ABSTRACTS OF PAPERS

(Abstract of a paper presented at the Western Regional meeting, Berkeley, California, July 19–21, 1965. Additional abstracts appeared in earlier issues.)

28. On robust procedures. Joseph L. Gastwirth, The Johns Hopkins University. (Invited)

This paper discusses a procedure for finding robust estimators of the location parameter of symmetric unimodal distributions. The estimators are based on robust rank tests and the methods used are applicable to more general situations. To every density function there corresponds an asymptotically most powerful rank test (amprt). For a set $(f_1, \cdots, f_n)$ of density functions the maximum robust rank test, R, maximizes the minimum limiting Pitman efficiency for samples from any one of the f_i, $i = 1, \cdots, n$. This maximum test can be used to construct estimators according to the proposal of Hodges and Lehman (*Ann. Math. Statist.* **34** 598–611); the robust test, R, generates another estimator T in the following manner. If the test based on R is the amprt for samples from a density function $g(x - \theta)$, then the estimator T will be the best linear unbiased estimate (blue) of the location parameter for samples from $g(x)$. Explicit formulas for the maximum robust test and the corresponding estimate T are derived. The relationship of the present paper to the work of Huber (*Ann. Math. Statist.* **35** 73–102) is discussed and it is shown that the blue corresponding to his least favorable distribution is the trimmed mean.

(Abstract of a paper presented at the Annual meeting, Philadelphia, Pennsylvania, September 8–11, 1965. Additional abstracts appeared in earlier issues.)

55. Some results on lower bounds for ASN. Gordon Simons, University of Minnesota. (By title)

Consider a test of hypotheses where we are to choose among K densities f_i, $i = 1, \cdots, K$; $K = 2, 3, \cdots$. Let $A = (\alpha_{ij})$ be the $K \times K$ error matrix with $\alpha_{ij} = P$ (accepting $f_j \mid f_i$ valid) and let f_0 be another density. Let E_0 denote expectation under f_0 and $L_i = \log (f_0/f_i)$, $i = 1, \cdots, K$. Using modest assumptions, two lower bounds for ASN, under f_0, have been shown. Bound 1: $\inf_{\{b_j\}} \max_{1 \leqq i \leqq K} [\sum_{j=1, K} b_j \log (b_j/\alpha_{ij})]/E_0 L_i$, where $\sum_{j=1, K} b_j = 1$ and $b_j > 0$. Bound 2: $[(T^2 - R \log S)^{\frac{1}{2}} - T]^2/R^2$, where R, S, and T are defined for subsets (size two or larger) of the first K positive integers. Let D be such a subset with v members; let $C = \{C_i \mid i \varepsilon D\}$ be a set of v real numbers for which $\sum_{i \varepsilon D} C_i = 0$ and $\sum_{i \varepsilon D} |C_i| = 1$; and let $\Phi(D)$ be the permutations of D with typical member φ (a v-dimensional vector). Then, $R(D) = \max_{i \varepsilon D} E_0 L_i$, $S(D) = \sum_{j=1, K} \min_{i \varepsilon D} \alpha_{ij}$, $T(D) = \inf_C \tau(C)/2v(v - 2)!$, where $\tau(C) = \sum_{\varphi \varepsilon \Phi(D)} \tau_\varphi(C)$, and where $\tau_\varphi(C) = E_0[\sum_{i \varepsilon D} C_{\varphi_i}(L_i - E_0 L_i)]^2$. The latter bound is really several bounds when $K > 2$ (one for each subset D). For $K = 2$, the latter bound is the same as one given by W. Hoeffding [*Amer. Math. Soc.* **31** (1960) 352–368] and the former is a rather disguised form of one also given by him [*Amer. Math. Soc.* **24** (1953) 127–130]. Subsequent reports will compare these bounds with some sequential tests.

(Abstracts of papers not connected with any meeting of the Institute.)

1. Multivariate two sample normal scores test for shift (preliminary report). Gouri Kanta Bhattacharyya, University of California, Berkeley.

For testing the identity of two multivariate populations against shift alternatives an asymptotically distribution-free test is proposed using the normal scores statistics coordinatewise and constructing a quadratic form in them parallel to the Wilcoxon type test proposed by Sen and Chatterjee. By an extension of the Chernoff-Savage theorem to multivariate distributions, the asymptotic distribution of the test statistic is found to be central χ^2 under the null hypothesis and noncentral χ^2 under a sequence of near translation alternatives. The Pitman efficiency of the test relative to Hotelling's T^2 test is obtained and compared to the efficiency of the Sen-Chatterjee test in various situations. Unlike the latter test, when the underlying distribution is multivariate normal the efficiency of our test is 1 irrespective of the direction in which the hypothesis is approached. In the family of all continuous multivariate distributions with pairwise independent coordinates the lower bound of the efficiency of our test is seen to be 1. The test is robust against gross errors when the underlying distributions are nondegenerate. However, the fact that in the univariate case the normal scores test has always efficiency at least 1 no longer holds in multidimensions. When the underlying distribution tends to degenerate to lower dimensions the efficiency of our test may be arbitrarily low.

2. On a minimal essentially complete class of experiments. S. EHRENFELD, New York University.

There are available linear experiments of n uncorrelated random variables $Y(x)$ with $v[Y(x)] = \sigma^2$ and $E[Y(x)] = x'\theta$. The k dimensional vectors x are to be chosen and θ is a parameter vector. The problem is how to choose the x's when they must lie in set A. In the asymptotic case, $\mathcal{E}[A]$ is the class of experiments with $x \varepsilon A$ and in the finite sample case, $\mathcal{E}_N[A]$ is the class where $x \varepsilon A$ and the sample size $n \leqq N$. A partial ordering of experiments $e_1 \geqq e_2$ is defined by $v_{e_1}[t'\hat{\theta}] \leqq v_{e_2}[t'\hat{\theta}]$ for all t, where $v_e[t'\hat{\theta}]$ is the variance of the least square estimate of $t'\theta$ using experiment e. The class of experiments $\mathcal{E}_1$ is essentially complete with respect to $\mathcal{E}_2$ when for any $e_2 \varepsilon \mathcal{E}_2$ there is $e_1 \varepsilon \mathcal{E}_1$ with $e_1 \geqq e_2$. $\mathcal{E}_1$ is minimal when no proper subset of $\mathcal{E}_1$ is essentially complete. Let $E(A)$ denote the extreme points of A. It is shown, that if A is compact then, in the asymptotic case, $\mathcal{E}[E(A)]$ is minimal essentially complete with respect to $\mathcal{E}[A]$. In the finite sample case, it is shown, by an example and some general theorems, that $\mathcal{E}_N[E(A)]$ is not necessarily even essentially complete with respect to $\mathcal{E}_N[A]$.

3. Maximal sets of anti-commuting skew-symmetric matrices. JOSEPH PUTTER, Volcani Institute of Agricultural Research, Rehovot.

The problem of constructing maximal-size sets of (non-vanishing) anti-commuting skew-symmetric matrices (SASM) of given order arises in connection with orthonormal bases of a regression error space (see abstract in *Ann. Math. Statist.* **36** 1083.) In this paper, the problem is solved using known solutions of a related problem arising in quantum theory. It is shown that the maximum size of a SASM of order n is $[n/2] + [n/4] + [n/8]$, and that all maximal SASM's of order n are orthogonally equivalent to a specified set consisting of matrices of ranks 2, 4 and 8.

4. Contributions to the theory of non-normality—I. Univariate case (preliminary report). K. SUBRAHMANIAM, University of Western Ontario.

We have studied here the distribution of the quadratic form $(\mathbf{x} - \boldsymbol{\mu})'A(\mathbf{x} - \boldsymbol{\mu})$, where A is a positive definite real symmetric matrix, $\mu_1, \mu_2, \cdots, \mu_n$ are n arbitrary real numbers and $x_1, x_2, \cdots, x_n$ are n independent observations from the non-normal population (Gayen, *Biometrika* **36**) $f(x) = [1 - (\lambda_3/6)D^3 + (\lambda_4/24)D^4 + (\lambda_3^2/72)D^6]\phi(x)$, where λ_3 and λ_4 are the standard measures of skewness and kurtosis, respectively. Here D^i stands for the

jth partial derivative with respect to x of the standard normal density $\phi(x)$. We have obtained some new integrals associated with the Hermite polynomials which could be of interest to mathematical statisticians. We have also obtained as a particular case the distribution of the random variable $(\chi^*)^2 = \sum_{i=1}^n x_i^2$. The density function of $(\chi^*)^2$ can be expressed as a linear function of the density functions of central chi-square variables. One sidelight of the work is the result that the determinant $|I - 2it \quad A|$ can be expressed in the form $\sum_{j=0}^n (-2it)^j \mathcal{G}(j)$, where $\mathcal{G}(j)$ is the *generalized trace* of order j of the matrix A. The latter quantity is defined as *the sum of the* $\binom{n}{j}$ *determinants of the submatrices of order* $(j \times j)$ *that can be formed from A by taking j diagonal elements at a time*. In particular we have $\mathcal{G}(1)$ = trace of the matrix A, $\mathcal{G}(n)$ = determinant of A, and by definition $\mathcal{G}(0) = 1$.

5. One-sample normal scores test null distribution and power (preliminary report). RORY THOMPSON, Z. GOVINDARAJULU and K. A. DOKSUM, Massachusetts Institute of Technology, Case Institute of Technology and University of California, Berkeley.

Some percentage points of the normal scores test for symmetry are extended to sample size 20 and compared with normal and Edgeworth approximations. The powers of the normal scores (N), Wilcoxon (W) sign (S), and t (T) tests are estimated from monte carlo trials for shifts of symmetrical hypothesized populations which are normal, logistic, double exponential, and uniform. Perhaps the most striking feature of the power results is the relative goodness of the t-test, and perhaps also of the normal scores test. The t-test power seemed to come out significantly lower than the best of the four considered only for small double exponential shift; the normal scores test only for uniform shift. In most cases, the powers of T, N, and W were virtually indistinguishable, but there were some tendencies. From best to worst, with a comma marking a definite break, the tests seemed to stack somewhat as follows: for normal shift TNW, S; for logistic shift TWN, S; for small double exponential shift SWNT, for large WTNS; for uniform shift T, NW, S.