in their applications. He has in mind the creation of what he calls a géométrie analytique arithmétique which shall be to the theory of numbers what ordinary analytic geometry is to algebra and analysis. He does not claim to have made more than a beginning in the working out of his idea, but clearly states that he has tried only to point out the possibility and the usefulness of such an attainment.

In the book under review the author uses coordinates more extensively than in his earlier work. He finds that the general idea of geometric transformation may be conveniently applied to arithmetic spaces, and that the general linear transformation in a two-dimensional arithmetic space changes arithmetic lines into arithmetic lines; that is, it is a collineation. This is true whether the arithmetic space be infinite or modular. In an infinite arithmetic space, the domain for the coordinates of points is the totality of integers, but in an arithmetic space of modulus m the domain consists of integers, modulo m. If m is a prime number, these integers form a field or finite algebra, and consequently the analytic geometry of the space is more like ordinary analytic geometry. In particular, the transformation called inversion is possible. Arnoux calls attention to this fact.

The reviewer takes this opportunity to mention the fact that Arnoux's k-dimensional arithmetic space of prime modulus p is what Veblen and Bussey have called a finite euclidean geometry and have denoted by the symbol $EG[k, p]$.* Arnoux has not given the theorem that the linear transformation in a k-dimensional modular arithmetic space ($k > 2$) of prime modulus is a collineation, nor has he made any study of groups of transformations in these finite euclidean geometries.

The book closes with a chapter of applications to number theory, magic squares, and Euler's problem of the 36 officers.

W. H. Bussey.

Fragen der Elementargeometrie, gesammelt und zusammengestellt von Federigo Enriques. Deutsche Ausgabe von Dr. Hermann Fleischer. II Teil. pp. xii + 348. Leipzig, B. G. Teubner, 1907.

One of the few unique books of the last few years on elementary mathematics was the collection of related monographs

* "Finite projective geometries," *Transactions Amer. Math. Society*, vol. 7 (1906), pp. 241–259. In particular, see §7.

on "Questioni riguardanti la geometria elementare" which appeared in 1900 under the editorship of F. Enriques. The volume under review is more than a translation, it is a thorough revision of the portion of the original that deals with the theory of geometrical construction. The other volume will contain a discussion of postulates, definitions, theory of area, the parallel axiom, and related topics.

In every respect the German edition is an improvement on the original. The text is less involved, and assumes little more than should be acquired in an ordinary college course in the calculus. Even this degree of proficiency is unnecessary for most of the book.

The first chapter contains a discussion of the various methods of attacking original problems of construction, and follows rather closely the excellent little book of Petersen on this subject. The next ninety pages are devoted to constructions that can be performed by the use of particular instruments, as, for example, the compasses alone, where the spirit of Mascheroni's work is very clearly presented; the ruler alone, showing the nature of the projective properties of figures; the parallel ruler; and various instruments in connection with a fixed curve. It is almost a pity that Lill's very elegant constructions of cubic and higher irrationalities by means of the movable right angle are not mentioned.

The analytic side of the subject is not neglected, and a clear demonstration of the impossibility of making constructions of higher order than the second with ruler and compasses is presented. The construction of the 17-gon is, perhaps, given more prominence than is necessary, but this is offset by the concise discussion of the various exact and approximate solutions of the classical problems of trisecting the angle, and duplicating the cube. Several approximate constructions are given for π, and the transcendence of π is demonstrated.

The book is a most welcome contribution to the literature that should be of interest to teachers in secondary schools. It makes no pretense of showing how to teach, but gives a background of fact and theory that no teacher should lack. The writer uses it as required reading in an undergraduate course for prospective teachers, and believes that most of the book can be appreciated by undergraduates. Though the style and aim of most of the chapters is for reading by professional teachers rather than professional mathematicians, at the same time the number of important ideas to which the reader is introduced is surprisingly large, and what is of as great importance, a breadth of outlook pervades the volume that teachers will certainly appreciate.

H. E. HAWKES.

Leçons de Mécanique céleste professeés à la Sorbonne. Par H. POINCARÉ. Paris, Gauthier-Villars. Tome I, 1905, 365 pp.; tome II, 1re partie, 1907, 165 pp.

TISSERAND has written an excellent four volume treatise on celestial mechanics, containing most of the classic contributions to the subject in the spirit and often in the notations of their original authors. Poincaré's Les Méthodes nouvelles de la Mécanique céleste was devoted to establishing, with the rigor of modern mathematical methods, the existence of various kinds of periodic orbits, to determining their properties, to proving the non-existence of new uniform integrals analytic in the masses and the existence of asymptotic solutions, etc. These profound and important researches have almost nothing in common with earlier investigations either in method or subject matter. The Leçons of Poincaré occupy ground between that covered in Tisserand's treatise and in Les Méthodes nouvelles. They are largely expositions of certain parts of standard theories, but the methods of development are those which the author considers as best suited to reaching the desired end regardless of what other writers may have used. It is doubtful if Poincaré could have compelled himself to take the little steps his predecessors have sometimes taken, or to follow their often circuitous paths. At any rate it is fortunate for us that we are able to see how the most penetrating and brilliant mathematician who has written on celestial mechanics reacts on some of our standard theories.

The first volume of the Leçons is devoted to the general theory of planetary perturbations. The first chapter contains an account of canonical equations, their properties, and their transformations. The canonical equations are used throughout the work. The problem of elliptic motion and the equations for the variations of the elliptic elements are developed in these variables, and Lagrange's method of variation of parameters is explained and applied. It is easily shown that the general term in the expression for either a coordinate or an element is of the form

$$\mu^{a} A M t^{m} \cos (\nu t + h),$$