The book, as a whole, commends itself for its simplicity of presentation. The treatment of the logarithm is a doubtful pedagogical expedient but there is no lax rigor about it. The responsibility is shifted to the numerical calculation of logarithms, just as was done in Olney's Calculus over thirty years ago.* A student who has had elementary training in algebra and trigonometry can read the book without difficulty and, in the main, it presents enough of the calculus and its applications to serve that body of students of which we have spoken at the beginning. But it does not represent what we, in America, have come to consider as a first course in the calculus.

L. WAYLAND DOWLING.

Vorlesungen über die Weierstrasssche Theorie der irrationalen Zahlen. Von VICTOR VON DANTSCHER. Leipzig und Berlin, Teubner, 1908. vi + 79 pp.

IN the preface we are told that these lectures are based upon a course given by Weierstrass in the summer semester of 1872, which was followed by the author of the present volume, and upon an elaboration of a later course given in 1884. The work under review is, however, not a mere reproduction of things given by Weierstrass, but it is the direct outcome of a course given repeatedly at the University of Gratz by Professor von Dantscher. It furnishes an easy and clear introduction to that theory of irrational numbers which was first developed by Weierstrass in his lectures at the University of Berlin, and it has decided pedagogic as well as scientific value.

C. Méray was the first to give a purely arithmetic meaning to the term irrational number,† and the theories developed by him, G. Cantor, Heine, and Dedekind have perhaps become better known than the theory of Weierstrass. This may be partly due to the fact that no expository publication relating to this theory was ever prepared under the direction of Weierstrass, and only the fundamental elements of this theory have been accessible in the works of Kossak, Pincherle, Biermann, and others. The first of these was based upon a course of lectures given by Weierstrass during the winter semester of 1865–6, and it was published in 1872 under the title "Die Elemente

* It should be stated that Olney sought only to avoid infinite series. The Watson proof of the rule for differentiating the logarithm tacitly assumes that $dx^n/dx = nx^{n-1}$ holds for all real values of n.

† Encyclopédie des Sciences mathématiques, tome I, vol. 1, p. 149.

der Arithmetik, Programm-Abhandlung des Werder'schen Gymnasiums."

The subject matter of the present volume may be sketched as follows: After a brief historical introduction and a proof of the fact that it is impossible, in the domain of rational numbers, to extract the nth root of a natural number which is not an nth power of a natural number, the fundamental concept "additive aggregate of an infinite number of positive rational numbers" is introduced, and we are reminded that the terms "additive" and "multiplicative," as applied to aggregates, are not due to Weierstrass. It seems desirable to use the term aggregate instead of the term set (Menge, ensemble) since a set of numbers generally implies that the numbers considered are discrete, while this is not commonly the case as regards the aggregates of numbers used by Weierstrass.* The term "additive" as applied to an aggregate of an infinite number of positive rational numbers implies that every possible sum of a finite number of the "members" of the aggregate is to be formed.

To make it possible to operate with these aggregates it is necessary to give a definition for the equality of two aggregates. In framing such a definition the term part (Bestandteil, partie) of a positive rational number a is used for every positive rational number smaller than a. Hence two rational positive numbers are equal if every part of each is a part of the other, and, similarly, two additive aggregates are said to be equal whenever every part of each is a part of the other. As there are additive aggregates whose equality may be established according to this definition irrespective of whether one of these aggregates is increased or diminished by any finite number, the author restricts himself to the consideration of additive aggregates which are such that every part is less than a given finite number. These aggregates are said to be convergent while all other aggregates are said to be divergent.

The explanation of these fundamental notions is followed by the representation of a convergent additive aggregate as a systematic fraction, and the proof that these aggregates may be combined by addition, subtraction, multiplication, and division after the definitions of these operations have been properly formulated. In view of the fact that the ordinary rules of arithmetic apply to these aggregates, they are called numbers, and the domain of rational numbers is extended by adding to

* J. Tannery, *Bulletin des Sciences math.*, vol. 32 (1908), p. 103.

it these additive convergent aggregates, which may also be called irrational numbers. It is then proved that, in this enlarged domain, it is possible to extract the nth root of any natural number, and to represent the ratio of two lines, whether they are commensurable or incommensurable.

The subject matter of the rest of the volume may perhaps be sufficiently evident from the general headings of the last three lectures. They read as follows : Additive aggregates of an infinity of positive and negative rational numbers; additive aggregates of an infinity of complex numbers of the form $a + bi$; multiplicative aggregates of an infinity of numbers. The value of the volume is greatly enhanced by illustrative examples, and it may be heartily recommended even to those who are just beginning graduate work in our universities. It need scarcely be added that a clear comprehension of this theory of irrational numbers will clear up many difficulties as regards the theory of absolutely convergent series with numerical terms.

G. A. MILLER.

Magic Squares and Cubes. By W. S. ANDREWS. With Chapters by PAUL CARUS, L. S. FRIERSON, C. A. BROWNE, JR., and an Introduction by PAUL CARUS: Chicago, The Open Court Publishing Company, 1908. vi + 199 pp.

Among the Arabians magic squares were known in the ninth century of our era and about this time they played an important rôle in Arabian astrology. A special work on the subject is attributed to an Arabian mathematician named Tâbit ben Korrah who died in 901,* and H. Suter mentions several other early Arabian writers on this subject in his work entitled Die Mathematiker und Astronomen der Araber und ihre Werke. These facts are not in accord with the statement on page 1 of the book under review, which reads as follows : "The earliest record of a magic square is found in Chinese literature dated about 1125 A. D."

The present work is, in the main, a direct reprint of articles which appeared in the *Monist* during recent years. Its author is an electrical engineer who, during his leisure hours, "has given considerable thought to the working out in his own original way the construction of magic squares and cubes of various styles and sizes." As may be inferred from this excerpt

* Encyclopédie des Sciences mathématiques, t. 1, vol. 3 (1906), p. 63.