# THE SIXTH ANNUAL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE Sixth Annual Meeting of the AMERICAN MATHEMATICAL SOCIETY was held in New York City on Thursday, December 28, 1899. Beside the usual election of officers and the presentation of annual reports, the occasion was especially marked by the presidential address of President R. S. Woodward, entitled "The century's progress in applied mathematics." An invitation to attend the reading of the address had been extended to the American Physical Society, which was present in a body, the total attendance reaching about eighty persons, of whom the Mathematical Society furnished nearly one-half. President H. A. Rowland of the Physical Society was placed in the chair during the joint session, which was extended to include the reading of the paper by Professor Pupin, noted below. The event afforded a pleasant renewal of the cordial relations of the two scientific bodies and a continued assurance of their hearty coöperation.

The following thirty-eight members of the Society were registered as in attendance at the two sessions :

Dr. E. M. Blake, Professor Maxime Bôcher, Dr. W. G. Bullard, Professor J. E. Clark, Dr. A. Cohen, Professor F. N. Cole, Dr. W. S. Dennett, Dr. F. Durell, Professor A. M. Ely, Professor T. S. Fiske, Mr. A. S. Gale, Miss Carrie Hammerslough, Dr. G. W. Hill, Dr. A. A. Himowich, Professor Harold Jacoby, Mr. S. A. Joffe, Professor Pomeroy Ladue, Professor P. A. Lambert, Professor Gustave Legras, Dr. Emory McClintock, Dr. James Maclay, Dr. Isabel Maddison, Professor Mansfield Merriman, Dr. G. A. Miller, Dr. D. A. Murray, Professor G. D. Olds, Mr. J. C. Pfister, Professor James Pierpont, Professor M. I. Pupin, Professor J. K. Rees, Mr. C. H. Rockwell, Dr. Virgil Snyder, Professor Henry Taber, Miss Mary Underhill, Professor J. H. Van Amringe, Professor L. A. Wait, Professor A. G. Webster, Professor R. S. Woodward.

The President of the Society, Professor R. S. Woodward, occupied the chair during the separate sessions of the Society. The Council announced the election of the following persons to membership : Professor William Beebe, Yale University, New Haven, Conn.; Dr. J. V. Collins, State Normal School, Stevens Point, Wis.; Professor A. R. Forsyth, Trinity College, Cambridge, England ; Professor M. W. Haskell, University of California, Berkeley, Cal.; Dr. Emilie N. Martin, Philadelphia, Pa.; Mr. C. A. Noble, University of California, Berkeley, Cal.; Mr. E. B. Wilson, Yale University, New Haven, Conn.; Miss R. G. Wood, New Haven, Conn. Four applications for membership were reported.

Reports were received from the Treasurer, the Librarian, and the Auditing Committee. These reports will be printed in the Annual Register, now in preparation. The Secretary reported that the total number of members of the Society was now 337, including twelve life members. Thirty-six new members joined the Society during the year 1899. The total attendance of members at the meetings during the year was 197, and the number of members attending at least one meeting was 110. The number of papers presented was 106, as against 88 in 1898.

On the occasion of the retirement of Professor Harold Jacoby from the position of Treasurer, appropriate resolutions were adopted expressing recognition of his valuable services, extending with a brief interruption from the founding of the Society.

At the annual election the following officers and members of the Council were chosen :

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| *President,* | Professor R. S. WOODWARD. |
| *First Vice-President,* | Professor E. H. MOORE. |
| *Second Vice-President,* | Professor T. S. FISKE. |
| *Secretary,* | Professor F. N. COLE. |
| *Treasurer,* | Dr. W. S. DENNETT. |
| *Librarian,* | Professor POMEROY LADUE. |

*Committee of Publication,*

Professor F. N. COLE,
Professor ALEXANDER ZIWET,
Professor FRANK MORLEY.

*Members of the Council to serve until December, 1902,*

Professor OSKAR BOLZA,
Professor SIMON NEWCOMB,
Professor L. A. WAIT.

The following papers were presented :

(1) Dr. G. A. MILLER : "On the groups which have the same group of isomorphisms."

(2) Professor MAXIME BÔCHER: "On regular singular points of linear differential equations of the second order whose coefficients are not necessarily analytic."

(3) Dr. VIRGIL SNYDER: "On cyclical quartic surfaces in space of $n$ dimensions."

(4) Dr. VIRGIL SNYDER: "On the geometry of the circle."

(5) Mr. W. B. FITE: "A proof that the commutator subgroup of a group may contain operators which are not commutators."

(6) J. E. CAMPBELL, M.A.: "On the types of linear partial differential equations of the second order (in three independent variables) which are unaltered by the transformations of a continuous group."

(7) Professor L. E. DICKSON: "Proof of the existence of the Galois field of order $p^r$ for every integer $r$ and prime number $p$."

(8) Dr. E. M. BLAKE: "On plane movements whose point loci are of order not greater than four."

(9) Professor R. S. WOODWARD: Presidential address, "The century's progress in applied mathematics."

(10) Professor M. I. PUPIN: "The propagation of electrical waves over non-uniform conductors."

(11) Professor HENRY TABER: "On the singular transformations of groups generated by infinitesimal transformations."

(12) Dr. J. I. HUTCHINSON: "On certain relations among the theta constants."

(13) Professor E. O. LOVETT: "Singular solutions of Monge equations."

Mr. Fite's paper was presented to the Society through Dr. G. A. Miller, and Mr. Campbell's paper through the Secretary. In the absence of the authors, Mr. Fite's paper was read by Dr. Miller, Dr. Hutchinson's by Dr. Virgil Snyder, and those of Mr. Campbell, Professor Dickson, and Professor Lovett were read by title.

The presidential address appeared in the January number of the BULLETIN (pp. 133–163), and has also been published in *Science*, vol. 11 (new series), no. 263, pp. 41–51; no. 264, pp. 81–92. The papers of Professor Dickson, Dr. Snyder, and Professor Taber are contained in the present number of the BULLETIN. Mr Campbell's paper will be published in the *Transactions*. Abstracts of the remaining papers are given below.

The main object of Dr. Miller's paper was the determination of all the possible groups whose group of isomorphisms or whose group of cogredient isomorphisms is the symmetric group of order six. It was shown that these groups are the direct products of abelian groups and an infinite system of groups which contains just one group for every power of two and is thus analogous to the system of Hamiltonian groups whose order is a power of two.

The following theorems were also proved: If a group is generated by two characteristic subgroups that have only identity in common, its group of isomorphisms is the direct product of the groups of isomorphisms of these two characteristic subgroups. The group of isomorphisms of an abelian group $A$ is abelian whenever $A$ is cyclical, and it is non-abelian when $A$ is non-cyclical. The necessary and sufficient condition that a cyclical group of order $n$ is the group of isomorphisms of some group is that $n$ is of the form $p^\alpha(p-1)$, $p$ being an odd prime number. If the group of cogredient isomorphisms $G_1$ of a group $G$ transforms one of its operators $A_1$ of order $p^\alpha$, $p$ being any prime number, into its $k^{\text{th}}$ power and if $k \not\equiv 1 \bmod p$, then at least one operator of order $p^\alpha$ corresponds to $A_1$ in the isomorphism between $G$ and $G_1$; if $k \equiv 1 \bmod p^\gamma$, but $\not\equiv 1 \bmod p^{\gamma+1}$, then at least one operator of order $p^{\alpha+\gamma'}$ $(\gamma' \leqq \gamma)$ corresponds to $A_1$ in the isomorphism between $G$ and $G_1$.

In Professor Bôcher's paper, which has now appeared in the *Transactions* (Volume 1, number 1, pp. 40–52), the questions which the author considered on pp. 280–281 of the last volume of the BULLETIN, are discussed by another method by means of which it is possible to treat the subject much more exhaustively.

In the geometry of the circle, as discussed by Dr. Snyder, the circle is taken for generating element analogous to the sphere in Lie's higher spherical geometry. A linear equation between the five coördinates defines the $\infty^2$ circles which cut a fixed circle at a constant angle. Several problems in construction were easily solved, *e. g.*, to find a circle cutting three given ones at given angles. The elements of differential geometry were given, and the differential equation of all envelopes of circles expressed. In this geometry a quadratic complex gives the bicircular quartics when the point circles form one coördinate complex, and a new system of circular curves of order 24 when the coördinates are unrestricted. Some important properties of both systems of curves were obtained.

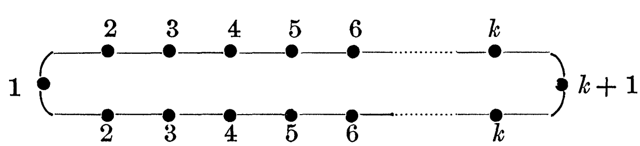
Mr. Fite's paper was in abstract as follows: Let $G$ be a group of order $p^m$, $p$ being a prime number, which has an abelian group of cogredient isomorphisms, generated by four independent operators $P$, $Q$, $R$, $S$; and let $A$, $B$, $C$, $D$ be the operators of $G$ that correspond to $P$, $Q$, $R$, $S$, respectively. The commutator subgroup of $G$ will contain operators which are not commutators of $G$ whenever the following conditions are satisfied:

$$B^{-1}AB = h_1A, \quad C^{-1}AC = h_2A, \quad D^{-1}AD = h_3A,$$
$$C^{-1}BC = h_4B, \quad D^{-1}BD = h_5B, \quad D^{-1}CD = h_6C,$$

where $h_1$, $h_2$, $\cdots$, $h_6$ are independent self-conjugate operators of $G$ different from identity. There is a group of order 1024, obtained by establishing a certain isomorphism for $G_8^4$ written in six distinct systems of elements, which satisfies all of the given conditions. There is also a group of order 256 whose commutator subgroup contains an operator which is not a commutator.

Dr. Blake considered the movements of a rigid body or system one of whose planes slides upon a fixed plane and is further restricted to those movements which cause all points of the system to generate curves of order not greater than four. A list of all known movements of this character is given in Part I. and the problem of a complete enumeration is proposed. The locus of a straight line oblique to the plane of movement may be a scroll of order either not greater than four or greater than four. The author has discussed these scrolls for all known movements of the former kind. They are the movements of the ellipsograph (article by the author in manuscript); movements whose centrodes are congruent conics (*American Journal of Mathematics*, vol. 21, p. 257); the degenerate case of three-bar movement whose centrodes are limaçons; and Roberts's generalization of the conchoid mechanism of Nicomedes. The author's present paper treats the last two cases in Parts II. and III.

In Professor Pupin's paper the conductor is a long wire having coils 1, 2, $\cdots$, $k$, $k+1$ interposed at equidistant points, as in the figure.



An electromotive force of type $Ee^{ipt}$ is impressed at point 1. The problem discussed leads to the integration of the differential equation of a plane wave in an absorbing medium, namely,

$$L\frac{dx^2}{dt^2} + R\frac{dx}{dt} = \frac{1}{C}\frac{d^2x}{ds^2},$$

where $s$ is the distance of any point from point 1, $x$ is the current at that point, $t$ is the time, and $L, R, C$ are the inductance, ohmic resistance, and capacity, respectively, of the wire per unit length.

The integral has to satisfy $k + 2$ boundary conditions, one at each coil. To formulate these, let $x_1, x_s, \cdots, x_{ks}, x_{(k+1)s}$ be the currents at points $1, 2, \cdots, k, k+1$. Also let $L_0, R_0, C_0$, be the inductance, capacity, and resistance, respectively, of any one of the $k + 2$ equal coils. Then at point 1

$$\frac{\delta x_1}{\delta s} = \frac{1}{2}ipCEe^{ipt} - \frac{C}{2}(-p^2L_0 + ipR_0)x_1 = \frac{1}{2}D_0 - \frac{H}{2}x_1;$$

at point $k + 1$.

$$\frac{\delta x_{(k+1)s}}{\delta s} = -\frac{H}{2}x_{(k+1)s};$$

and at any other point $a$

$$\left(\frac{\delta x_{as}}{\delta s}\right)_1 - \left(\frac{\delta x_{as}}{\delta s}\right)_2 = -Hx_a,$$

where $a$ may be any integer from 1 to $k$.

The integral does not seem to be obtainable by ordinary methods. The method of successive eliminations which the author developed in his paper on "Electrical oscillations in a loaded conductor," read at the meeting of the Society, February 25, 1899, succeeds in finding an integral.

Thus the current in the section between the points $a$ and $a + 1$ at a distance $\xi$ from $a$ is given by

$$x_{as+\xi} = \frac{[\frac{1}{2}h_0 + h_0'] \cos\left(\frac{al}{k+1} + \xi\right)\mu - d\cos\left\{\frac{(k+1-a)l}{k+1} - \xi\right\}\mu}{\mu \sin \mu l}$$
$$+ \frac{h_0}{\mu}\left\{x_s \sin\left[\frac{(a-1)l}{k+1} + \xi\right]\mu + x_{2s}\sin\left[\frac{(a-2)l}{k+1} + \xi\right]\mu + \cdots + x_{as}\sin\mu\xi\right\},$$

where

$$h_0 = C(-p^2L_0 + ipR_0), \quad -\mu^2 = C(-p^2L + ipR),$$

$$h_0' = h_0 \left\{ x_s \cos\frac{k\mu l}{k+1} + x_{2s}\cos\frac{(k-1)l\mu}{k+1} + \cdots + x_{ks}\cos\frac{l\mu}{k+1} \right\}$$

The currents $x_1, x_s, \cdots x_{ks}, x_{(k+1)s}$ are determined from the following $k+2$ simultaneous equations:

$$(\sigma_1 + 1)\, x_1 - x_s = \frac{\rho}{2} + \frac{\sigma_1}{2} x_1,$$

$$(\sigma_1 + 2)\, x_s - x_{2s} - x_1 = 0,$$

$$(\sigma_1 + 2)\, x_{2s} - x_{3s} - x_{2s} = 0,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$(\sigma_1 + 2)\, x_{ks} - x_{(k+1)s} - x_{(k-1)s} = 0,$$

$$(\sigma_1 + 1)\, x_{(k+1)s} - x_{ks} = +\frac{\sigma_1}{2} x_{(k+1)s},$$

where

$$\sigma_1 = \frac{h_0 \sin\dfrac{\mu l}{k+1}}{\mu} - 4\sin^2\tfrac{1}{2}\frac{\mu l}{k+1}, \qquad \rho = \frac{\tfrac{1}{2}pcEe^{ipt}\sin\dfrac{\mu l}{k+1}}{\mu}$$

By putting $\sigma_1 = -4\sin^2\psi$, we obtain any current

$$x_{ms} = -\frac{\frac{\rho}{2}\cos 2(k - m + 2)\psi}{\sin 2\psi \sin(2k + 2)\psi}.$$

This disposes of the waves of forced period. The waves of free period can be obtained by putting

$$\sin 2\psi \sin(2k + 2)\psi = 0, \qquad \therefore\ (2k + 2)\psi = \nu\pi,$$

where $\nu$ may be any integer from 0 to $2k + 2$.

The theta constants considered by Dr. Hutchinson are the values of the theta functions with half integer characteristics for zero arguments. Relations among the thetas of two different Göpel systems are considered. From these can be derived the expression of all the theta constants in terms of those belonging to the same Göpel group (or system). This is the direct generalization of the results already well known for the hyperelliptic theta functions (depending upon two arguments).

Professor Lovett's paper, which is intended for publication in the *Transactions*, employs Lie's theory of infinitesimal transformations to construct a method for determining the singular solutions of Monge and Pfaff equations.

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# THE DECEMBER MEETING OF THE CHICAGO SECTION.

THE Sixth Semi-Annual Meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY was held on December 28 and 29, 1899, at the University of Chicago. The following members of the Society were in attendance :

Professor Oskar Bolza, Professor E. W. Davis, Professor Thomas F. Holgate, Dr. Kurt Laves, Professor H. Maschke, Professor John A. Miller, Professor E. H. Moore, Professor Alexander Pell, Professor D. A. Rothrock, Professor G. T. Sellew, Professor E. B. Skinner, Dr. H. E. Slaught, Dr. H. F. Stecker, Professor C. A. Waldo, Dr. J. V. Westfall, Professor H. S. White, Professor Mary F. Winston, Professor J. W. A. Young.

Professor E. H. Moore, Vice-President of the Society, occupied the chair during the first of the four sessions, after which Professor E. W. Davis presided. The Christmas meeting being the regular time for the election of officers of the Section, the Secretary was re-elected and Professors H. B. Newson and C. A. Waldo were elected members of the programme committee. The time and place of the next meeting were fixed for Saturday, April 14, 1900, at Northwestern University, Evanston, Ill.

The following papers were presented :

(1) Mr. R. E. MORITZ : "A generalization of the process of differentiation."

(2) Professor E. D. ROE : "On the transcendental form of the resultant."

(3) Dr. E. J. WILCZYNSKI : "An application of Lie's theory to hydrodynamics."