# Output-Feedback Stabilization Control of Systems with Random Switchings and State Jumps 

Wei Qian, ${ }^{1}$ Shen Cong, ${ }^{2}$ and Zheng Zheng ${ }^{1}$<br>${ }^{1}$ School of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo 454000, China<br>${ }^{2}$ Department of Mechanical and Electrical Engineering, Heilongjiang University, Harbin 150080, China<br>Correspondence should be addressed to Wei Qian; qwei@hpu.edu.cn

Received 18 March 2014; Revised 5 May 2014; Accepted 6 May 2014; Published 19 May 2014
Academic Editor: Peng Shi
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#### Abstract

The work is concerned with output-feedback stabilization control problem for a class of systems with random switchings and state jumps. The switching signal is supposed to obey Poisson distribution. Firstly, based on the asymptotical property of the distribution of switching points, we derive some sufficient conditions to guarantee the closed-loop system to be almost surely exponentially stable. Then, we pose a parametrization approach to convert the construction conditions of the output-feedback control into a family of matrix inequalities. Finally, a simulation example is given to demonstrate the effectiveness of our method.


## 1. Introduction

It is one of the fundamental ways to represent the uncertainties in modeling practical systems in terms of random processes, since the stochastic analysis theory provides an unified framework to account for the uncertainties and to study their influences. In particular, it is conventional to describe the environment noise by Wiener processes; see, for example, $[1,2]$ and the references therein. Since the last decade, considerable attentions have been devoted to investigate the effect of environment noise on stability. The researchers gain a comprehensive insight into the influences of the statistic properties of the sample path of Wiener processes. For example, as pointed out by Deng et al. in series of their works, the noise can be used to stabilize a given unstable system or to make a system more stable even when it is already stable; see, for example, [3] and the references therein.

In this paper, we consider to use certain randomness to represent the switching signal which results in the abrupt changes in parameters and states of a dynamical system. In the stochastic framework to deal with such systems, it is a common assumption that the time evolution of switching signal is determined by Markovian transition matrix; see, for example, [4-11]. From this perspective, roughly speaking,
the stability relies on two aspects, namely, the embedded Poisson distribution and the embedded discrete-time Markov chain; please refer to [12]. Indeed, the embedded Poisson distribution represents the density of switching points, while the embedded Markov chain indicates the likelihood of a subsystem to be activated at a switching point. In this sense, focusing on the aspect of how the varying rate of switching signal influences stability, we might purely suppose the time evolution of switching signal to obey Poisson distribution; see, for example, [13-16]. A merit of supposing switching signal to obey the Poisson distribution lies in that it naturally implies that all the switching points are uniformly distributed; see [1]. In contrast, in the deterministic framework to deal with switching signal, the property of uniform distribution of switching signal must be imposed in terms of average dwelltime technique for that it is essential for deriving stability conditions.

By means of Poisson processes, we actually have a counter for the switching points that are located within a certain interval of time. In other words, we can indicate the density of the switching points in terms of Poisson exponent. Therefore, by using the statistic property of the distribution of switching points, we are able to establish the constraint conditions on Poisson exponent so as to guarantee the system to be stable almost surely. Motivated by the observation, the goal
of this paper is to design output-feedback controller to stabilize the systems with abrupt changes in parameters and states, which are triggered by the switching signal obeying Poisson distribution. After establishing stability conditions for the closed-loop system, we construct the controllers via a parametrization approach so that we can check their existence in terms of a family of matrix inequalities. We will demonstrate the effectiveness of the method via a numerical example.

## 2. Problem Formulation

Consider the system that is composed of the following linear subsystems:

$$
\begin{gather*}
\dot{x}(t)=A_{i} x(t)+B_{i} u(t) \\
y(t)=C_{i} x(t), \tag{1}
\end{gather*}
$$

where $x(t) \in \mathbf{R}^{n}, u(t) \in \mathbf{R}^{p}$, and $y(t) \in \mathbf{R}^{q}$ are the state vector, control input, and measured output, respectively. The matrices $A_{i}, B_{i}, C_{i}$ are of appropriate dimensions. The system under consideration is generated by the right-continuous switching signal $\sigma(t)$, which takes value from the index set $\mathfrak{F}:=\{1, \ldots, N\}$ to orchestrate among the subsystems in (1). Synchronously, at a switching point, there is a state jump. The overall system then is described as follows :

$$
\begin{gather*}
\dot{x}(t)=A(\sigma(t)) x(t)+B(\sigma(t)) u(t)  \tag{2}\\
x\left(t^{+}\right)=H\left(\sigma(t), \sigma\left(t^{+}\right)\right) x(t)+G\left(\sigma(t), \sigma\left(t^{+}\right)\right) u(t) \tag{3}
\end{gather*}
$$

$$
\begin{equation*}
y(t)=C(\sigma(t)) x(t), \quad t \geq 0 \tag{4}
\end{equation*}
$$

Alternatively, (3) can be rewritten as

$$
\begin{align*}
\Delta x(t)= & {\left[H\left(\sigma(t)=i, \sigma\left(t^{+}\right)=j\right) x(t)-x(t)\right] } \\
& +\left[G\left(\sigma(t)=i, \sigma\left(t^{+}\right)=j\right) u(t)-u(t)\right]  \tag{5}\\
= & {\left[H_{i j} x(t)-x(t)\right]+\left[G_{i j} u(t)-u(t)\right], }
\end{align*}
$$

which indicates the impulse effect that occurs at the switching point of $\sigma(t)$ being from $i$ to $j$. Therefore, the state-transition of such a system is jointly determined by the continuous dynamics in (2) and the discontinuous dynamics in (3). The former is intended to indicate the abrupt changes in parameters, while the latter corresponds to state jumps.

The dynamical behavior of the system under investigation in strong way relies on the switching mechanism. One way to interpret the switching mechanism is to characterize the time evolution of switching signal. To this end, we expand it into the following sequential form:

$$
\begin{equation*}
\left\{\left(\sigma\left(t_{0}\right), t_{0}=0\right),\left(\sigma\left(t_{1}\right), t_{1}\right), \ldots,\left(\sigma\left(t_{k}\right), t_{k}\right), \ldots\right\} \tag{6}
\end{equation*}
$$

It means that the $\sigma\left(t_{k}\right)$ th subsystem is activated during the interval $\left[t_{k}, t_{k+1}\right)$. We now suppose the switching signal in (6) to obey Poisson distribution. In order to put the argument on a firm footing, we define the random switching signal
on a complete probability space $\{\Omega, \mathscr{F}, \mathscr{P}\}$. Let $\mathscr{E}$ denote the mathematical expectation operator. Thus, given $\Delta>0$, the Poisson distribution requires the length between two successive switching points to satisfy

$$
\begin{equation*}
\mathscr{P}\left\{t_{k+1}-t_{k} \geq \Delta\right\}=e^{-\lambda \Delta}, \quad k=0,1,2, \ldots \tag{7}
\end{equation*}
$$

where $\lambda>0$ is referred to as Poisson exponent. Equivalently, it reads that

$$
\begin{equation*}
\mathscr{P}\{\sigma(t+\Delta) \neq \sigma(t)\}=1-e^{-\lambda \Delta} \tag{8}
\end{equation*}
$$

In this way, we define a statistic property for the sequence of switching points $t_{0}<t_{1}<\cdots<t_{k}<\cdots$, which tends to infinity. Actually, the Poisson exponent $\lambda$ defines the mean value of the number of the switching points distributed within the time interval of unit length. Therefore, by means of Poisson exponent, we can make a sense of the varying rate of a switching signal. Accordingly, we categorize switching signals in such a way that we write $\delta_{\lambda}$ for the collection of all the switching signals obeying Poisson distribution exactly with the exponent $\lambda$.

We aim at designing output-feedback control to stabilize the overall system with respect to certain classes $\mathcal{S}_{\lambda}$. For this purpose, we construct the following full-order outputfeedback compensator:

$$
\begin{gather*}
\dot{\hat{x}}(t)=\widehat{A}_{i} \widehat{x}(t)+\widehat{B}_{i} y(t)  \tag{9}\\
u(t)=\widehat{C}_{i} \widehat{x}(t) \tag{10}
\end{gather*}
$$

for each subsystem and

$$
\begin{equation*}
\widehat{x}\left(t^{+}\right)=\widehat{H}\left(\sigma(t), \sigma\left(t^{+}\right)\right) \widehat{x}(t)+\widehat{G}\left(\sigma(t), \sigma\left(t^{+}\right)\right) y(t) \tag{11}
\end{equation*}
$$

corresponding to the state jump. The matrices $\widehat{A}_{i}, \widehat{B}_{i}, \widehat{C}_{i}$ and $\widehat{G}_{i j}=\widehat{G}\left(\sigma(t)=i, \sigma\left(t^{+}\right)=j\right), \widehat{H}_{i j}=\widehat{H}\left(\sigma(t)=i, \sigma\left(t^{+}\right)=j\right)$ of appropriate dimensions are left to be solved.

Therefore, with the controller in the form of (9)-(11), we get the following closed-loop system:

$$
\begin{gather*}
\dot{\xi}(t)=\bar{A}(\sigma(t)) \xi(t) \\
\xi\left(t^{+}\right)=\bar{H}\left(\sigma(t), \sigma\left(t^{+}\right)\right) \xi(t), \quad t \geq 0 \tag{12}
\end{gather*}
$$

where $\xi=\left[\begin{array}{l}x \\ \bar{x}\end{array}\right]$. Correspondingly, the system matrices of each closed-loop subsystem are defined by $\bar{A}_{i}=\bar{A}(\sigma(t)=i)=$ $\left[\begin{array}{cc}A_{i} & B_{i} \widehat{C}_{i} \\ \widehat{B}_{i} C_{i} & \widehat{A}_{i}\end{array}\right]$ and $\bar{H}_{i j}=\bar{H}\left(\sigma(t)=i, \sigma\left(t^{+}\right)=j\right)=\left[\begin{array}{cc}H_{i j} & G_{i j} \widehat{C}_{i} \\ \widehat{G}_{i j} C_{i} & \widehat{H}_{i j}\end{array}\right]$.

For given switching signal $\sigma$, we denote by $\xi\left(t ; x_{0}, \sigma\right)$ the corresponding motion of system (12) at time $t$ starting from $\xi_{0}$ at initial time $t_{0}$.

Definition 1. The closed-loop system in (12) is said to be almost surely exponentially stable, if

$$
\begin{equation*}
\mathscr{P}\left\{\limsup _{t \rightarrow \infty} \frac{\ln \left|\xi\left(t ; \xi_{0}, \sigma\right)\right|}{t}<0\right\}=1 \tag{13}
\end{equation*}
$$

for all $\xi_{0} \in \mathbf{R}^{2 n}$.

## 3. Main Results

In this section, we first establish the conditions guaranteeing the closed-loop system in (12) to be almost surely exponentially stable and then propose a parametrization approach to solve the output-feedback controller in the form of (9), (10), and (11).

Theorem 2. System (12) is almost surely exponentially stable for all the switching signals belonging to $\mathcal{S}_{\lambda}$, if there exist scalars $\chi$ and $\beta$, and a family of positive-definite matrices $P_{i}$ such that

$$
\begin{gather*}
\bar{A}_{i}^{\prime} P_{i}+P_{i} \bar{A}_{i}-\beta P_{i} \leq 0, \quad i \in \mathfrak{\Im}  \tag{14}\\
\bar{H}_{i j}^{\prime} P_{j} \bar{H}_{i j}-\chi P_{i} \leq 0, \quad i \neq j  \tag{15}\\
\lambda \ln \chi+\beta<0 \tag{16}
\end{gather*}
$$

Proof. Construct the Lyapunov function as follows:

$$
\begin{equation*}
V(\xi(t), \sigma(t)=i)=\xi^{\prime}(t) P_{i} \xi(t) \tag{17}
\end{equation*}
$$

For any $t \geq t_{0}$, let $N\left(\left[t_{0}, t\right]\right)$ be the counter of the switching points of $\sigma(t)$ distributed within the interval $\left[t_{0}, t\right]$ and let it equal an integer value, say, $k$. At the same time, let the sequence

$$
\begin{equation*}
t_{0}<t_{1}<t_{2}<\cdots<t_{k} \leq t \tag{18}
\end{equation*}
$$

denote the corresponding switching points.
Indeed, (14) implies that

$$
\begin{array}{r}
V\left(\xi\left(t_{i}\right), \sigma\left(t_{i-1}^{+}\right)\right) \leq e^{\beta\left(t_{i}-t_{i-1}\right)} V\left(\xi\left(t_{i-1}^{+}\right), \sigma\left(t_{i-1}^{+}\right)\right)  \tag{19}\\
1 \leq i \leq k
\end{array}
$$

This together with (15) yields

$$
\begin{align*}
V\left(\xi\left(t_{i}\right), \sigma\left(t_{i-1}^{+}\right)\right) & \leq \chi e^{\beta\left(t_{i}-t_{i-1}\right)} V\left(\xi\left(t_{i-1}\right), \sigma\left(t_{i-2}^{+}\right)\right) \\
& \leq \chi e^{\beta\left(t_{i}-t_{i-2}\right)} V\left(\xi\left(t_{i-2}^{+}\right), \sigma\left(t_{i-2}^{+}\right)\right) \tag{20}
\end{align*}
$$

Therefore, we get

$$
\begin{align*}
& V\left(\xi(t), \sigma\left(t_{k}^{+}\right)\right) \\
& \quad \leq \chi^{k} e^{\beta\left(t-t_{0}\right)} V\left(\xi\left(t_{0}\right), \sigma\left(t_{0}\right)\right) \\
& \quad=\exp \left[N\left(\left[t_{0}, t\right]\right) \ln \chi+\beta\left(t-t_{0}\right)\right] V\left(\xi\left(t_{0}\right), \sigma\left(t_{0}\right)\right) \tag{21}
\end{align*}
$$

It then follows that

$$
\begin{equation*}
\limsup _{t \rightarrow \infty} \frac{\ln \left|\xi\left(t ; \xi_{0}, \sigma\right)\right|}{t}=\lim _{t \rightarrow \infty} \frac{N\left(\left[t_{0}, t\right]\right) \ln \chi+\beta\left(t-t_{0}\right)}{t} \tag{22}
\end{equation*}
$$

Furthermore, by the law of large number of Poisson processes (cf. page $214,[1]$ ), there is probability 1 that

$$
\begin{equation*}
\lim _{t \rightarrow \infty} \frac{N\left(\left[t_{0}, t\right]\right)}{t}=\lambda \tag{23}
\end{equation*}
$$

Hence, in view of (16), we arrive at

$$
\begin{equation*}
\mathscr{P}\left\{\limsup _{t \rightarrow \infty} \frac{\ln \left|\xi\left(t ; \xi_{0}, \sigma\right)\right|}{t}=\lambda \ln \chi+\beta<0\right\}=1 . \tag{24}
\end{equation*}
$$

The proof is thus completed.
With the analysis result in hand, we now turn to consider the output-feedback controller design problem. For this purpose, we introduce a set of auxiliary matrices as follows:

$$
\begin{align*}
\Psi= & \left\{X_{i}, Y_{i}, W_{i} \in \mathbf{R}^{n \times n}, U_{i} \in \mathbf{R}^{p \times n}, V_{i} \in \mathbf{R}^{n \times q}: i \in \mathfrak{J}\right\} \\
& \cup\left\{N_{i j} \in \mathbf{R}^{n \times n}, M_{i j} \in \mathbf{R}^{n \times q}: i \neq j\right\} \tag{25}
\end{align*}
$$

where $X_{i}$ and $Y_{i}$ are symmetric matrices.
Theorem 3. The closed-loop system (12) can be exponentially stabilized with respect to certain class $\mathcal{S}_{\lambda}$, if there exist scalars $\chi>0$ and $\beta$ satisfying (16), and a set of matrices as in (25) satisfying the following inequalities:

$$
\begin{gather*}
{\left[\begin{array}{cc}
A_{i} Y_{i}+B_{i} U_{i}+\star & A_{i}-W_{i}^{\prime} \\
\star & X_{i} A_{i}+V_{i} C_{i}+\star
\end{array}\right]-\beta Q_{i}(\Psi)<0}  \tag{26}\\
{\left[\begin{array}{cc}
\chi Q_{i}(\Psi) & \star \\
R_{i j}(\Psi) & Q_{j}(\Psi)
\end{array}\right] \geq 0, \quad i \neq j} \tag{27}
\end{gather*}
$$

where $*$ represents the entries obtained by symmetry, and

$$
\begin{align*}
& Q_{i}(\Psi)=\left[\begin{array}{cc}
Y_{i} & I_{n} \\
\star & X_{i}
\end{array}\right],  \tag{28}\\
& R_{i j}(\Psi)=\left[\begin{array}{cc}
H_{i j} Y_{i}+G_{i j} U_{i} & H_{i j} \\
N_{i j} & X_{j} H_{i j}-M_{i j} C_{i}
\end{array}\right] .
\end{align*}
$$

Proof. According to the $\Psi$ defined in (25), the outputfeedback compensator in (9)-(11) can be parameterized by

$$
\begin{align*}
& \widehat{A}_{i}(\Psi)=\left(X_{i}-Y_{i}^{-1}\right)^{-1} \\
& \times\left(W_{i}+X_{i} A_{i} Y_{i}+X_{i} B_{i} U_{i}+V_{i} C_{i} Y_{i}\right) Y_{i}^{-1} \\
& \widehat{B}_{i}(\Psi)=-\left(X_{i}-Y_{i}^{-1}\right)^{-1} V_{i} \\
& \widehat{C}_{i}(\Psi)=U_{i} Y_{i}^{-1} \\
& \widehat{G}_{i j}(\Psi)=\left(X_{j}-Y_{j}^{-1}\right)^{-1} M_{i j} \\
& \widehat{H}_{i j}(\Psi)=\left(X_{j}-Y_{j}^{-1}\right)^{-1} \\
& \times\left(X_{j} H_{i j} Y_{i}+X_{j} G_{i j} U_{i}-M_{i j} C_{i} Y_{i}-N_{i j}\right) Y_{i}^{-1} . \tag{29}
\end{align*}
$$

Correspondingly, the substitution of the parameterized matrices in (29) gives the closed-loop matrices as follows:

$$
\begin{align*}
& \bar{A}_{i}(\Psi)=\left[\begin{array}{cc}
A_{i} & B_{i} \widehat{C}_{i}(\Psi) \\
\widehat{B}_{i}(\Psi) C_{i} & \widehat{A}_{i}(\Psi)
\end{array}\right], \quad i \in \mathfrak{J},  \tag{30}\\
& \bar{H}_{i j}(\Psi)=\left[\begin{array}{cc}
H_{i j} & G_{i j} \widehat{C}_{i}(\Psi) \\
\widehat{G}_{i j}(\Psi) C_{i} & \widehat{H}_{i j}(\Psi)
\end{array}\right], \quad i \neq j . \tag{31}
\end{align*}
$$

Additionally, in order to be consistent with the construction of the compensator as in (29), the Lyapunov matrix is parameterized as

$$
P_{i}(\Psi)=\left[\begin{array}{cc}
X_{i} & \star  \tag{32}\\
Y_{i}^{-1}-X_{i} & X_{i}-Y_{i}^{-1}
\end{array}\right] .
$$

With the parameterized matrices in (29)-(32), the conditions in (14) and (15) then become

$$
\begin{gather*}
\bar{A}_{i}^{\prime}(\Psi) P_{i}(\Psi)+P_{i}(\Psi) \bar{A}_{i}(\Psi)-\beta P_{i}(\Psi)<0, \quad i \in \mathfrak{J}  \tag{33}\\
{\left[\begin{array}{cc}
\chi P_{i}(\Psi) & \star \\
P_{j}(\Psi) & \bar{H}_{i j}(\Psi) \\
P_{j}(\Psi)
\end{array}\right] \geq 0, \quad i \neq j} \tag{34}
\end{gather*}
$$

Let $S_{i}(\Psi)=\left[\begin{array}{cc}Y_{i} & I_{n} \\ Y_{i} & 0\end{array}\right]$. Substituting the parameterized matrices in (30) and (32) into (33) and then pre- and postmultiplying the obtained expression by $S_{i}^{\prime}(\Psi)$ and $S_{i}(\Psi)$ yield

$$
\left[\begin{array}{cc}
A_{i} Y_{i}+B_{i} U_{i}+\star & A_{i}-W_{i}^{\prime}  \tag{35}\\
\star & X_{i} A_{i}+V_{i} C_{i}+\star
\end{array}\right]-\beta\left[\begin{array}{cc}
Y_{i} & I_{n} \\
\star & X_{i}
\end{array}\right]<0 .
$$

It proves the equivalence between (26) and (33).
Furthermore, let $T_{i j}(\Psi)=\left[\begin{array}{cc}S_{i}(\Psi) & 0 \\ 0 & S_{j}(\Psi)\end{array}\right]$. Substituting the parameterized matrices in (31) and (32) into (34) and then pre- and postmultiplying the obtained expression by $T_{i j}^{\prime}(\Psi)$ and $T_{i j}(\Psi)$, we get

$$
\begin{gather*}
T_{i j}^{\prime}(\Psi)\left[\begin{array}{cc}
\chi P_{i}(\Psi) & \star \\
P_{j}(\Psi) \bar{H}_{i j}(\Psi) & P_{j}(\Psi)
\end{array}\right] T_{i j}(\Psi)  \tag{36}\\
\quad=\left[\begin{array}{cc}
\chi Q_{i}(\Psi) & \star \\
R_{i j}(\Psi) & Q_{j}(\Psi)
\end{array}\right] \geq 0 .
\end{gather*}
$$

The equivalence between (27) and (34) then is proven, while the positiveness of the parameterized Lyapunov matrix in (32) is guaranteed. The proof is thus completed.

The parametrization has been proven a successful design methodology; see, for example, [5]. It enables us to convert the nonconvex constraint conditions into the convex ones. In this paper, we propose a family of auxiliary matrices that are defined in (25) and, moreover, parameterize the outputfeedback controllers. In this way, based on some matrix transformation techniques, we convert the nonconvex constraint conditions on the controllers into convex ones, which can be solved efficiently. Moreover, the posed parametrization method can be extended to other kinds of systems with switching and state jumps, such as periodical switching systems and impulsive systems.

## 4. Illustrative Examples

We now use an example to illustrate the stability analysis result.

Example 1. Let the switching signal obey Poisson distribution with its exponent $\lambda=1.0$. Consider the system with random switchings and state jumps for the following parameters:

$$
\begin{gather*}
A_{1}=\left[\begin{array}{cc}
0 & 1 \\
0.5 & 0
\end{array}\right], \quad B_{1}=\left[\begin{array}{c}
1 \\
0.5
\end{array}\right], \quad C_{1}=\left[\begin{array}{c}
1 \\
0.5
\end{array}\right]^{\prime} \\
A_{2}=\left[\begin{array}{cc}
0 & 1 \\
-1 & 1
\end{array}\right], \quad B_{2}=\left[\begin{array}{c}
-1 \\
0.5
\end{array}\right], \quad C_{2}=\left[\begin{array}{l}
0 \\
1
\end{array}\right]^{\prime}  \tag{37}\\
H_{12}=\left[\begin{array}{cc}
-0.5 & 0.5 \\
0.5 & 1
\end{array}\right], \quad G_{12}=\left[\begin{array}{l}
1 \\
0
\end{array}\right] \\
H_{21}=\left[\begin{array}{cc}
1 & 0.5 \\
-1 & 1
\end{array}\right], \quad G_{21}=\left[\begin{array}{c}
-0.5 \\
1
\end{array}\right]
\end{gather*}
$$

According to Theorem 3, by setting $\chi=1.2$ and $\beta=-0.185$ to meet the condition in (16), we can solve a set of the parameters defined as in (25) and, correspondingly, present the outputfeedback compensators as follows:

$$
\begin{align*}
& \dot{\hat{x}}(t)=\widehat{A}_{1} \widehat{x}(t)+\widehat{B}_{1} y(t) \\
& =\left[\begin{array}{ll}
-2.0154 & -0.8209 \\
-1.3904 & -0.4459
\end{array}\right] \widehat{x}(t)+\left[\begin{array}{l}
2.4499 \\
1.9873
\end{array}\right] y(t) \\
& \widehat{x}\left(t^{+}\right)=\widehat{H}_{12} \widehat{x}(t)+\widehat{G}_{12} y(t) \\
& =\left[\begin{array}{cc}
-0.7140 & -0.0803 \\
-0.4240 & 0.5026
\end{array}\right] \widehat{x}(t)+\left[\begin{array}{c}
-0.2406 \\
0.7340
\end{array}\right] y(t) \\
& u(t)=\widehat{C}_{1} \widehat{x}(t)=\left[\begin{array}{ll}
-0.4420 & -0.7257
\end{array}\right] \widehat{x}(t),  \tag{38}\\
& \dot{\hat{x}}(t)=\widehat{A}_{2} \widehat{x}(t)+\widehat{B}_{2} y(t) \\
& =\left[\begin{array}{cc}
-0.5187 & 1.4607 \\
-0.0858 & -2.2714
\end{array}\right] \widehat{x}(t)+\left[\begin{array}{c}
-0.1375 \\
2.1958
\end{array}\right] y(t) \\
& \widehat{x}\left(t^{+}\right)=\widehat{H}_{21} \widehat{x}(t)+\widehat{G}_{21} y(t) \\
& =\left[\begin{array}{cc}
0.6799 & 0.9497 \\
-0.2400 & -1.5745
\end{array}\right] \widehat{x}(t)+\left[\begin{array}{c}
0.3297 \\
0.7073
\end{array}\right] y(t) \\
& u(t)=\widehat{C}_{2} \widehat{x}(t)=\left[\begin{array}{ll}
0.6270 & -1.5785
\end{array}\right] \widehat{x}(t),
\end{align*}
$$

which stabilizes the closed-loop system.

## 5. Conclusion

We considered the output-feedback stabilization problem for a kind of dynamical systems undergoing random switchings and state jumps. We supposed the time evolution of switching signal to obey Poisson distribution. Therefore, based on characterizing the asymptotical behavior of the distribution of switching points, we established the almost surely exponentially stable conditions for the closed-loop system. We then proposed a parametrization approach, which allows to solve the dynamical output-feedback control in terms of matrix inequalities. Finally, a numerical example was presented to demonstrate the effectiveness of the method.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

## Acknowledgments

The authors would like to thank the anonymous reviewers for their helpful and insightful comments for improving this paper. This work is supported by the National Nature Science Foundation of China under Grants nos. 61104119 and 61340015, the Science and Technology Innovation Talents Project of Henan University under Grant no. 13HASTIT044, and the Young Core Instructor Foundation from Department of Education of Henan Province under Grant no. 2011GGJS054.

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