ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Monterey Meeting of the Institute, November 14–15, 1958)

1. On Computing Expectations in Sequential Analysis. Fred C. Andrews, University of Oregon, and J. R. Blum, Indiana University.

Consider an arbitrary sequence of random variables $X_1, X_2, \cdots$, to be observed sequentially and a corresponding sequence of statistics $f_1(X_1)$, $f_2(X_1, X_2), \cdots$, $f_j(X_1, \cdots X_j), \cdots$ each of the latter with zero expectations. With m denoting the number of random variables observed, determined by a sequential stopping rule, a necessary and sufficient condition that $E(f_m) = 0$ for all truncated sequential stopping rules is that the sequence $f_1, f_2, \cdots$ be a martingale. Applications of this result is made to expectations of sums and products including a form of the fundamental identity of sequential analysis which is valid for unbounded stopping rules.

2. Exact Nonparametric Tests for Randomized Blocks. John E. Walsh, Systems Development Corporation, Santa Monica, California. (By title)

A class of nonparametric procedures for testing the statistical identity of treatments in randomized block experiments is suggested and discussed. The suggested procedures are squarely based on experimental within-block randomizations, and they may be chosen so as to have special power against particular alternatives. The blocks are assumed to be statistically independent but no assumption is made concerning the dependence within the various blocks. The basic idea is to obtain from each block a statistic that is, under the null hypothesis, symmetrically distributed about zero and then apply a nonparametric test of symmetry about zero. The observational data can be of any quantitative type.

3. On the Determination of Joint Distributions from the Marginal Distributions of Linear Combinations. Thomas S. Ferguson, University of California, Los Angeles.

Let $Z_n = \alpha_n X + \beta_n Y$ where $\gamma_n = \alpha_n/\beta_n$ are all distinct for $n = 1, 2, \cdots$. A sufficient condition that the joint distribution of (X, Y) be determined uniquely by the distributions of the Z_n is that there exist an integer m such that (1) $E \exp \{t \mid Z_m \mid\} < \infty$ for some $t > 0$ and (2) there is a limit point of the γ_n's (possibly $\pm \infty$) other than γ_m. Conversely when the joint distribution has a piece of positive continuous density somewhere, the distributions of a finite number of the Z_n's do not determine the joint distribution. Thus in particular the bivariate normal distribution is determined uniquely by any infinite collection of distinct linear combinations of the variables and by no finite number of them. These results extend to many dimensions, with suitable modifications.

4. Approximation to the Probability Density of Zero—Crossings Intervals of a Gaussian Process. Sylvain Ehrenfeld, New York University. (By title)

Let $x(t)$ be a stationary Gaussian process with a given spectrum $w(f)$, and let $P_O(t)$ be the probability density of the lengths of intervals between successive zeros in this process. In the present paper several approximations to $P_O(t)$ are obtained. This is achieved by the evaluation of multiple integrals whose evaluation are equivalent to finding "volumes" of hyperspherical simplices in n-dimensional space. The above geometrical problem is solved for certain cases ($n = 3, 4$) and given the desired approximations. The problem of finding the volume of a hyperspherical simplex reduces (for $n = 4$) to the problem of finding the 3-dimensional volume of a tetrahedron-like figure (the analogue of a spherical triangle) on the surface of a 4-dimensional sphere. The final part of the paper is concerned with a comparison between the various approximations to a count of zero crossings, taken from a series of ocean pressure records. There also is a discussion of computational problems.

5. The Probability in the Extreme Tail of a Convolution. DAVID BLACKWELL AND J. L. HODGES, JR., University of California, Berkeley. (By title)

Let $X_1, X_2, \cdots$ be independent identically distributed integer-valued random variables, such that 0 is a possible value and the g.c.d. of possible values is 1. Suppose $E(e^{tX_1})$ is finite for some $t > 0$. For any number a with $E(X_1) < a < \sup X_1$ there are a distribution ρ on the values of X_1 and a number m, $0 < m < 1$ such that Prob $\{X_1 + \cdots + X_n = na\} = \pi_n^{**}(1 + 0(n^{-2}))$ as $n \to \infty$ through those values for which na is a possible value of $X_1 + \cdots + X_n$, where $\pi_n^{**} = m^n[1 + ((\mu_4/\mu_2^2) - 3 - (5\mu_3^2/3\mu_2^3))1/8n]/\sigma\sqrt{2\pi n}$, and σ, μ_2, μ_3, μ_4 are the standard deviation and central moments of the ρ distribution. By ignoring certain error terms, an approximation Prob $\{X_1 + \cdots + X_n \geq na\} = c\pi_n^{**}(1 - d/n)(1 + 0(n^{-2}))$ is obtained. It is noted that Prob $\{X_1 + \cdots + X_n = na\} = m^n$ Prob $\{Y_1 + \cdots + Y_n = na\}$, where $Y_1, Y_2, \cdots$ are independent variables with the ρ distribution. Some numerical illustrations of the accuracy of the approximations are given.

6. Asymptotic Methods of Evaluating the Integral from a to ∞ of $f(x)$. WYMAN RICHARDSON, University of North Carolina. (By title)

Three iterative procedures are considered. Procedure A: a transformation is applied carrying ∞ into 0 and a into b. Then the integral, "$F(a)$", is expanded in a Taylor series about 0 or b. Procedure B: $F_2(a) = F_1(a)f(a)/f_1(a)$, where $f_1(a) = -F_1'(a)$. Procedure C: $F(a) = f(a)[v_1(a) + \cdots + v_n(a)] + \int_a^\infty (x)v_n'(x)dx$, where $v_1(a) = -f(a)/f'(a)$ and $v_n(a) = v_1(a)v_{n-1}'(a)$, (Laplace, Winckler). A battery of order theorems are proved, using the Cauchy definition: "order of $f = r$" means "$x^{-r-\epsilon} < f(x) < x^{-r+\epsilon}$ for x large". The order of the "relative error", $F_1(x)/F(x) - 1$, equals that of the "relative frequency error", $f_1(x)/f(x) - 1$. If the order of $f = \infty$, B and C are usually "asymptotic I": the order of the relative error $\to \infty$ as the number of terms increases; and "asymptotic II": (the relative error for $n + 1$ terms)/(the relative error for n terms) $\to 0$. If the order of f is finite, linear combinations of the terms are taken to make the procedure asymptotic II. Useful formulae, such as $F(a) \cong -r[f(a)]^2/f'(a)(r - 1)$, and $-[f(a)]^2/f'(a)[2 - u(a)]$, where $u(x) = f(x)f''(x)/[f'(x)]^2$, are obtained from a few terms of these procedures and applied to statistical distributions. They are asymptotic as $a \to \infty$. Analogous procedures for sums of series and finite tails are considered.

7. Some Properties of Binary Arrays Which Are Generated by Iterated Sequences and Reversals. H. VON GUERARD, Lockheed Aircraft Corporation.

From a sufficiently extended binary event another one is deduced, by assigning 0's to sequences and 1's to reversals. This operation satisfies the group of additivity of a binary (modulation 2) ring algebra (R. D. Bose and R. R. Kuebler, Univ. N.C., Inst. Stat., Mimeo. Ser. 199 (1958)). Repeated application generates an array of binary numbers, whose frequency ratios (= No. of 1's/No. of 0's) (a) converge to 1, (b) repeat periodically, (c) assume relative extrema by geometric progression of iteration order, (d) behave irregularly, i.e. neither (a), (b) nor (c),—if, and only if, the initiating process is (a) randomized, incl. conditioned processes, (b) periodic, (c) transient (single peaks or steps), (d) inductive. To (b): M. Kochen and E. H. Galanter (*Inform. Contr.* 1, 267–288 (1958)) determined the elements of minimal generating sets (mgs) for λ-placed binary numbers (here: periods), from which all others could be deduced by completion or by translation. By iterated addition (mod. 2), any periodic process generates periodically repeated arrays (of periodic processes), whose elements are exactly those of the mgs's, none of them occurring more than once. Since these arrays, in the average, imply more than 1 element, as λ increases, their number becomes progressively small if compared to the number of elements of the mgs's.

(Additional abstract for the Cambridge Meeting of the Institute, August 25–29, 1958)

30. On the Bounds for the Variance of Mann-Whitney Statistic. J. S. RUSTAGI, Michigan State University (By title)

Let $X_1, X_2, \cdots, X_m$ and $Y_1, Y_2, \cdots, Y_n$ be two random samples from strictly increasing continuous cumulative distribution functions (cdf's) $F(x)$ and $G(y)$ respectively. Then the Mann-Whitney statistic U is given by U = number of pairs (X_i, Y_j) such that $Y_j < X_i$, $i = 1, 2, \cdots, m$; $j = 1, 2, \cdots, n$. Let $L(t) = F(G^{-1}(t))$. Then variance of U, $V(U) = mn[(m-1)\int_0^1 (L(t) - kt)^2\, dt + A]$ where A is a constant free of $L(t)$ and $k = (n-1)/(m-1)$. Utilizing the techniques and results of an earlier paper by the author (*Ann. Math. Stat.*, Vol. 28, pp. 309–328), lower and upper bounds for $V(U)$ are determined in terms of $p = P(Y < X) = \int_{-\infty}^{\infty} F(t)\, dG(t) = 1 - \int_0^1 L(t)\, dt$. The problem essentially is that of minimizing and maximizing $\int_0^1 (L(t) - kt)^2\, dt$ over a class of cdf's $L(t)$ defined over $[0, 1]$ such that $\int_0^1 L(t)\, dt = 1 - p$. Lower bounds are also obtained for $V(U)$ under an additional restriction that X is stochastically smaller than Y or $L(t) \geq t$ for $0 \leq t \leq 1$. (Received July 7, 1958, revised November 24, 1958).

NEWS AND NOTICES

Readers are invited to submit to the Secretary of The Institute news items of interest

Personal Items

Frances Campbell Ameniya, formerly chairman of the Department of Mathematics at George Pepperdine College in Los Angeles, California, has been appointed Associate Professor of Mathematics at California Western University in San Diego, California.

R. E. Barlow is now working on a doctorate at Stanford while employed at Sylvania Electronic Defense Laboratory, Mt. View, California, as a mathematical statistician.

Ishu Bangdiwala is on a leave of absence from his position as Head of the Department of the Statistics Section of the Agricultural Experiment Station of the University of Puerto Rico, to accept the position as Assistant Director of Research in the Superior Council on Education, which is the governing board of the University.

Jerome Cornfield, assistant chief of the Biometrics Branch, Division of Re-