NOTES

ON THE UNBIASEDNESS OF YATES' METHOD OF ESTIMATION USING INTERBLOCK INFORMATION[1]

By Franklin A. Graybill and V. Seshadri

Oklahoma State University

In a balanced incomplete block model with blocks and errors random normal variables, Yates has shown that there are two independent unbiased estimates for any treatment contrast. These are referred to as intrablock and interblock estimators. Yates has also given a method for combining these two estimators which depends on the variances (unknown) and has shown how to estimate the variances from an analysis of variance [1]. Since this combined estimator is used quite extensively, it seems desirable to study its properties. Graybill and Weeks [2] have shown that Yates' combined estimator is based on a set of minimal sufficient statistics and have presented an estimator which is unbiased.

The purpose of this note is to show that Yates' estimator, which is based on intrablock and interblock information, is unbiased.

The model and distributional assumptions in this paper are exactly those given in [2], and the same notations are used and will not be repeated here.

In [2] it is shown that Yates' estimator (denoted by $\bar{\tau}_i$) of τ_i is

$$(1) \qquad \begin{aligned} \bar{\tau}_i &= x_i + \gamma(u_i - x_i) && \text{if} \quad \hat{\sigma}_\beta^2 > 0 \\ &= x_i + \lambda t/rk(u_i - x_i) && \text{if} \quad \hat{\sigma}_\beta^2 \leqq 0 \end{aligned}$$

where

$$(2) \quad \gamma = \frac{\dfrac{\lambda^2 t(r-\lambda)}{rk(r-1)}(U-X)'(U-X) + \dfrac{\lambda k}{(r-1)}S^{*2} + \dfrac{\lambda(k-t)}{f(r-1)}S^2}{\dfrac{\lambda^2 t(r-\lambda)}{rk(r-1)}(U-X)'(U-X) + \dfrac{\lambda k}{(r-1)}S^{*2} + \left[\dfrac{\lambda(k-t)}{f(r-1)} + \dfrac{(r-\lambda)}{f}\right]S^2}$$

and where

$$(3) \quad \hat{\sigma}_\beta^2 = 1/t(r-1)[\lambda t(r-\lambda)/rk^2(U-X)'(U-X) + S^{*2} - (b-1)/fS^2]$$

We now define $\phi(\hat{\sigma}_\beta^2)$ such that

$$\begin{aligned} \phi(\hat{\sigma}_\beta^2) &= 0 && \text{if} \quad \hat{\sigma}_\beta^2 > 0 \\ &= 1 && \text{if} \quad \hat{\sigma}_\beta^2 \leqq 0 \end{aligned}$$

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Yates' estimate can now be written as

$$(4)\quad \bar{\tau}_i = [1 - \phi(\hat{\sigma}_\beta^2)][x_i + \gamma(u_i - x_i)] + \phi(\hat{\sigma}_\beta^2)[x_i + (\lambda t/rk)(u_i - x_i)]$$

Clearly (4) is equivalent to (1). Rearranging and simplifying (4) we get

$$\bar{\tau}_i = [x_i + \gamma(u_i - x_i)] + \phi(\hat{\sigma}_\beta^2)[(\lambda t/rk) - \gamma](u_i - x_i)$$

Graybill and Weeks have shown in [2] that $E[x_i + \gamma(u_i - x_i)] = \tau_i$. Therefore in order to show that Yates' estimate is unbiased we need only show that

$$E[\phi(\hat{\sigma}_\beta^2)((\lambda t/rk) - \gamma)(u_i - x_i)] = 0$$

Let $z_i = (u_i - x_i)$ where $i = 1, 2, \cdots, t - 1$. Now $\hat{\sigma}_\beta^2$ is a function of z_i, S^{*2}, and S^2. So let

$$\hat{\sigma}_\beta^2 = g(z_1, z_2, \cdots, z_{t-1}, S^{*2}, S^2).$$

γ is also a function of z_i, S^{*2}, and S^2. Therefore, let

$$\gamma = h(z_1, z_2, \cdots, z_{t-1}, S^{*2}, S^2).$$

Denote the joint density of the $t + 1$ random variables $z_1, z_2, \cdots, z_{t-1}, S^{*2}, S^2$ by $f(z_1, z_2, \cdots, z_{t-1}, S^{*2}, S^2)$. From (2) it is clear that γ is an even function of the z_i and from (3) we see that $\hat{\sigma}_\beta^2$ is also an even function of the z_i. Therefore, $\phi(\hat{\sigma}_\beta^2)$ is an even function of z_i, $(i = 1, 2, \cdots, t - 1)$ and $\phi(\hat{\sigma}_\beta^2)[(\lambda t/rk) - \gamma]$ is also an even function of z_i. Hence $\phi(\hat{\sigma}_\beta^2)[(\lambda t/rk) - \gamma](u_i - x_i)$ is an odd function of z_i. Therefore,

$$E[\phi(\hat{\sigma}_\beta^2)((\lambda t/rk) - \gamma)(u_i - x_i)] = 0,$$

since z_i are independent normal variables with mean zero and are independent of S^2 and S^{*2}. Thus Yates' estimator, which is based on intrablock and interblock information, is unbiased.

REFERENCES

[1] YATES, F. "The recovery of interblock information in balanced incomplete block designs," *Ann. Eugenics*. Vol. 10, (1940), pp. 317–325.

[2] GRAYBILL, F. A., AND D. L. WEEKS, "Combining interblock and intrablock information in balanced incomplete blocks," *Ann. Math. Stat.* Vol. 30, (1959), pp. 799–805.

ON THE BLOCK STRUCTURE OF CERTAIN PBIB DESIGNS WITH TWO ASSOCIATE CLASSES HAVING TRIANGULAR AND L_2 ASSOCIATION SCHEMES

BY DAMARAJU RAGHAVARAO

University of Bombay

0. Summary. The PBIB designs [2] with two associate classes are classified in [3] as 1. Group Divisible, 2. Simple, 3. Triangular, 4. Latin Square type with i

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