# GRADUATION BY A TRUNCATED NORMAL

By Nathan Keyfitz

Below is a table for finding the constants of a truncated normal by the equation of moments. Karl Pearson* gives such a table for the case in which the data are to be fitted to the "tail" (i.e. less than half) of a normal curve but I do not believe that the formulae for a distribution consisting of more than half of a normal curve have before been tabulated.

The table below was calculated primarily for an investigation being carried out on the duration of unemployment. The Canadian Census of 1931 reported the number of persons losing 1–4, 5–8, ··· 49–52 weeks in the course of the year June 1st, 1930 to June 1st, 1931, by various classifications, (industry, province, age, etc.).

The tendency to report even numbers of months on the part of the enumerated population was evident in the result, and some kind of graduation was necessary for an interpretation. After some experiment a part of a normal curve was settled upon as the simplest and generally most satisfactory representation.

It was found that among the classes of workers in which unemployment is high the curve is more advanced,—i.e. the mode is at a higher number of weeks,—than in the classes where unemployment is low. In many cases, (in most groupings of female workers for instance) where unemployment is relatively very low the modal point of the uncurtailed normal stands at a negative number of weeks,—for these cases the fitting is to a true tail and the tables of the Biometric Laboratory were used.

Details of the results of the investigation will be published shortly in the Unemployment Monograph of the Dominion Bureau of Statistics. Meanwhile, this table will be of use as the complement of Pearson's tabulation which is only suitable for $\psi_1 \geqq .5708$.

*Table for finding the constants of a truncated normal by the equation of moments*

| $x'$ | $\psi_1$ | $\Delta\psi_1$ | $\psi_2$ | $\Delta\psi_2$ |
|---|---|---|---|---|
| 0 | .5708 | −.0180 | 1.2533 | −.0562 |
| .1 | .5528 | −.0183 | 1.1971 | −.0543 |
| .2 | .5345 | −.0188 | 1.1428 | −.0526 |
| .3 | .5157 | −.0190 | 1.0902 | −.0506 |
| .4 | .4967 | −.0193 | 1.0396 | −.0487 |
| .5 | .4774 | −.0195 | .9909 | −.0467 |

* Tables for Statisticians and Biometricians, page 25.

| $\chi'$ | $\psi_1$ | $\Delta\psi_1$ | $\psi_2$ | $\Delta\psi_2$ |
|---|---|---|---|---|
| .6 | .4579 | −.0195 | .9442 | −.0449 |
| .7 | .4384 | −.0196 | .8993 | −.0428 |
| .8 | .4188 | −.0196 | .8565 | −.0409 |
| .9 | .3992 | −.0194 | .8156 | −.0390 |
| 1.0 | .3798 | −.0192 | .7766 | −.0370 |
| 1.1 | .3606 | −.0189 | .7396 | −.0351 |
| 1.2 | .3417 | −.0185 | .7045 | −.0332 |
| 1.3 | .3232 | −.0180 | .6713 | −.0315 |
| 1.4 | .3052 | −.0175 | .6398 | −.0296 |
| 1.5 | .2877 | −.0170 | .6102 | −.0279 |
| 1.6 | .2707 | −.0163 | .5823 | −.0263 |
| 1.7 | .2544 | −.0156 | .5560 | −.0246 |
| 1.8 | .2388 | −.0148 | .5314 | −.0232 |
| 1.9 | .2240 | −.0141 | .5082 | −.0216 |
| 2.0 | .2099 | −.0134 | .4866 | −.0204 |
| 2.1 | .1965 | −.0126 | .4662 | −.0190 |
| 2.2 | .1839 | −.0118 | .4472 | −.0178 |
| 2.3 | .1721 | −.0110 | .4294 | −.0166 |
| 2.4 | .1611 | −.0103 | .4128 | −.0156 |
| 2.5 | .1508 | −.0096 | .3972 | −.0146 |
| 2.6 | .1412 | −.0089 | .3826 | −.0137 |
| 2.7 | .1323 | −.0083 | .3689 | −.0128 |
| 2.8 | .1240 | −.0076 | .3561 | −.0120 |
| 2.9 | .1164 | −.0071 | .3441 | −.0113 |
| 3.0 | .1093 | | .3328 | |
| 3.5 | .0813 | | .2856 | |
| 4.0 | .06246 | | .24999 | |
| 4.5 | .049379 | | .222221 | |
| 5.0 | .0399997 | | .1999999 | |

Let $d$ = distance of centroid of actual distribution from point of truncation.
Let $\Sigma$ = standard deviation of distribution about its mean. Then $\psi_1 = \frac{\Sigma^2}{d^2}$.
Hence corresponding $\chi'$ and $\psi_2$ may be found.
Then $\sigma = d\psi_2$, where $\sigma$ = standard deviation of uncurtailed normal.
And $\chi = \chi'\sigma$, where $\chi$ = origin of uncurtailed normal.
*N.B.* The point of truncation is taken for the origin in the original distribution.

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