CORRECTION TO "DESIGN OF OPTIMAL CONTROL FOR A REGRESSION PROBLEM"

By Der-Shin Chang and Chi Song Wong

National Tsing Hua University and University of Windsor

For brevity, we shall try to use the notations in [1].

In Theorem (iii) of [1], Chang generalizes his result for A-optimal models to weighted optimal designs. However his proof is incorrect: let $g = P'f$ (page 1081 of [1]), where P is orthogonal and $\|f_j\|_R^2 \leq L, j = 1, 2, \cdots, n$. Since $f_1, f_2, \cdots, f_n$ are not necessarily orthogonal in $H(R)$, it may not be true that $\|g_i\|^2 \leq L$, a statement that is used in the proof of Theorem 1 (iii) in [1]. The following example shows that Theorem 1 (iii) is not true: Let $n = 2, \lambda_1, \lambda_2 > 0, \lambda_1 \neq \lambda_2$,

$$W = \frac{1}{2}\begin{pmatrix} \lambda_1 + \lambda_2 & \lambda_1 - \lambda_2 \\ \lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 \end{pmatrix}, \qquad P = \frac{1}{2^{1/2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

Then P is orthogonal and $W = PDP'$. Let $j \in \{1, 2\}, f_j = (\lambda_1)^{1/2} \cos \Theta_j \, \phi_1 + (\lambda_2)^{1/2} \sin \Theta_j \, \phi_2$. Then $\|f_j\|_R = 1$. Let a $= \cos(\Theta_1 - \Theta_2)$. Then

$$M(f) = \begin{pmatrix} 1 & a \\ a & 1 \end{pmatrix}$$

and with a suitable choice of (Θ_1, Θ_2), a can be any given number in $(-1, 1)$. So we may choose f such that $a = ((\lambda_1)^{1/2} - (\lambda_2)^{1/2})((\lambda_1)^{1/2} + (\lambda_2)^{1/2})$. By a direct calculation,

$$\text{tr } W\Sigma(f) = ((\lambda_1)^{1/2} + (\lambda_2)^{1/2})^2 2 < \lambda_1 + \lambda_2.$$

So

$$\min_{g \in X} \text{tr } W\Sigma(g) < \lambda_1 + \lambda_2,$$

a contradiction to Chang's conclusion that

$$\min_{g \in X} W\Sigma(g) = \lambda_1 + \lambda_2.$$

In [2], the second author will use a simpler argument to prove that when W is diagonal, Theorem (iii) in [1] holds (in a more general setting).

REFERENCES

[1] Chang, D. S. (1979). Design of optimal control for a regression problem. *Ann. Statist.* **7** 1078–1085.
[2] Wong, C. S. (1980). Linear estimates and optimal control of a regression experiment. Unpublished manuscript.

Department of Mathematics,
National Tsing Hua University,
Hsinchu, Taiwan 300,
Republic of China.

Department of Mathematics,
University of Windsor,
Windsor, Ontario, N9B 3P4,
Canada.