. MOORE: "Some infinitesimal properties of five-parameter families of lines in space of four dimensions."

(12) Professor EDWARD KASNER: "Forces depending on the time, and a related transformation group."

(13) Professor F. H. SAFFORD: "Sturm's method of integrating $dx/\sqrt{X} + dy/\sqrt{Y} = 0$."

(14) Dr. G. F. GUNDELFINGER: "On the geometry of line elements in the plane with reference to osculating circles."

In the absence of the authors Dr. Taylor's paper was presented by Professor Osgood, and the papers of Professors Snyder, Carver, Miller, Fite, and Safford were read by title. Abstracts of the papers follow below.

1. In his vice-presidential address before the American Asso-

ciation for the Advancement of Science (1886) Gibbs suggested a method for obtaining the results of Grassmann's linear algebra by the use of what he called an indeterminate product. Following this method Dr. Phillips gives proofs of a number of the fundamental theorems of Grassmann.

2. If the lines of a polygon of $2n$ sides lie on a quadric surface, it follows from a theorem of Poncelet that the lines joining the odd vertices to the non-adjacent even ones lie on a surface of class $n - 2$. Dr. Phillips discusses these surfaces and the configurations of lines for the cases $n = 5$ and $n = 6$.

3. In Professor Snyder's paper a method was developed for determining all the congruences formed by the generators of a bundle of quadric surfaces, and for distinguishing the necessary and sufficient condition that the two systems generate congruences that are rationally separable. The lines l, l' through a basic point B_i of a bundle of quadrics $\Sigma\lambda_i H_i$ are arranged in pairs, each of which uniquely determines the other, defining an involution I. A cone K with vertex at B_i fixes a congruence σ in Σ and its image K' in I fixes another congruence τ; the lines of σ, τ are formed by conjugate systems of generators of the same family of quadrics. The singular cones at B_i, B_k are birationally equivalent. The congruences σ, τ define an infinite discontinuous birational group of point transformations which leave their common focal surface invariant. Finally, both congruences define a relation $f(\lambda) = 0$ which is the equation of a contact curve of the discriminant $\Delta(\lambda)$ in the λ plane. All the contact curves of $\Delta(\lambda)$ having an odd characteristic can be obtained in this way.

4. In his Vorlesungen über Zahlentheorie, Sommer shows that any ideal of the quadratic number field $k(\sqrt{m})$ can be reduced to the canonic form $(i, i_1 + i_2\omega)$, where i, i_1, and i_2 are rational integers, i_2 a factor of i and i_1, and ω is $\sqrt{m}$ or $\frac{1}{2}(1 + \sqrt{m})$ according as $m \not\equiv 1 \pmod 4$ or $m \equiv 1 \pmod 4$. In the present paper Dr. Carver uses a slightly modified canonic form, viz., $r(s, t + \omega)$, r, s and t being rational integers and ω having the meaning given above, with the conditions $s > t \geqq 0$ and

$$t^2 \equiv m \pmod s, \qquad \text{if } m \not\equiv 1 \pmod 4;$$
$$(2t + 1)^2 \equiv m \pmod{4s}, \qquad \text{if } m \equiv 1 \pmod 4.$$

A method is developed for factoring, multiplying, and dividing (when division is possible) ideals in this canonic form.

5. If H represents any subgroup of a group G, all the operators of G may be represented, without repetition, in either of the following two ways:

$$G = H + HS_2 + HS_3 + \cdots + HS_\gamma$$
$$= H + T_2H + T_3H + \cdots + T_\gamma H.$$

If for each HS_α ($\alpha = 2, 3, \cdots, \gamma$) it is possible to find some $T_\beta H$ so that all the operators of HS_α coincide with those of $T_\beta H$, then H is an invariant subgroup of G, and vice versa. Galois called attention to this important case and named it a proper decomposition of G. Professor Miller considers the general case where HS_α has exactly ρ operators in common with $T_\beta H$ and proves that, if this condition is satisfied, the operators of both HS_α and $T_\beta H$ transform H into a group which has exactly ρ operators in common with H. In particular, if all the operators of HS_α coincide with those of $T_\beta H$ then H is invariant under each of the operators of HS_α. He also proves that in any group whatsoever it is possible to select the operators $S_2, S_3, \cdots, S_\gamma$ in such a way that all the operators of G may be represented, without repetition, in either of the following ways, H being any arbitrary subgroup of G:

$$G = H + HS_2 + HS_3 + \cdots + HS_\gamma$$
$$= H + S_2H + S_3H + \cdots + S_\gamma H.$$

In Weber's Lehrbuch der Algebra, volume 2, 1896, page 8, it is observed that the second of these decompositions is also equal to

$$H + S_2^{-1}H + S_3^{-1}H + \cdots + S_\gamma^{-1}H.$$

Hence the S's may also be replaced by their inverses without affecting either decomposition as a whole.

6. The solution of the problem of mapping the interior of a simply connected region S on the interior of a circle was completed by Professor Osgood in his proof of the existence of the Green's function of the most general simply connected plane region. The question as to whether the boundary of S, when this boundary is of the most general nature, will be transformed

continuously into the circumference of the circle, was answered by Professor Osgood in a set of theorems published in volume 9 of the BULLETIN. The major part of Dr. Taylor's paper is given to the proofs of these theorems.

7. In view of the erroneous statement by Reye in his Geometrie der Lage that a space collineation A with just four invariant points can be commutative only with collineations that have the same invariant points, Professor Fite determines in detail what must be the invariant points of collineations commutative with A. He extends the results for three dimensional space to collineations in space of $n - 1$ dimensions, and determines the nature and number of the collineations that are commutative with both A and B, B being any collineation commutative with A.

8. Professor Richardson shows that the minimum of the function $f(x, y, z)$ for those values of the variables that satisfy the relation $g(x, y, z) + \pi h(x, y, z) = 0$ is a function $M(\pi)$ of the parameter π, which when maximized gives the same constant as the minimum of the function $f(x, y, z)$ for those values of the variables which satisfy the relations $g(x, y, z) = 0$, $h(x, y, z) = 0$.

In the calculus of variations the two problems corresponding are also found to be equivalent:

1. To find a function $y(x)$ which satisfies the boundary conditions

$$(A) \qquad y(0) = y(1) = 0$$

and the integral relations

$$\int_0^1 g(x, y, dy/dx)dx = 0, \quad \int_0^1 h(x, y, dy/dx) = 0,$$

and minimizes the integral

$$(B) \qquad \int_0^1 f(x, y, dy/dx).$$

2. The minimum of the integral (B) for those functions $y(x)$ which satisfy the boundary conditions (A) and the integral condition

$$\int_0^1 \{g(x, y, dy/dx) + \pi h(x, y, dy/dx)\}dx = 0$$

is a function of the parameter π. To determine π in such a way that the minimum is maximized.

In both theories the results admit of immediate generalization.

9. Moore and Wiman have determined the subgroups of the linear fractional group $LF(2, p^n)$. Mr. Mitchell gives another method for the determination of those subgroups which contain additive groups. It is based on the solution of the Diophantine equation

$$\Omega = 1 + (p^m - 1)\frac{\Omega}{d_1 p^m} + \sum_{i=1}^{r}(d_i - 1)\frac{\Omega}{f_i d_i} \quad (f_i = 1, 2).$$

10. In this paper Mr. Mitchell extends the results obtained by him concerning the subgroups of the linear group $LF(3, p)$, to the more general group $LF(3, p^n)$. The group is represented as a collineation group in the finite plane $PG(2, p^n)$.

The maximal subgroups are: the $LF(3, p^k)$, where k is a divisor of n; the hyperorthogonal groups $HO(3, p^k)$, which appear for n even and k a factor of $\frac{1}{2}n$; groups leaving invariant a point, line, triangle, and conic; a G_{720} (for $p = 5$), $G_{\frac{1}{2}7!}$ ($p = 5$), the G_{360}, G_{216}, and G_{168}.

11. Professor Moore makes use of the ten coordinates of a line in S_4 to discuss some properties of five-parameter families which involve second and higher differentials. The interpretation in the geometry of circles in ordinary space is then given.

12. The forces considered by Professor Kasner lead to differential equations of the form $\ddot{x} = \phi(x, y, t)$, $\ddot{y} = \psi(x, y, t)$. In connection with the trajectories (xy curves), it is of interest to introduce the xt curves and the yt curves, t being represented as a space coordinate. The transformations of x, y, t converting a pair of equations of the above type into one of the same type form an infinite group involving three arbitrary functions.

13. In Professor Safford's paper Sturm's method of integrating $dx/\sqrt{X} + dy/\sqrt{Y} = 0$ is discussed for a somewhat more general form of X than usual. The integrating factor is found as a function of xy and the coefficients of X. The determination of this factor has previously been in most cases dependent upon a knowledge of the integral to be obtained instead of upon the given differential equation.

14. By interpreting the projective geometry within a linear line complex as a geometry of line elements in the plane by

means of a transformation of Lie, Dr. Gundelfinger effects a classification of ordinary differential equations of the first order with respect to the arrangement of the ∞^2 osculating circles to their integral curves and develops a theory of reciprocal line element loci.

F. N. COLE,
Secretary.

THE APRIL MEETING OF THE CHICAGO SECTION.

THE twenty-seventh regular meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY was held at the University of Chicago on Friday and Saturday, April 8–9, 1910. Professor L. E. Dickson, Vice-President of the Society and Chairman of the Section, presided at the three sessions held on Friday morning and afternoon and Saturday morning. The attendance at the various sessions included twenty visitors and the following forty-four members of the Society:

Mr. R. P. Baker, Mr. W. H. Bates, Professor G. A. Bliss, Professor Oskar Bolza, Dr. R. L. Börger, Dr. H. E. Buchanan, Dr. Thomas Buck, Dr. H. T. Burgess, Dr. A. R. Crathorne, Professor D. R. Curtiss, Professor L. E. Dickson, Dr. Arnold Dresden, Professor W. B. Ford, Professor A. G. Hall, Professor E. R. Hedrick, Mr. T. H. Hildebrandt, Professor O. D. Kellogg, Professor Kurt Laves, Dr. A. C. Lunn, Dr. W. D. MacMillan, Mr. E. J. Miles, Professor G. A. Miller, Dr. R. L. Moore, Professor E. H. Moore, Professor J. C. Morehead, Professor F. R. Moulton, Dr. L. I. Neikirk, Dr. Anna J. Pell, Professor Alexander Pell, Professor H. L. Rietz, Miss Ida M. Schottenfels, Mr. A. R. Schweitzer, Professor J. B. Shaw, Mr. R. R. Shumway, Professor C. H. Sisam, Professor H. E. Slaught, Professor E. J. Townsend, Professor A. L. Underhill, Professor E. B. Van Vleck, Professor E. J. Wilczynski, Professor B. F. Yanney, Professor J. W. Young, Professor J. W. A. Young, Professor Alexander Ziwet.

On Friday evening nearly all of the members present at the meeting dined together at the Quadrangle Club, at which time the question of holding the next meeting of the Section at Minneapolis was discussed. A letter was read from the President of the University of Minnesota urging the Section to meet