$$F\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) = 0,$$

in which the first member is an analytic function of its five arguments, there exists in general through an analytic curve an analytic surface $z = z(x, y)$ which satisfies the equation. In the paper of Professor Bliss it is shown that a similar theorem is true when the function F is required to have continuous first and second derivatives only. The proof is made with the help of the theory of characteristic curves and the existence theorems for a set of ordinary differential equations

$$\frac{dx_1}{dt} = f_1(x_1, x_2, \cdots, x_n), \cdots, \frac{dx_n}{dt} = f_n(x_1, x_2, \cdots, x_n).$$

The relation between the characteristic curves and integral surfaces is well known when everything is analytic. But in order to work the other way and derive existence theorems from the properties of characteristic curves, it is necessary to use the theorems on the differentiability of solutions of a system of ordinary equations with respect to the constants of integration. These theorems seem to have been overlooked in this connection.

F. N. Cole,
Secretary.

THE DECEMBER MEETING OF THE CHICAGO SECTION.

The twenty-second regular meeting of the Chicago Section of the American Mathematical Society was held at the University of Chicago on Monday, Tuesday and Wednesday, December 30–31, 1907, and January 1st, 1908, in connection with the fifty-eighth convocation of the American association for the advancement of science. The great gathering of scientists in other lines doubtless attracted unusual numbers of mathematicians, resulting in a wide representation of members and the largest attendance ever recorded at any meeting of the Society.

Monday afternoon and Tuesday morning were devoted to meetings with Sections A and D of the American association, for discussion of the teaching of mathematics to engineering students. These meetings proved of so great interest that an additional half-day was set apart for the informal discussion, and this session was finally brought to a most delightful climax by the vice-presidential address of Professor Edward Kasner of Section A, on "Geometry and mechanics," which proved equally fascinating to both engineers and mathematicians. A detailed report of these joint meetings, with abstracts of the papers, appears elsewhere in the BULLETIN, and the papers will be published in full in *Science* during the year.

The sessions on Monday morning, Wednesday morning, and Wednesday afternoon were given to the reading of papers. Professor Heinrich Maschke, Vice-President of the Society, presided at the first session, and Professor E. B. Van Vleck, Chairman of the Section, relieved at times by Professor L. E. Dickson and Professor H. B. Newson, presided at the other sessions.

The officers of the Section elected for the ensuing year were: Professor G. A. Miller, Chairman; Professor H. E. Slaught, Secretary, and Professor D. R. Curtiss, third member of the programme committee.

The promotion of acquaintance and good fellowship was an unusually prominent feature of the meetings, owing to the complete arrangements for entertainment and especially to the subscription dinner at the Hotel Del Prado, where just one hundred mathematicians and engineers were gathered. It has long been the custom on these occasions to send postal card greetings to absent members — a custom inaugurated by Professor E. H. Moore — and in this instance the hundred greetings were sent to him, on board ship departing for his year's vacation.

The attendance at the several sessions included the following eighty-four members of the Society:

Professor D. P. Bartlett, Professor S. M. Barton, Professor W. W. Beman, Dr. G. D. Birkhoff, Professor O. Bolza, Professor W. C. Brenke, Professor W. E. Brooke, Mr. Thomas Buck, Professor W. H. Bussey, Professor D. F. Campbell, Professor H. E. Cobb, Mr. E. H. Comstock, Professor C. E. Comstock, Professor D. R. Curtiss, Dr. H. N. Davis, Professor E. W. Davis, Professor S. C. Davisson, Professor L. E. Dickson, Professor J. F. Downey, Professor L. W. Dowling, Mr. Arnold Dresden, Professor H. T. Eddy, Professor C. C. Engberg, Mr. E. B. Escott, Professor H. B. Evans, Professor T. M. Focke, Mr. R. M. Ginnings, Miss Harriet E. Glazier, Professor B. F. Groat, Professor A. G. Hall, Professor G. B. Halsted, Professor Harris Hancock, Dr. Charles Haseman, Professor C. N. Haskins, Professor A. S. Hathaway, Professor E. R. Hedrick, Mr. T. H. Hildebrandt, Mr. F. H. Hodge, President C. S. Howe, Professor E. V. Huntington, Professor Edward Kasner, Professor O. D. Kellogg, Professor Kurt Laves, Dr. A. C. Lunn, Mr. W. A. Luby, Professor C. R. Mann, Mr. W. D. MacMillan, Professor J. L. Markley, Professor Heinrich Maschke, Professor G. A. Miller, Professor F. R. Moulton, Professor G. W. Myers, Dr. L. I. Neikirk, Professor B. L. Newkirk, Professor H. B. Newson, Mr. E. W. Ponzer, Professor Charles Puryear, Professor N. C. Riggs, Professor D. A. Rothrock, Mr. A. R. Schweitzer, Miss Ida M. Schottenfels, Professor G. T. Sellew, Professor J. B. Shaw, Mr. R. L. Short, Dr. C. H. Sisam, Mr. C. G. Simpson, Professor E. B. Skinner, Professor H. E. Slaught, Professor C. S. Slichter, Professor S. E. Slocum, Mr. E. R. Smith, Professor A. W. Smith, Mr. E. H. Taylor, Professor E. J. Townsend, Dr. A. L. Underhill, Professor E. B. Van Vleck, Professor C. A. Waldo, Professor C. B. Williams, Professor D. T. Wilson, Professor E. J. Wilczynski, Professor F. S. Woods, President R. S. Woodward, Mr. A. E. Young, Professor J. W. A. Young, Professor Alexander Ziwet.

The following papers were read :

(1) Dr. J. C. MOREHEAD: "The rapid computation of power residues."

(2) Dr. C. H. SISAM : "On the inflectional tangents to a quartic curve."

(3) Dr. C. H. SISAM : "Some loci connected with plane curves, II."

(4) Professor J. B. SHAW : "Standard forms of certain types of Peirce algebras."

(5) Professor G. A. MILLER : "On the multiple holomorphs of a group."

(6) Professor L. W. DOWLING : "On the generation of plane quintics with ordinary double points."

(7) Professor E. J. WILCZYNSKI : "Projective differential geometry of curved surfaces, III."

(8) Dr. CHARLES HASEMAN : "Some boundary problems in the theory of functions."

(9) Dr. A. R. CRATHORNE : "Hilbert's invariant integral in the general isoperimetric problems."

(10) Mr. E. B. ESCOTT: "The converse of Fermat's theorem."

(11) Professor W. C. BRENKE: "Convergence, summability and differentiability of trigonometric series."

(12) Mr. A. W. SCHWEITZER: "On Moore's 'Tactical memoranda.'"

(13) Dr. G. D. BIRKHOFF: "On a certain existence and oscillation theorem."

(14) Professor E. H. MOORE: "Note on a form of general analysis."

(15) Professor L. E. DICKSON: "Reduction of families of quadratic forms in a general field."

(16) Professor L. E. DICKSON: "On commutative linear groups."

(17) Mr. H. E. BUCHANAN: "Note on the convergence of a sequence of functions of a certain type."

(18) Dr. A. L. UNDERHILL: "Note on the calculus of variations."

(19) Professor E. R. HEDRICK: "Note on the structure of point sets" (preliminary communication).

(20) Professor C. N. HASKINS: "Note on the generalized law of the mean."

(21) Professor F. R. MOULTON: "On certain relations, due to tidal friction, in the motion of two mutually attracting bodies."

(22) Professor A. S. HATHAWAY: "Motion of three bodies with fixed center of gravity."

Mr. Buchanan was introduced by Professor Moulton. In the absence of the authors the papers of Dr. Morehead, Dr. Crathorne, and Professor Moore were read by title. Abstracts of the papers follow below; the numbering is the same as that attached to the titles in the above list.

1. Dr. Morehead's paper describes a means of solving easily the congruence $a^{kn}x \equiv b \pmod{m}$ for successive values of k. The solutions x are nth power residues or non-residues, modulo m, according as b is an nth power residue or non-residue. The power residues, modulo m, of all degrees may be obtained by the solution of $\phi(m)$ (the totient function) congruences of the above form. The case $a = 2$ is the most important, because of its simplicity.

2. In this paper Dr. Sisam proves that the inflectional tangents to an arbitrary quartic curve form the complete set of tangents common to a curve of class four and one of class six. The curve of class four in question is the envelope of the lines meeting the quartic in equianharmonic points; the one of class six is the envelope of the lines meeting the quartic in harmonic points.

3. In a paper previously presented* Dr. Sisam discussed the locus of the points of intersection of the tangents to a curve when the lines joining their points of tangency touch a second given curve. In the present paper, Plücker's numbers for these loci are completely determined, several important special cases are considered, and the dual loci are discussed.

4. The following types of Peirce algebras (that is, nonquaternion algebras with no skew units) have previously been considered and the subtypes exhibited:

$(i, i^2, \cdots, i^{r-1})$, deficiency zero, by Benjamin Peirce.
$(i, j^2, \cdots, j^{r-2})$, deficiency unity, by Benjamin Peirce.
$(i, i^2, j, \cdots, j^{r-3})$, deficiency two, by Starkweather.
$(i, j, ij, j^2, \cdots, j^{r-3})$, deficiency two, by Starkweather.
$(i, j, k, k^2, \cdots, k^{r-3})$, deficiency two, by Starkweather.

It is the purpose of Professor Shaw's paper to consider further types and reduce the same to their simplest forms; in particular the types $(i, i^2, \cdots, i^m, j, \cdots, j^{r-m-1})$, $(i, j, \cdots, j^{m-1}, k, \cdots, k^{r-m-1})$, $(i, i^2, \cdots, i^m, j, ij, \cdots, j^{r-m-2})$, $(i, i^2, \cdots, i^m, j, ij, j^2, ij^2, j^3, \cdots, j^{r-m-3})$.

5. In considering the holomorph of an abelian group G of odd order, Burnside proved a theorem under the assumption that G is a characteristic subgroup of its holomorph (Theory of groups of finite order, page 238). Professor Miller proves that this assumption is superfluous, by establishing the theorem that an abelian group of odd order is always a characteristic subgroup of its holomorph. He considers the general question, under what condition an abelian group is a characteristic subgroup of its holomorph. Some of his results are expressed in the following theorems: If the order of an abelian group is not divisible by 8, it is a characteristic subgroup of its holomorph. If an abelian group K of order 2^m contains only one largest in-

* BULLETIN, vol. 13, p. 277.

variant exceeding 2^3 and if its holomorph contains another invariant subgroup K_1 which is similar to K, then K, K_1 will generate a group whose commutator subgroup is of order 2. When K contains only one second largest invariant, its holomorph contains exactly three other invariant subgroups which are similar to K; but its holomorph contains only one other invariant subgroup similar to K when K contains more than one second largest invariant. The holomorph of any abelian group K cannot involve more than four invariant subgroups which are similar to K.

6. Castelnuovo (*Mathematische Annalen*, volume 44), has proved that every plane involution is rational, *i. e.*, that every such involution may be set up by a net of curves. Given, therefore, any involution I_n of order n, it is possible to find a net of curves such that any two curves of the net intersect in a set of points of the involution, and any two sets of points determine a curve of the net. The plane may be considered as containing two sorts of elements, the point sets of I_n and the curves of the net. The geometry in such a plane includes the discussion of all plane curves which can be the loci of singly infinite series of point sets of I_n or the envelopes of singly infinite series of curves of the net.

Professor Dowling considers the I_{6-d} set up by a net of cubics having three collinear points in common and d other points, where $d \leqq 4$. Quintic curves possessing simple double points appear among the involution curves. The converse problem, whether a given quintic, can be considered as an involution curve in an I_{6-d}, is answered in the affirmative.

Other involutions are considered with a view to applying the notion of an involution in the plane to the generation of any algebraic plane curve.

7. In a previous paper Professor Wilczynski has shown that the equation of every non-ruled surface can be developed, in the vicinity of an ordinary point, in the canonical form

$$(1) \qquad z = xy + \tfrac{1}{6}(x^3 + y^3) + \tfrac{1}{24}(Ix^4 + Jy^4) + \cdots,$$

where I, J and all further coefficients are absolute invariants of the surface. No development of this form exists for a ruled surface. It is the purpose of the present paper to find a canonical development for this exceptional case and to discuss it in detail. The canonical form found is

(2) $z = xy + x^3 + x^5 + Ax^4y + Bx^3y^2 +$ terms of the sixth order,

where A and B are absolute invariants. Certain exceptional cases are also discussed in which this new development also breaks down.

The question as to the meaning of the canonical development (2), *i. e.*, primarily as to the geometric significance of the tetrahedron of reference which gives rise to this development, is then taken up and completely answered. A number of new and important results are found on the way. It is found that there exists a single infinity of cubic scrolls which have contact of the fourth order with the given ruled surface at the given point. Each of these scrolls has a nodal straight line with two pinch points upon it. The locus of the nodal lines is the osculating hyperboloid. The locus of the pinch points is a twisted cubic curve upon the osculating hyperboloid.

Among this single infinity of cubic scrolls there is one whose pinch points coincide (a Cayley cubic scroll). Thus there exists one and only one Cayley cubic scroll which has contact of the fourth order with a given ruled surface at a given point. The pinch point, nodal line, and singular tangent plane of this surface are respectively a vertex, an edge, and a face of the canonical tetrahedron. Thereby the tetrahedron is completely determined. For the two edges which meet at the given point of the surface are the two asymptotic tangents of the surface at that point.

There is thus a unique Cayley cubic scroll associated with every point of a ruled surface. The question arises as to the relation between those which belong to points of the same generator. It is found that this relation is as follows: the locus of the pinch points of the osculating Cayley cubic scrolls, which belong to the various points of any generator, is a twisted cubic curve which is situated on the osculating hyperboloid, and which intersects the generator in its flecnodes.

8. In this paper Dr. Haseman first shows how to prove the existence of a regular analytic function of a complex variable $f(z) = u(x, y) + iv(x, y)$ on a multiply connected as well as on a simply connected surface, where the relation

$$a(s)\, u(s) + b(s)\, v(s) + c(s) = 0$$

between the boundary values $u(s)$ and $v(s)$ of the real and imaginary parts respectively of $f(z)$ is given, and where $a(s)$, $b(s)$, $c(s)$ may each have a finite number of finite discontinuities. This is done by choosing a regular analytic function $g(z)$ in the given region such that $f^*(z) = f(z)g(z)$, where $f^*(z)$ may be gotten from the relation $a^*(s)\, u^*(s) + b^*(s)\, v^*(s) + c^*(s) = 0$, and where the coefficients in the relation are all continuous throughout.

A number of special cases arise from this theory. For example, a regular analytic function $f(z)$ may be found, where the real part $u(s)$ is given for certain segments of the bounding curve of the given region and the imaginary part $v(s)$ for the rest of the boundary.

A similar problem may be solved for multiply connected surfaces; for example, we may find $f(z)$ for the region between two concentric circles, where $u(s)$ is given on the outer circle and $v(s)$ on the inner circle.

It is next shown how to find a regular analytic function of a complex argument $f_a(z)$ exterior to a given region and another function of the same character $f_j(z)$ interior to the region, when the relation $f_a[\sigma(s)] = c(s)f_j(s)$ is given between the boundary values $f_a(\sigma)$ and $f_j(s)$ of our functions at the points $\sigma = \sigma(s)$ and s respectively, where $\sigma(s)$ is a regular differentiable function of s and where $c(s)$ may have a finite number of finite discontinuities.

The required functions are found by choosing two other functions $g_a(z)$, $g_j(z)$ of the same characters such that $f_a^*(z) = f_a(z)g_a(z)$ and $f_j^*(z) = f_j(z)g_j(z)$, in which the starred functions are gotten from the relation $f_a^*[\sigma(s)] = c^*(s)f_j^*(s)$, where $c^*(s)$ is everywhere continuous. The theory may be extended to multiply connected surfaces.

Finally the existence is proved of two regular analytic complex functions $f_a(z)$, $f'_a(z)$ exterior to a given region and two others of the same character $f_j(z)$, $f'_j(z)$ inside the given region, where the following relations

$$f_a(\sigma) = c_1(s)f_j(\rho) + c_2(s)f'_j(s), \quad f'_a(\sigma) = c'_1(s)f_j(\rho) + c'_2(s)f'_j(s)$$

are given between the boundary values of $f_a(z)$ and $f'_a(z)$ at the point $\sigma = \sigma(s)$, of $f_j(z)$ at the point $\rho = \rho(s)$ and of $f'_j(z)$ at the point s of the boundary, where σ is a regular differentiable function of ρ, and ρ in turn the same kind of a function of s. The method of determining these functions is to build up two integral equations in f_j and f'_j similar to those found by Hilbert (*Göttinger Nachrichten*, 1905) for a somewhat simpler problem.

9. Dr. Crathorne considers the problem of determining the minimum of

$$J \equiv \int_1^2 F(y_1', \cdots, y_n'; y_1, \cdots, y_n; x)dx = \text{minimum}$$

under the conditions

$$J_i \equiv \int_1^2 G_i(y_1', \cdots, y_n'; y_1, \cdots, y_n; x)dx = l_i \quad (i = 1, 2, \cdots, r).$$

From the $(2n + r)$-parameter set of solutions of the Euler equations of the problem, it is possible to choose an n parameter set satisfying certain conditions analogous to the condition of transversality in the plane problem. The integral

$$J^* \equiv \int_1^x \{F^* + \sum F^*_{P_i}(y_i' - P_i)\}dx \quad (i = 1, 2, \cdots, n),$$

where $F^* = F + \sum \lambda_i G_i$, is the solution of the Hamilton equation of the problem. Each solution of this equation determines a set $P_1, P_2, \cdots, P_n$ such that the solutions of $y_i' = P_i$ form an n-parameter set which is shown to be equivalent to the above n-parameter set of solutions of Euler's equations. A "field" is thus established in which J^* is a point function.

10. The paper by Mr. Escott discusses the converse of the theorem of Fermat. If p is any prime and e any integer not divisible by p, $e^{p-1} - 1 \equiv 0 \pmod{p}$. According to Ball's Mathematical recreations, the Chinese have a theorem that the condition $2^{p-1} - 1 \equiv 0 \pmod{p}$ is a sufficient condition for a prime number. This theorem has been shown to be untrue by a number of persons. In this paper, which is based on one published in the *Messenger of Mathematics* in March, 1907, Mr. Escott gives a method by which composite numbers can be found containing as many factors as desired and which satisfy the above congruence.

11. In this paper Professor Brenke considers the series

$$\sum_1^\infty (a_u \cos ux + b_u \sin ux),$$

and determines sufficient conditions for convergence, summability (based on the definition of Cesàro) and existence of derivatives of the function defined by the series. The results are obtained by transformation by a factor

$$\phi(x) = R(\cos \tfrac{1}{2}bx, \sin \tfrac{1}{2}bx),$$

R denoting a rational integral function of the arguments indicated. Application is made to increasing the rate of convergence of trigonometric and other series, and the sums of certain classes of series are obtained in closed form.

12. In his "Tactical memoranda"* Moore defines systems of k-ads, k-ids, $\underset{\cdot}{k}$-ads, $\underset{\cdot}{k}$-ids. A k-ad may be thought of as any one of a symmetric class of k-ids in k elements and hence may be called a symmetric k-id. A k-id is some one of the sets of $\underset{\cdot}{2}$-ids obtained from the open linear chain $x_1x_2 \;\; x_2x_3 \cdots x_{k-1}x_k$ of 2-ids in k distinct variables, each variable having the range $a_1a_2 \cdots a_k$ independently. Thus a $\underset{\cdot}{k}$-id is what Mr. Schweitzer elsewhere has called an ordered $\underset{\cdot}{k}$-ad. A k-id is simply an open linear chain of 2-ids in k elements. A closed linear chain of 2-ids in k elements is a Moore $S\{2, 1k\}$ but not necessarily conversely.

Between the proper k-idic, and the symmetric k-idic is the alternating k-idic standpoint; the latter is considered chiefly in Mr. Schweitzer's paper. A set of connected k-ids with the index of connection (k, p), $p = 1, 2, \cdots, k$ is defined; every k-id is a set of connected k-ids, index (k, k). For example, the 3-ids *mbc, amc, abm,* have the index (3, 1); the 3-ids *mbn, mnc, amn,* have the index (3, 2), etc. Corresponding to each type of connection, types of open and closed chains of k-ids are defined. A set of alternating k-ids with index of connection $(k, 1)$ is reduced formally to chains with index (2, 1), and these chains are also formed by the suitable superposition of linear chains of 2-ids. Mr. Schweitzer has elsewhere defined an n-dimensional permutation ($n = 1, 2, 3, \cdots$) and shown that every such permutation contains an n-dimensional open chain. It is here shown that every n-dimensional chain is reducible to chains with index (2, 1). Finally, the resolution of an alternating set of k-ids in m elements into chains of alternating k-ids is dis-

* *Amer. Jour. of Math.*, vol. 18, pp. 264–303. The possibility of a connection between the tactical systems here set forth and the above memoir was suggested by Professor Moore. The author takes great pleasure in acknowledging the stimulating influence of Professor Moore and the unusual advantages which have thus been most generously placed at his disposal.

cussed. This leads to infinitudes of finite systems which are important for independence considerations in the author's geometric systems. Particular cases of Moore's $S\{k, l, m\}$ arise frequently and in an interesting synthetic manner.

13. Dr. Birkhoff's paper develops an existence and oscillation theorem for the solutions of a differential equation

$$\frac{d^2u}{dx^2} + q(x, \lambda)u = 0$$

fulfilling self-adjoint boundary conditions at a and b. The function $q(x, \lambda)$ increases with the parameter λ. The methods are similar to those which Professor Bôcher has employed to consider a special case. The paper will be offered to the *Transactions*.

14. The note of Professor Moore will appear in full in an early number of the BULLETIN. The form of analysis in question is that in which one at least of the independent variables enters without direct specification of nature and range of variation.

15. The first paper by Professor Dickson is devoted, in the main, to the reduction, to non-equivalent types, of families of ternary quadratic forms based on two linearly independent forms. The analysis applies to a general field or domain of rationality. The paper has been offered for publication in the *Quarterly Journal of Mathematics*.

16. The paper by Professor Dickson on commutative linear groups is a continuation of the one presented at the last summer meeting (abstract in BULLETIN, November, 1907, pages 61, 62). The complete article has been offered for publication in the *Annals of Mathematics*.

17. In this note Mr. Buchanan states and proves the following theorems: (1) If a sequence of functions, monotonic non-decreasing, converges to a continuous limit function, then the limit function is monotonic non-decreasing and the convergence is uniform. (2) If a sequence of functions $f_n(x)$ $(n = 1, 2, 3, \cdots, n)$ be of finite variation in an interval ab and

convergent to a continuous limit function $f(x)$, also of finite variation in ab, and if at every two points x_1, x_2, $a \leqq x_1 < x_2 \leqq b$, the limit of the variation of $f_n(x)$ is equal to the variation of $f(x)$, then the convergence is uniform in ab.

18. In a paper presented to the Society in April, 1907, Dr. Underhill showed an invariantive normal form of the second variation of

$$J = \int_{t_0}^{t_1} F(x, y, x', y')dt,$$

namely,

$$\delta^2 J = \int_{\alpha_0}^{\alpha_1} \left[\left(\frac{dv}{d\alpha} \right)^2 - \bar{\kappa}_0 \nu^2 \right] d\alpha, \text{ where } \alpha = \int_{t_0}^{t} F(x, y, x', y')dt$$

and where ν and $\bar{\kappa}_0$ are absolute invariants under extended point and parameter transformations connected with the integrand function of J.

The Jacobi equation in this case is

$$\psi(\nu) \equiv - \bar{\kappa}_0 \nu - \frac{d^2\nu}{d\alpha^2} = 0.$$

Let $\nu = \phi(\nu)$ be a solution vanishing for $\alpha = \alpha_0$. In order to draw conclusions as to the next zero of $\nu = \varphi(\alpha)$, use is made of the equation

$$\frac{d^2\nu}{d\alpha^2} + \frac{1}{a^2}\nu = 0,$$

which has as a solution vanishing at $\alpha = \alpha_0$

$$\nu = c \sin\left(\frac{\alpha - \alpha_0}{a}\right),$$

and with the next zero at $\alpha = \alpha_0 + \pi a$.

By means of two theorems on differential equations (cf. Darboux, Théorie des surfaces, III, § 629), by which the solutions of the Jacobi equation are compared with those of the last equation, the following theorems may be at once formulated :

I. For the problem of minimizing J, if $1/\bar{\kappa}_0$ is positive and less than a^2 along the extremal, then the extremal cannot be a minimizing curve in an interval greater than πa.

II. If $1/\bar{\kappa}_0$ is positive and greater than b^2 along the extremal, then the extremal furnishes a weak minimum in an interval at least equal to πb.

III. In case the problem is that of geodesics on a surface, $\bar{\kappa}_0$ is the gaussian curvature and the above theorems reduce to those already stated by Bonnet, *Comptes rendus*, volume 40, page 1311, also volume 40, page 32.

19. In this preliminary paper Professor Hedrick discusses certain consequences of the Lindelöf definition of a point of condensation (see Borel, Leçons, 1905, Chapter I). Many of the results are not new and a detailed statement is withheld pending further investigation along the lines indicated.

20. The paper of Professor Haskins is a geometric interpretation of L' Hôpital's theorem on the generalized law of the mean. It will be offered to the *Annals of Mathematics* for publication.

21. The motion of a system of tidally disturbing bodies is conditioned by the conservation of the moment of momentum with respect to any plane, and by the fact that the total kinetic and potential energy of the system can only decrease. Professor Moulton's paper develops the consequences of these conditions in the various cases which can arise. Applications are made chiefly to the earth-moon system, and to double stars. Among other things it is shown that tidal friction can not have been an important factor in developing the distances between the stars in binary systems.

22. Professor Hathaway considers a reduced problem for arbitrary motion, analogous to Lagrange's reduced problem for gravitational motion. The reduced elements are the three mutual distances and three relative velocities of the bodies (in magnitude only). When these are given, we can determine algebraically the angle between the plane of the bodies and the plane parallel to their relative velocities, the angular velocities of these planes, their inclinations to the axis of moment of momentum, etc. The complete solution of the problem is by linear differential equations in terms of the reduced elements whose complete solutions are arbitrary constant rotations of a particular solution. When one set of these differential equations is

solved, the other set may be reduced to quadratures. If the axis of moment of momentum is fixed, the complete solution is by quadrature, thus paralleling the gravitational problem.

It also appears that Lagrange's investigations in the problem of three bodies are generally independent of the law of gravitation. For example, those motions in which the triangle of the bodies is always similar to itself require that triangle to be either equilateral or with collinear vertices, and the sides to revolve in a fixed plane through the center of gravity. It is shown here that the angular velocity of the triangle varies inversely as the square of a side and so is constant for rigid configurations. Also that no other solution than the above is possible for constantly collinear bodies whose co-line is not a fixed line.

Lagrange's biquadratic equation for ρ holds also for arbitrary motion, and his differential equation for the same in any gravitational motion has the roots of that biquadratic for particular, and not singular, solutions.

H. E. SLAUGHT,
Secretary of the Section.

JOINT MEETINGS OF MATHEMATICIANS AND ENGINEERS AT THE UNIVERSITY OF CHICAGO.

A SERIES of meetings of mathematicians and engineers was held at the University of Chicago, December 30–31, 1907, under the auspices of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY and conjointly with Sections A (mathematics and astronomy) and D (mechanical science and engineering) of the American Association for the Advancement of Science.

The invitation to join in the discussion of the teaching of mathematics to students of engineering had been widely distributed among those engaged in the practice of engineering as well as among professors in technical schools. The attendance was large and representative, including one hundred men especially interested on the mathematical side and fifty on the engineering side. Among the institutions represented were the State Universities of Illinois, Indiana, Iowa, Kansas, Minnesota, Missouri, Nebraska, Ohio, Pennsylvania, Vermont,