ABSTRACTS

(Abstracts of papers presented at the Annual meeting, New Brunswick, New Jersey, August 30–September 2, 1966. Additional abstracts appeared in earlier issues.)

50. Multidimensional partially balanced designs. DONALD A. ANDERSON, University of Nebraska. (By title)

The construction and analysis of three and four dimensional designs have been discussed by Potoff [*Technometrics* (1962)]. J. N. Srivastava [*Sankhyā* (1964)] has defined a multidimensional partially balanced, MDPB, association scheme and has introduced a class of multidimensional partially balanced designs. A general procedure for the analysis of MDPB designs is given by Srivastava; however, the construction of such designs was not studied. Consider the following 6 sets, for a cube take S_1, the set of edges; S_2, the set of verticies; S_3, the set of faces; S_4, the set of lines joining the midpoints of diagonally opposite edges; S_5, the set of diagonals; and S_6, the set of lines joining the midpoints of opposite faces. Let C be the class of $m = \sum_{k=1}^{6} m_k$ sets consisting of the above 6 sets where the kth set is included in the class m_k times. A MDPB association scheme has been defined on C and several three and four dimensional designs have been constructed which involve a very small fraction of a full replication. MDPB association schemes which are extensions of ordinary triangular and cyclic schemes have been defined for other classes of sets and MDPB designs corresponding to these schemes have been constructed. (Received 18 July 1966.)

51. On the non-existence of some partially balanced arrays with 2 symbols (preliminary report). DHARAM VIR CHOPRA, University of Nebraska. (By title)

A partially balanced array of strength d, m constraints, N assemblies, 2 symbols [see Chakravarty "Fractional replications in asymmetrical factorial designs and partially balanced arrays," *Sankhyā* **17** (1956)] is an $(m \times N)$ matrix T with elements 0 and 1, with the following property: Let T^* be any d-rowed submatrix of T and let $\mathbf{v}_1 = (j_{11}, j_{12}, \cdots, j_{1d})'$, $\mathbf{v}_2 = (j_{21}, \cdots, j_{2d})'$ be any two column vectors of T^* where the j's $= 0$ or 1 and $\mathbf{v}_1$ is obtainable from $\mathbf{v}_2$ by permuting its elements. Then $\lambda(\mathbf{v}_1) = \lambda(\mathbf{v}_2)$ where $\lambda(\mathbf{v}_i)$ is the number of times $\mathbf{v}_i$ occurs as a column in T^*. Let μ_i be the number of times each distinct column vector of weight i appears in T^*, and put $\boldsymbol{\mu}' = (\mu_0, \mu_1, \mu_2, \cdots, \mu_d)$. In the paper "Optimal balanced 2^m fractional factorial designs" (to be published in S. N. Roy memorial volume, Univ. of North Carolina), J. N. Srivastava shows that an array S with parameters $m = 7, N = 44, d = 4$ and $\boldsymbol{\mu}' = (4, 3, 2, 3, 4)$, considered as a fractional design for 2^7 factorial, minimizes the trace of the covariance matrix of the estimates (among the class of all PB array with $N = 44$). He obtains S by taking all 7-place column vectors, with weights 0, 2, 5 and 7. In this paper the array S has been shown to be unique. As a consequence of this the non-existence of arrays with parameters $N = 44$, $m = 8$, $d = 4$ and values of $\boldsymbol{\mu}$ equal to (4, 3, 2, 3, 4), (3, 3, 2, 3, 5), (1, 3, 3, 3, 1), (5, 2, 3, 2, 5), (7, 2, 3, 2, 3) and (6, 2, 2, 4, 2) has been established. (Received 18 July 1966.)

52. Sequential selection of the best of k population. JOHN DEELY, Purdue University.

Let $\pi_1, \pi_2, \cdots, \pi_k$ be k populations and let $w = (\theta_1, \theta_2, \cdots \theta_k)$ be the vector of unknown parameters. We assume the ordering to be random permutation so that even if the entries of w were known, the exact location of the largest one would still be unknown. We observe sequentially the random variables $Y_1, Y_2, \cdots$ until we stop; always stopping at Y_k. Y_i is assumed to have a density $f(y \mid \theta_i)$ having a monotone likelihood ratio and mean θ_i. Using results of Chow and Robbins [*Z. Wahrscheinlichkits theorie* (1963)], maximin stopping rules are obtained over the set $\Omega_d = \{w: \theta_i \geqq d, i = 1, 2, \cdots, k\}$ for a payoff function which is increasing in each of its arguments. Further if there exists an *a priori* distribution G_i on θ_i, then a Bayes stopping rule is obtained. The case $G_i = G, i = 1, 2, \cdots, k$, is also treated. In each of the cases mentioned above, the specific rules are obtained for two payoff functions: $\tilde{X}_n = y_n - cn$ and $X_n = \max(y_1, y_2, \cdots, y_n) - cn$. An example using the normal density is also given. (Received 22 July 1966.)

53. An optimum πPS sampling. T. V. Hanurav, Michigan State University.

In sampling a finite population, to estimate the total $Y = \sum Y_i$ we encounter the problem of π PS of sampling i.e. of selecting a sample such that the inclusion probability π_i for the ith unit equals νP_i where ν is a positive constant and P_i is the proportionate size measure of an auxiliary variable. When ν is an integer n, we need to select the sample such that every sample contains just n distinct units. (This, together with the corresponding Horvitz and Thompson estimator $\hat{Y}$ of Y is "optimum" in a well-defined sense.) For $n = 2$ and for general values of P_i's earlier the author gave a solution (to appear in *J. Roy. Statist. Soc.*) which incidentally furnishes a stable non-negative unbiased estimator of $V(\hat{Y})$. Here we extend this procedure to higher values of n, retaining the above advantages. Without loss of generality, let $P_i \leqq P_{i+1}$, and consider the case $P_{N-n+1} = P_N$. Let $Q_{i1}(\mathbf{P}) = Q_{i1}^{(n)}(\mathbf{P})$ be defined, for $K = 1, 2, \cdots, n-1$, recursively by

$$Q_{i1}^{(K+1)}(\mathbf{P}) = P_i\{Q_1^{(K)}(\mathbf{P}) - (K-1)Q_{i1}^{(K)}(\mathbf{P})\}$$

where $Q_1^{(K)}(\mathbf{P}) = \sum_i Q_{i1}^{(K)}(\mathbf{P})$ and $Q_{i1}^{(1)}(\mathbf{P}) = P_i$. Let $Q_{i,t+1} = Q_{i,t+1}(\mathbf{P}) = Q_{i1}(\mathbf{Q}_t)$ and $Q_t = \sum_i Q_{it}$, where $\mathbf{Q}_t = (Q_{1t}(\mathbf{P}), Q_{2t}(\mathbf{P}), \cdots, Q_{Nt}(\mathbf{P}))$. Finally, let P_{it}'s be defined recursively by $P_{it} = (P_{i,t-1} - Q_{i,t-1})/(1 - Q_{t-1})$ for $1 \leqq t < \infty$, with $P_{i1} = P_i$. Sampling is done thus: Select a pps sample of size n, with replacement, with probability P_{i1} for the ith unit. If all the selected units are distinct, accept the sample; otherwise reject the sample and select in the second step, a pps sample with the probabilities P_{i22}'s. Accept the sample if all the units are distinct and otherwise reject and thus proceed using P_{it}'s as the probabilities in the tth step. This sequential sampling terminates, with probability 1, after a finite number of steps and the convergence is rapid. The π_i's are exactly equal to nP_i's, the joint inclusion probabilities π_{ij}'s are easily calculated; the Yates and Grundy estimator of $V(\hat{Y})$ turns out to be a non-negative and stable one. The case $P_{N-n+1} < P_N$ can be dealt with by a modification similar to the one given for the case $n = 2$. The computations are easily mechanizable. (Received 19 July 1966.)

54. A Pitman-type close estimator for the parameter of a Poisson distribution (preliminary report). James M. Maynard and Bryant Chow, Rutgers-The State University.

Pitman [*Proc. Camb. Phil. Soc.* **33** (1937), 212–222] defined "closeness" for an estimator thus: If X_1 and X_2 are (consistent) estimators for a parameter θ, then X_1 is a "closer" estimator for θ than X_2 if, for all θ, $\Pr\{|X_2 - \theta| < |X_1 - \theta|\} < \frac{1}{2}$. A "closest" estimator is one that is closer than any other estimator. Except for some results of Landau [*Univ. of Pittsburgh Bull.* **43** (1947), 143–150], all work on closest estimators has been restricted to continuous distributions. This paper presents the initial results of a study of closest estimators for parameters of discrete distributions. A simple "close" estimator for the parameter, λ, of a Poisson distribution is derived. This estimator (1) is a closer estimator of

λ than the maximum likelihood estimator, $\hat{\lambda}$, for all but very small values of λ; (2) has, to order $o(n^{-4})$ as $n \to \infty$, a smaller mean-squared error than $\hat{\lambda}$; (3) approaches $\hat{\lambda}$ as $n \to \infty$. A Monte-Carlo study has been carried out to demonstrate these results and to investigate further properties of this estimator. (Received 18 July 1966.)

55. A multiple decision procedure for ranking the means of normal populations: dependent sample case. K. M. LAL SAXENA, University of Nebraska. (By title)

The problem of ranking the means of k normal populations is considered when the samples from each population are dependent samples. Let X_{ijl} denote the lth observation on the jth experimental unit from the ith population, $i = 1, \cdots, k; j = 1, \cdots, m; l = 1, \cdots, n$. Suppose X_{ijl}'s are jointly normally distributed with $EX_{ijl} = \theta_i$ for all j and l, cov $(X_{ijl}, X_{ijl'}) = \sigma_{ll'}$ (known) for all i and j, and cov $(X_{ijl}, X_{i'j'l'}) = 0$ for $i \neq i'$ or $j \neq j'$ and all l and l'. Let the cost of one experimental unit be C_1 and the cost of taking one observation on any unit be C_2. Then the total cost is $mk(C_1 + C_2 n)$. An indifference zone procedure based on minimum variance unbiased estimators of θ_i is obtained to select the t largest means with the probability of correct selection larger than a preassigned value. The procedure also gives the optimum choices for m and n to minimize the total cost. Examples with covariance structure for (i) stationary autoregressive scheme of the first order, (ii) two step moving average, and (iii) equally correlated observations have been considered. (Received 18 July 1966.)

56. On the extensions of the Gauss-Markov theorem to complex multivariate linear models (preliminary report). J. N. SRIVASTAVA, University of Nebraska.

This concerns the problem of linear estimation under two general multivariate linear models M_1 and M_2. [For earlier references, see author's paper, "On Certain Generalisations of MANOVA", *Proc. Internat. Symp. Mult. Anal.* Dayton, (1965).] Under M_1, we have (i) $u(\geqq 1)$ sets of experimental units, the ith set S_i containing N_i units, (ii) p responses $(R_1, \cdots, R_p)$, out of which exactly those with subscripts $l_{i1}, \cdots, l_{ip_i}$ $(1 \leqq p_i \leqq p)$ are measured on each unit in S_i. Assumptions on means and variances include: (a) $E(Y_i) = A_i[\xi_1, \cdots, \xi_p] D_i$; Var $(\mathbf{Y}'_{ij}) = D_i' \Sigma D_i$, $(j = 1, \cdots, N_i; i = 1, \cdots, u)$; and (b) observations made on distinct units are independent. Here Y_i $(N_i \times p_i)$ is the observation matrix from the set S_i; A_i is a known (design) matrix; $\xi_r (m \times 1)$ are unknown parameters wrt R_r, $\mathbf{Y}'_{ij}$ is the jth row of Y_i, and corresponds to the p_i observations on the jth unit in S_i; $\Sigma(p \times p)$ is an unknown dispersion matrix; and $D_i(p \times p_i)$ contains 1 in the cells $(l_{i\beta}, \beta)$, $(\beta = 1, \cdots, p)$, and zero elsewhere. Assumptions under M_2 are: (a) $E(y_{lr}, \cdots, y_{Nr}) = \xi_r' A_r'$; (b) Var $(y_{jl}, \cdots, y_{jp}) = \Sigma$; (c) Cov $(y_{jr}, y_{j'r'}) = 0$, if $j \neq j'$. Here y_{jr} $(j = 1, \cdots, N; r = 1, \cdots, p)$ denotes the observed value for the rth response on the jth unit; and A_r, $\xi_r'(1 \times m_r)$ and Σ are defined analogous to M_1. In this paper, the main problem is of obtaining best linear unbiased estimates (BLUE) for $\theta (= \sum_{j=1}^{p} \mathbf{c}_j' \xi_j$, where $\mathbf{c}_j$ are given vectors). For both models M_1 and M_2, a necessary and sufficient condition (on the A_i's and $\mathbf{c}_j$'s) for the existence (and uniqueness) of a BLUE of θ is obtained, without any further assumption on the nature of the joint distribution of the observations. Various consequences and applications of these results are also studied. (Received 18 July 1966.)

57. A note on unbiased minimum variance estimates. K. TAKEUCHI, Texas A and M University. (By title)

Let $t(x) \varepsilon L_2$ space with respect to the weight function $\phi(x_i \theta)$. Our problem is to find the unbiased minimum variance estimator at parameter value $\theta = \theta_0$ which minimizes the functional $J[t] = \int_{R(x)} \{t(x) - g(\theta_0)\}^2 \phi(x_i\theta_0)\, d_x$ subject to the side conditions $\int_{R(x)} t(x)\phi(x_i\theta_j)\, d_x = g(\theta_j)$ $(j = 1, 2, \cdots, m)$. At first we consider the case where m is finite, then the case where m is infinite. Applying the direct method (Ritz method), we prove the existence of an unbiased minimum variance estimator $\bar{t}(x)$ when m is finite. Applying the theory of Lagrange multipliers, we derive the result: $\sum_{j=1}^{m} \lambda(\theta_0, \theta_j) \int_{R(x)} \phi(x_i\theta_j)h(x)\, d_x = \int_{R(x)} t(x)h(x)\phi(x_i\theta_0)\, dx$, where $h(x) \varepsilon L_2$ is arbitrary and the $\lambda(\theta_0, \theta_j)$ are Lagrange multipliers. In terms of linear operator T it can be expressed as follows: $T \int_{R(x)} \phi(x_i\theta)h(x)\, dx = \int_{R(x)} \bar{t}(x)h(x)\phi(x_i\theta_0)\, dx$. This corresponds to Stein's general operator [*Ann. Math. Statist.* **21** (1950) 406–415]. For the above relation hold for arbitrary $h(x) \varepsilon L_2$, it is necessary that $\bar{t}(x)\phi(x_i\theta_0) = \sum_{j=1}^{m} \lambda(\theta_0, \theta_j)\phi(x_i\theta_j)$ hold. Next we consider the case where m is infinite. In the similar way as above, we prove the existence of an unbiased minimum variance estimator $t^*(x)$. Then we establish the conditions under which $t^*(x)$ be expressed in terms of linear operator. (Received 19 August 1966.)

58. A covariance-like analysis for incorporating extra observations. GEORGE ZYSKIND, Iowa State University.

The analysis of covariance technique has long been used in treating problems in which some of the data are missing. In the case where a set of data has been analysed by least squares and an additional set of data becomes available a covariance-like analysis employing the computations already performed may be carried out in the following fashion. Let $y_1 = X_1\beta + e_1$ be the model for the observations for which a least squares fit has been obtained and let the model for the additional observations be $y_2 = X_2\beta + r_2$. Then the fit to the required model of interest

$$y' = (y_1, y_2) = (X_1, X_2)\beta + (e_1, r_2)$$
$$= X\beta + e$$

may be obtained by noting that this model is equivalent to the one given by $y' = (X_1, X_2)\beta + (\phi, I)\delta + e$ under the restriction that $\delta = \phi$. It is easy to verify, however, that if $\tilde{\beta}$ is any vector such that $X_1'X_1\tilde{\beta} = X_1'y_1$ then the best fit to the last preceding model under no restrictions is $(X_1\tilde{\beta}, y_2) = (X_1, X_2)\tilde{\beta} + (\phi, I)(y_2 - X_2\tilde{\beta})$. Adjustments in the estimators due to the condition $\delta = 0$ may be easily performed and thus a best fit obtained to the desired entire model $y = X\beta + e$. Convenient expressions for best estimators and their variances and for sums of squares appropriate to various tests have thus been worked out by use of restrictions in the known parametrically augmented analysis of covariance-like model $y = (X_1, X_2)\beta + (\phi, I)\delta + e$ for which least squares solutions in the case of no restrictions are available. (Received 22 July 1966.)

(Abstracts of papers presented at the European Regional meeting, London, England, September 5–10, 1966. Additional abstracts appeared in earlier issues.)

12. A distribution-free test for linearity of a regression curve. H. DALGAS CHRISTIANSEN, Polytechnical University of Denmark.

Let $(c_1, X_1), \cdots, (c_N, X_N)$ be stochastically independent observations, and let $X_i - g(c_i)$ have a symmetric, continuous distribution with median 0. For each triple (X_i, X_j, X_k), $c_i < c_j < c_k$, put $d = (c_k - c_j)/(c_k - c_i)$ and $p = P\{X_j < dX_i + (1 - d)X_k\}$. Then $p = \frac{1}{2}$ under the hypothesis $H_0: g(c)$ is linear in c, while $p > \frac{1}{2}$ for $H_1: g(c)$ is convex in c. From the N observations, $r \leqq N/3$ stochastically independent triples can be formed. Let Z be the number of such triples for which $X_j < dX_i + (1 - d)X_k$. Then Z is distributed binomially (r, p), and a test rejecting H_0 for $Z > z_{1-\alpha}$, Bin $(r, .5, z_{1-\alpha}) = 1 - \alpha$, is exact, unbiased against alternatives H_1, and invariant under a suitable group of transformations. The test can be generalized to situations with several observations for each c, to families of regression curves, and to classes of nonsymmetric continuous distributions. The power depends on the selection of triples. For the alternative $g(c) = c^2$, $F = N(0, \sigma)$ calculations have yielded small-sample efficiencies from 0.6 to above 1 versus an F-test of variation not explained by a regression line. (Received 21 July 1966.)

13. **Some aspects of a restricted random walk** (preliminary report). SRI GOPAL MOHANTY, McMaster University and University of Bonn. (By title)

In this paper a restricted random walk has been considered, where a particle starting from the origin, moves at any stage either one unit to the right or μ (a non-negative integer) units to the left of its position, such that it reaches $a - \mu b (a \geqq b)$ in $a + b$ steps. The distributions of the number of times the particle crosses the origin and the number of times it reaches the origin have been obtained. (Received 18 July 1966.)

14. **Multiparameter consistency and Bayesian indifference specifications** (preliminary report). MELVIN R. NOVICK, Educational Testing Service, Princeton.

The multi-parameter consistency requirement introduced by Novick and Hall [*J. Amer. Statist. Assoc.* **60** (1965) 1104–1117] is applied to multi-parameter normal regression and multinomial models. The requirement is that the *a posteriori* conditional distribution of parameters based on an initial indifference specification be consistent with the corresponding statements obtained when the indifference specification is made on the corresponding reduced parameter model. For the multinormal regression and multinomial models studied and within the usual natural conjugate classes it is shown that only the indifference specifications obtained by Novick and Hall satisfy this multi-parameter consistency requirement. An application of this requirement to a Bayesian model for the computerized administration of psychological tests is discussed. (Received 15 July 1966.)

15. **A general Gauss-Markoff theorem in the case of any non-negative covariance matrix of observations.** GEORGE ZYSKIND and FRANK B. MARTIN, Iowa State University.

Aitken (1934) has shown in the case of a full rank linear model $y = X\beta + e$, with nonsingular covariance matrix V, known up to a constant factor, that the best unbiased linear estimator (blue) of the components of the vector parameter β is given by the unique solution to the system of equations $X'V^{-1}X\beta = X'V^{-1}y$. The case when the matrix V is singular occurs under some circumstances, particularly under a finite additive randomization model, or, from one point of view, when parameters are subject to linear constraints. It is easy to verify that the analogous equations $X'V^+X\beta = X'V^+y$, where V^+ is the Moore-Penrose unique generalized inverse V^+ of V, do not abstract all the information in the data regarding the point estimation of β. We demonstrate the construction of a class of conditional inverses of V such that if $V^\sim$ is any member of the class then $\lambda'\beta$ is estimable if and only if it is expressible as a linear combination, say, $\delta'X'V^\sim X\beta$, of the left-hand sides of the normal-type equations $X'V^\sim X\beta = X'V^\sim y$, and that when $\lambda'\beta$ is so expressible its blue, analogously to the case of the simple Gauss-Markoff theorem, is given by $\delta'X'V^\sim y = \lambda'\hat{\beta}$, where $\hat{\beta}$ is any solution to these normal-type equations. (Received 25 July 1966.)

(Abstracts of papers to be presented at the Eastern Regional meeting, Atlanta, Georgia, April 3–5, 1967. Additional abstracts will appear in future issues.)

1. On a class of rank order tests for independence in multivariate distributions. MADAN L. PURI and PRANAB K. SEN, Courant Institute of Mathematical Sciences, New York University and University of North Carolina.

In this paper we offer nonparametric competitors of some classical tests of multidimensional independence considered by Wilks [*Ann. Math. Statistics* (1935)], Daly [*Ann. Math. Statistics* (1960)] and, Wald and Brookner [*Ann. Math. Statistics* (1961)]. In this context, the problems of testing (i) the mutual independence *of* q subsets of the totality of p variates ($p \geqq q \geqq 2$), and (ii) pair wise independence of p variates are considered, all against appropriate classes of stochastic dependence alternatives. Some strictly distribution free permutation tests are also developed here, and their asymptotic properties are studied with the aid of a theorem on the asymptotic distribution of a class of rank order statistics to the case of more than two variates. (Received 18 July 1966.)

2. Asymptotically most powerful rank order test for grouped data. P. K. SEN, University of North Carolina.

The present paper is concerned with the extensions of the findings of Hájek [*Ann. Math. Statist.* **33** (1962), 1124–1147] on asymptotically most powerful rank order tests to grouped data where the underlying distributions are essentially continuous but the observable random variables correspond to a finite or countable set of contiguous class intervals. In this context, the two sample problem for grouped data is considered and various efficiency results are also studied. (Received 1 August 1966.)

(Abstract of a paper to be presented at the Western Regional meeting, Missoula, Montana, June 15–17, 1967. Additional abstracts will appear in future issues.)

1. Asymptotic normalities under contiguity in a dependence case. K. L. MEHRA, University of Alberta.

Let $X_{\nu i} = (X_{\nu i1}, X_{\nu i2}, \cdots, X_{\nu ik_i})$, $i = 1, 2, \cdots, n_\nu$, with $n_\nu \to \infty$ as $\nu \to \infty$, be a sequence of n_ν independent random vectors where $x_{\nu ij} = \alpha + \beta C_{\nu ij} + \sigma Y_{\nu ij}$, with $-\infty < \alpha, \beta, C_{\nu ij} < \infty$ and $\sigma > 0$ as constants, and $y_{\nu i} = (y_{\nu i1}, \cdots, y_{\nu ik_i})$ are n_ν independent rv's, distributed for each i according to a distribution $F_i(x)$ which is continuous and jointly symmetric in the arguments and satisfies the condition: For any subset $A_\nu = \{i\} \subset \{1, 2, \cdots, n_\nu\}$, the marginal distribution of any m components of $y_{\nu i}$ with $m \leqq \min_{i \varepsilon A_\nu} k_i$ is the same for all $i \varepsilon A_\nu$. Let $N_\nu = \sum_{i=1}^{n_\nu} k_i$, $\{\xi_{\nu k} = k = 1, 2, \cdots, N_\nu\}$ be a double sequence of real numbers and consider the statistics of the type $S_\nu = \sum_{i=1}^{n_\nu} \sum_{j=1}^{k_i} d_{\nu ij} \xi_{\nu R_{\nu ij}}$, where $R_{\nu ij}$ is the rank of $X_{\nu ij}$ in a combined ranking of the N_ν random components $X_{\nu ij}$, and $d_{\nu ij}$ are constants satisfying $\sum_j d_{\nu ij} = 0$. The purpose of this paper is to prove, under a set of sufficient conditions on the df's F_i and the sequences $\{C_{\nu ij}\}$, $\{d_{\nu ij}\}$ and $\{\xi_{\nu k}\}$, the asymptotic normality of the sums S_ν. Our results extend the results of Hájek (*Ann. Math. Statist.* **32** 509–523 and **33** 1124–1147), both for $\beta = 0$ and $\beta \neq 0$, to the case when the components $X_{\nu ij}$ follow the above pattern of dependence. An application of the main theorem useful in nonparametric theory is also considered. (Received 6 September 1966.)

(Abstracts of papers not connected with any meeting of the Institute.)

1. Nonparametric tests for trend in multivariate time series (preliminary report). G. K. BHATTACHARYYA and JEROME KLOTZ, University of Wisconsin.

A multivariate time series consists of vector observations $\mathbf{X}_i = (X_i^{(1)}, X_i^{(2)}, \cdots, X_i^{(p)})$ taken at a sequence of equally spaced times $i = 1, 2, \cdots, n$. Denoting by $F_i(\mathbf{x})$ the unknown

p-variate continuous cdf of $\mathbf{X}_i$ we define $\{F_i(\mathbf{x}), i = 1, 2, \cdots\}$ to possess trend if for at least one $\alpha = 1, 2, \cdots, p$, the sequence of αth marginal cdf $\{F_i^{(\alpha)}(x), i = 1, 2, \cdots\}$ is stochastically ordered in either direction. To test the hypothesis $H_0: F_1(\mathbf{x}) = F_2(\mathbf{x}) = \cdots = F_n(\mathbf{x})$ against such trend alternatives an asymptotically distribution free rank test is proposed using the test statistic $L_n = n\mathbf{T}_n\hat{\Gamma}_n^{-1}\mathbf{T}_n'$ where $\mathbf{T}_n = (T_{1n}, T_{2n}, \cdots, T_{pn})$, $T_{\alpha n} = 12\sum_{i=1}^n (i - (n+1)/2)R_i^{(\alpha)}/(n^3 - n)$, $R_i^{(\alpha)}$ is the rank of $X_i^{(\alpha)}$ among $X_1^{(\alpha)}, X_2^{(\alpha)}, \cdots, X_n^{(\alpha)}$; $\gamma_{\alpha\beta} = 12\sum_{i=1}^n (R_i^{(\alpha)} - (n+1)/2)R_i^{(\beta)}/(n^3 - n)$ and the $p \times p$ matrix $\hat{\Gamma}_n$ is $(\gamma_{\alpha\beta})$ or I_p according as $(\gamma_{\alpha\beta})$ is nonsingular or singular. L_n is asymptotically distributed as χ_p^2 and the test which rejects for large values of L_n is shown to be consistent against a broad class of trend alternatives. For linear trend in multivariate normal population the classical test is UMP invariant and has the same distribution as Hotelling's T^2. The Pitman efficiency $e(\boldsymbol{\theta})$ of our test relative to the classical test corresponding to a sequence of local linear trend alternatives $F_{in}(\mathbf{x}) = F(\mathbf{x} - \boldsymbol{\alpha} - \boldsymbol{\theta}_n i)$, where $n^{3/2}\boldsymbol{\theta}_n \to \boldsymbol{\theta} \neq \mathbf{0}$ as $n \to \infty$, is found to be the cube root of the Pitman efficiency of the multivariate Wilcoxon test relative to Hotelling's T^2 (cf. Bickel *Amer. Math. Soc.* **36** 160–173). Mann's test for univariate trend is also extended to the multivariate case and its asymptotic efficiency relative to L_n is found to be 1. For $p = 1$ and F normal, $e(\boldsymbol{\theta})$ has the unique value .98. For $p = 2$ and bivariate normal F $.953 < e(\boldsymbol{\theta}) < .985$. An application of the procedure to lake freezing and thawing dates is being considered. (Received 15 September 1966.)

2. A decision theoretic notion of asymptotic efficiency (preliminary report). PETER J. BICKEL, University of California, Berkeley.

Efficiency of one sequence of decision procedures with respect to another is defined in terms of the limit of sample sizes required to reach equal risks for a given point θ. We show that under regularity conditions if the structure of the problem is left unchanged the efficiency if defined is independent of the loss function. In particular, in univariate estimation problems where the decision procedures are competing asymptotically normal estimates the efficiency is the ratio of asymptotic variances. Bayes procedures corresponding to different priors under suitable conditions have efficiency 1 with respect to each other as well as with respect to maximum likelihood procedures. The relation between the efficiency of related tests, estimates and finite multiple decision procedures is being examined. (Received 12 September 1966.)

3. A short table of the generalized hypergeometric distribution. A. M. MATHAI and R. K. SAXENA, McGill University.

In a previous paper 'On a generalized hypergeometric distribution' (to appear, *Metrika* **11** No. 3) the authors have introduced and studied a general probability distribution from which almost all the classical continuous distributions come as special cases. In this article it is shown that the same distribution can be used as a generating function for almost all the classical discrete distributions by interchanging the roles of the parameters and the variables. It is shown that a particular family of distributions defined by this probability functions is complete. A short table of this generalized hypergeometric distribution is also given. (Received 18 July 1966.)

4. The admissible linear estimates of the changing mean. CHANDAN K. MUSTAFI, Columbia University.

Let $\mathbf{X}' = (x_1 x_2 \cdots x_n)$ be n observations whose means are changing randomly according to the model suggested by Chernoff and Zacks [*Ann. Math. Statist.* **35** (1964) 999–1018]. For any two $n \times 1$ vectors $\mathbf{a}$ and $\mathbf{b}$ let $Q(\mathbf{a}, \mathbf{b}) = \mathbf{a}'\,\mathbf{V}^{-1}\,\mathbf{b}$ where the definition of the matrix

V is given in the same reference. We consider the problem of estimating the current mean by a linear function of the observations with respect to squared error loss. It is shown (i) any linear function $\mathbf{C}'\mathbf{X} = \sum_{i=1}^{n} c_i x_i$ is admissible if and only if $0 < c_n \leqq (1 + \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} v^{ij})^{-1}$ and $c_i = c_n \sum_{j=1}^{n-1} v^{ij}$ $(i = 1, 2, \cdots, n-1)$. Suppose x_i contains an additive component βt_i $(i = 1, 2, \cdots, n)$ where β is an unknown real constant and t_i's are known. Let $Q(\mathbf{t}, \mathbf{e}) \neq 0$ where $\mathbf{t}' = (t_1, t_2, \cdots, t_n)$ and $\mathbf{e}' = (11 \cdots 1)$. It is shown that (ii) (a) any estimate $\mathbf{C}'\mathbf{X}$ which is *not* of the form $a_0 Q(\mathbf{e}, \mathbf{X}) + a_1 Q(\mathbf{t}, \mathbf{X})$ for some real a_0 and a_1 is *inadmissible*; (b) if $\mathbf{C}'\mathbf{X} = a_0 Q(\mathbf{e}, \mathbf{X}) + a_1 Q(\mathbf{t}, \mathbf{X})$ but $(a_0 Q(\mathbf{e}, \mathbf{e}) + a_1 Q(\mathbf{e}, \mathbf{t}))^2 + (Q(\mathbf{t}, \mathbf{t}) Q(\mathbf{e}, \mathbf{e}) - Q^2(\mathbf{e}, \mathbf{t})) a_1^2 > 1 + Q^2(\mathbf{e}, \mathbf{t}) (Q(\mathbf{e}, \mathbf{e}) Q(\mathbf{t}, \mathbf{t}) - Q^2(\mathbf{e}, \mathbf{t}))^{-1}$ it is *inadmissible*; (c) if $\mathbf{C}'\mathbf{X} = a_0 Q(\mathbf{e}, \mathbf{X}) + a_1 Q(\mathbf{t}, \mathbf{X})$ and $0 \leqq -a_1 Q^{-1}(\mathbf{e}, \mathbf{t})(Q(\mathbf{t}, \mathbf{t}) Q(\mathbf{e}, \mathbf{e}) - Q^2(\mathbf{e}, t)) \leqq a_0 Q(\mathbf{e}, \mathbf{e}) + a_1 Q(\mathbf{e}, \mathbf{t}) \leqq 1$ it is *admissible*. (Received 18 July 1966.)

5. Binomial and hypergeometric group-testing. MILTON SOBEL, University of Minnesota.

In group-testing each of N units has to be classified as defective or satisfactory. Suppose the number of defectives D is known and each test on x units tells us whether all x are satisfactory or at least one defective is present. The problem is to find a procedure that minimizes the expected number of tests for classifying N units. A procedure R_1 is defined by recursion formulas. For $D = 1$ it is shown to be a known optimal procedure for any N. For $D = 2$ it is described explicitly and shown to be optimal for an infinite sequence of N-values. A subclass of "nonmixing" (or "nested") procedures is defined and procedure R_1 for $D = 2$ is shown to be optimal in this subclass for any N. (Received 29 July 1966.)

6. On a class of partially orthogonal 3^n and $2^m \times 3^n$ fractional factorial designs (preliminary report). J. N. SRIVASTAVA, Colorado State University.

In this body of work, we consider $2^m 3^n (m \geqq 0)$ fractional designs of resolution V. It is shown (by actual construction) that for $m = 0$, and any fixed n, there exists a non-singular (2, 0) symmetric (implying orthogonal estimates for linear and quadratic effects) partially balanced array T with N assemblies if and only if $N \geqq 2n^2 + 1$. (Note: T non-singular means that it is of resolution V). Also, non-singular arrays T with parameters $\theta = (m, n, N, t)$ are constructed with the following values of θ: (0, 3, 21, 1.449), (0, 4, 45, 1.133), (0, 5, 63, 2.004), (0, 6, 90, 1.778), (0, 7, 117, *tt*), (0, 8, 153, *tt*); (5, 1, 36, 1.209), (8, 1, 72, 2.987), (3, 2, 42, 0.696), (4, 2, 48, 0.831), (5, 2, 54, 1.411), (8, 2, 108, 1.413), (1, 3, 30, 1.728), (2, 3, 36, 1.222), (3, 3, 54, 1.210), (4, 3, 64, 2.341), (6, 3, 112, 3.288), (8, 3, 144, 4.008), (1, 5, 81, 1.131), (2, 5, 92, 3.099), (3, 5, 104, 6.068), (1, 6, 108, 1.990), (2, 6, 126, 3.042), and (3, 6, 162, 1.788). Here t denotes the trace of the covariance matrix of the estimates, and *tt* means that the numerical value of the corresponding t is not yet calculated, though the non-singularity of the design is established. The definition of the effects is the (non-normalized) analytical one, if $m \neq 0$, and the usual geometric one otherwise. The above fractions (except the case $m = 4$, $n = 3$) are all (2, 0) symmetric wrt 3-level factors, and/or (1, 0) symmetric wrt 2-level ones, and therefore the vector of estimates is generally divisible in 2 or 4 or more parts of not too different sizes, such that any two parts are mutually orthogonal. The designs in this paper involve a reduction in N or t or both over the corresponding ones known to the author. (Received 15 September 1966.)

7. Some contributions to the theory of non-normality—III (preliminary report). K. SUBRAHMANIAM, University of Manitoba.

In this paper we consider the distributions of the ratios of quadratic forms. Some examples referring to the analysis of variance are also considered. The results obtained are based on papers by Gurland [*Ann. Math. Statist.* **19** (1948) 228–237 and *Sankhyā* **17** (1956) 37–50] and generalize this results to the non-normality of the form introduced by Gayen (*Biometrika* **36** 353–369). We have also given an expansion ror $|I - 2i(t_1A + t_2B)|$ in powers of t_1 and t_2 as

$$|I - 2i(t_1A + t_2B)| = \sum_{k=0}^{p} \sum_{r=0}^{p} \{(-2it_1)^r(-2it_2)^{k-r}G_{AB}(r, k - r)\}.$$

Where A and B are square matrices of order p and where $G_{AB}(r, k - r)$ is a *mixed generalized trace* of order $(r, k - r)$ with respect to the matrices A and B. We define this quantity as follows: Let A and B be any two square matrices of order p. Consider the square sub-matrices or orders $k(\leqq p)$ that can be formed from A by taking k diagonal elements at a time. There are $\binom{p}{k}$ such submatrices that can be so formed. Now replace in each one of these sub-matrices any j rows by the corresponding elements of the matrix B. We have a *mixed matrix* of order $(i, k - i)$. There are in all $\binom{p}{i,k-i}$ such *mixed matrices* of order $(i, k - i)$. The sum of the determinants of all such *mixed matrices* is defined as a *mixed generalized trace* of order $(i, k - i)$ with respect to A and B. (Received 23 August 1966.)

8. The behavior of some tests for ordered alternatives under interior slippage.

HANS K. URY, San Francisco Medical Center, University of California.

For testing the standard null hypothesis against ordered alternatives in a one-way analysis of variance with k samples, Bartholomew [*Biometrika.* (1959) 36–48 and (1961) 325–332] proposed some test statistics for the case of underlying normal distributions, and Chacko [*Ann. Math. Statist.* **34** (1963) 945–956] gave the corresponding nonparametric test. It is shown that under "interior slippage" (i.e., when one population other than the first or kth is larger or smaller than the others), the probability that these tests will reject the null hypothesis simultaneously in favor of both the alternatives of upward and downward ordering goes to 1 in the limit, as the sample sizes grow sufficiently large, regardless of the significance level. The same result holds for interior slippage of several populations, provided the amounts of upward and downward slippage, weighted by sample sizes, are unequal. Therefore, if a significant result is obtained when testing against upward ordering (say), it would appear advisable to carry out a subsequent test against downward ordering. Since a nonparametric test against trend by Terpstra [*Indag. Math.* (1952), 327–333] is shown to behave somewhat more "normally" in slippage situations, it is suggested that this faster test might well be preferable for small k. On the other hand, the Bartholomew and Chacko tests, applied at level $\frac{1}{2}\alpha$ against *both* ordered alternatives, provide consistent level α tests against interior slippage. (Received 29 August 1966.)

9. Generalized maximum likelihood estimators in certain cases.

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Let $X_1, \cdots, X_n$ be independently distributed with the common density (Lebesgue measure) $f(x; \theta)$, where $f(x; \theta) = 0$ if $x < \theta$ or $x > B(\theta)$, $f(\theta+; \theta) = g(\theta) > 0$, $f(B(\theta)-; \theta) = h(\theta) > 0$. Under reasonable regularity conditions the authors [*Teor. Verojatnost i Primenen* **11** (1966), 68–93] obtained asymptotically efficient estimators of θ except when $g(\theta) - h(\theta)B'(\theta) = 0$. The authors now prove that any convex linear combination (with constant coefficients) of $(w_n - r/2n)$ and $(B^{-1}(v_n) + r/2n)$ is an asymptotically efficient estimator. Here $w_n = \min_i X_i$, $v_n = \max_i X_i$, and r is the arbitrary positive constant used in the definition of efficiency as in the authors' paper cited. The proof subsumes the cases already studied; similar results are obtained for cases when the density is positive in several disjoint intervals. (Received 25 July 1966.)