

of the more general equation of the first kind where $K(x, y)$ is assumed to have a discontinuity along a curve $y = \phi(x)$.

In the section just described the kernel was assumed to be finite. The earliest integral equation of the first kind, that of Abel, was however one in which the kernel became infinite along the line $x = y$. The concluding section of the book is devoted to equations of the type

$$f(x) = \int_a^x \frac{G(x, \xi)}{(x-\xi)^\lambda} u(\xi) d\xi,$$

which has a kernel with an infinite discontinuity including Abel's kernel as a special case for $G = 1$, and to a number of examples not falling under the previous theory. Especially interesting is the explanation of the relation of Fourier's integral

$$f(x) = \frac{2}{\pi} \int_0^\infty \int_0^\infty \cos(x\xi) \cos(\xi\xi_1) f(\xi_1) d\xi_1 d\xi$$

to the theory of integral equations in which the limits are infinite.

G. A. BLISS.

SHORTER NOTICES.

Grundlagen der Analysis. Von MORITZ PASCH. Ausgearbeitet unter Mitwirkung von CLEMENS THAER. Leipzig, Teubner, 1908. 8vo. vi + 140 pp.

THIS book presents an admirable attempt to develop the concept of the real number in a more exact logical fashion. There is no attempt to reduce the assumptions to a categorical set, and even their consistency is not considered; but they are everywhere clearly stated, the theorems follow by ready deductions, and the large number of definitions would seem to be put in an unusually clear way, and one especially well adapted to the purpose of the general argument.

The book opens with a consideration of the relation of things to names, of the notions of precede and follow, and of methods of mathematical proofs. This is followed by a treatment of sets, sequences, and series, leading up to integers. By subjecting the integers to the four fundamental operations, fractions, including decimal fractions, and negative numbers are intro-

duced, and the laws of calculation are shown to be still valid in the enlarged set. The author then extends the concept of the set to non-enumerably infinite aggregates, which allows him to introduce irrational numbers in the usual Dedekind fashion. Powers with both bases and exponents, arbitrary real numbers, and logarithms of arbitrary positive real numbers to arbitrary positive real bases are then treated. The book proper concludes with an exposition of some of the principal theorems of combinations, and the binomial theorem for a positive real exponent. An appendix is added to the book, giving extracts from some of the author's previous writings.

On account of the abstract nature of the subject matter, the book is not suitable for the beginner, but it should appeal strongly to every teacher of mathematics. Notwithstanding the abstractness of the subject, the matter is attractively arranged, and may be perused with profit by one who possesses general maturity, even without an extensive knowledge of the technique of mathematics. It is unfortunate that books of this type are so inaccessible to English readers.

F. W. OWENS.

Eine konforme Abbildung als zweidimensionale Logarithmentafel zur Rechnung mit komplexen Zahlen. By Dr. F. BENNECKE, Professor at the Victoria-Gymnasium in Potsdam, 1907.

THE above is the title of the Festschrift by Dr. Bennecke at the three hundredth anniversary of the establishment of the Royal Joachimsthal Gymnasium at Berlin. In the early paragraphs is given a statement of the literature of the subject. While the author makes no pretension of presenting an exhaustive bibliography, nevertheless the citations are suggestive of the historical development and of the general interest in graphic methods, particularly as applied to operations with complex numbers. The purpose of the pamphlet is to establish a method by which the logarithms of complex numbers may be calculated with sufficient accuracy for practical purposes by means of graphs. This purpose is accomplished by a conformal representation upon the (X, Y) -plane of the two systems of curves given by $x = \text{constant}$, $y = \text{constant}$, where

$$W = X + iY = \log z = \log (x + iy).$$

It is shown that any curve of either of the two resulting systems