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Research Article

Fault Detection for Wireless Networked Control Systems with Stochastic Uncertainties and Multiple Time Delays

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The fault detection problem for a class of wireless networked control systems is investigated. A Bernoulli distributed parameter is introduced in modeling the system dynamics; moreover, multiple time delays arising in the communication are taken into account. The detection observer for tracking the system states is designed, which generates both the state errors and the output errors. By adopting the linear matrix inequality method, a sufficient condition for the stability of wireless networked control systems with stochastic uncertainties and multiple time delays is proposed, and the gain of the fault detection observer is obtained. Finally, an illustrated example is provided to show that the observer designed in this paper tracks the system states well when there is no fault in the systems; however, when fault happens, the observer residual signal rises rapidly and the fault can be quickly detected, which demonstrate the effectiveness of the theoretical results.

1. Introduction

The network technology has received compelling attention during the past decades [1]. The networked control system, which plays an important role in modern industry such as the car industry and the health care, can usually be classified into the wired networked control system (WNCS) and the wireless networked control system (WiNCS) [2, 3]. Compared with WNCS, WiNCS is a comparatively new technology, which is widely used in military, monitory, and other complex situations. In WiNCS, large numbers of sensor nodes are arranged in the region of interest; due to the characteristics of the wireless communication, information flows among nodes are dynamic. As the structure of WiNCS becomes increasingly modular, system faults may result in fatal damage to the whole system. As a result, the fault detection problem for WiNCS deserves to be investigated.

The fault detection problem for WiNCS has been studied extensively in recent years [4–10]. Reference [7] considers the fault detection problem for WiNCS with access constraints and random packet dropouts. The residual generation is carried out based on a deterministic formulation and a residual evaluation is proposed by considering the random packet dropouts. In addition, with the help of Chebyshev's inequality, the fault detection threshold value is obtained. Reference [8] investigates the fault detection problem for a class of linear time invariant systems with limited network quality of services (QoS). The probabilistic switching between different situations is required to obey a homogeneous Markovian chain. In [9], the adaptive observer-based fault estimation problem is studied; by exploring the augmented matrix, error dynamic systems are transferred to Markov jumping systems. In [10], the T-S fuzzy model is adopted to describe the system model, with the benefit that the exact value of network-induced delay is not necessarily known; a fuzzy observer-based approach for the fault detection is developed.

In WiNCS, the sensor nodes gather information from the plant and pass the information to the designed controller via the bus structure. However, as the sensor nodes may be influenced by several unexpected factors such as temperature and moisture, not all the sensor nodes are in the working status. When some nodes are not working, the information flow may be transmitted from other signal channels, which arouses in the uncertainties of WiNCS. On the other hand, as the information transmission is time consuming, time delays arise naturally in WiNCS. The fault detection problem for wireless networked control systems with both stochastic uncertainties and multiple time delays has not been considered in the literature, which motivates the work in this paper. In this paper, we investigate the fault detection for a class of wireless networked control systems. A Bernoulli distributed parameter is introduced in modeling the system dynamics and the detection observer for tracking the system states is designed. By adopting the linear matrix inequality method, a sufficient condition for the stability of wireless networked control systems with stochastic uncertainties and multiple time delays is proposed, and the gain of the fault detection observer is obtained. Finally, an example is given to show the effectiveness of proposed method.

The rest of this paper is organized as follows. In Section 2, we provide the problem formulation. In Section 3, the sufficient condition for the stability of WiNCS is obtained by the linear matrix inequality method. An illustrative example is given in Section 4 and Section 5 is a brief conclusion of this paper.

2. Problem Formulation

Consider a class of WiNCS. The system model is given as follows:

$$x(k+1)=\left(A+\alpha_k\widetilde{A}\right)x(k)+A_d\sum_{i=1}^{N}x(k-i)+Bu(k)+E_f f(k), \quad y(k)=Cx(k), \tag{1}$$

where $x(k)\in\mathbf{R}^{n_x}$ denotes the state without delays, $x(k-i)\in\mathbf{R}^{n_x}$ denotes the delayed state of the system, $u(k)\in\mathbf{R}^{n_u}$ denotes the system input, $f(k)\in\mathbf{R}^{n_f}$ denotes the fault of the system, $y(k)\in\mathbf{R}^{n_y}$ denotes the system output, α_k is the stochastic variable, and A, $\widetilde{A}$, A_d, B, E_f, and C are constant matrices with appropriate dimensions. The control law $u(k)$ has the following form:

$$u(k)=Kx(k), \tag{2}$$

where K is the parameter matrix.

In this paper, the stochastic variable α_k is assumed to be a Bernoulli distributed sequence, which represents whether the communication environment changes or not at each nonnegative integer time k. We assume that $\alpha_k = 0$ when there is no change in communication environment, and $\alpha_k = 1$ when the network environment changes. Moreover, the following equations hold:

$$P\{\alpha_k=1\}=\alpha, \quad P\{\alpha_k=0\}=1-\alpha, \tag{3}$$

where $\alpha\in[0,1]$ is a given constant.

In order to generate a residual signal, we design the fault detection observer for model (1) as follows:

$$\begin{aligned}\hat{x}(k+1)&=A\hat{x}(k)+A_d\sum_{i=1}^{N}\hat{x}(k-i)+BK\hat{x}(k)+L\left[y(k)-\hat{y}(k)\right],\\ \hat{y}(k)&=C\hat{x}(k),\end{aligned} \tag{4}$$

where $\hat{x}(k)$ and $\hat{y}(k)$ are the state and the output of the observer and L is the parameter of the observer to be designed.

Let

$$e_x(k)=x(k)-\hat{x}(k), \quad e_y(k)=y(k)-\hat{y}(k). \tag{5}$$

The error model of the system is given as

$$\begin{aligned}e_x(k+1)&=(A+BK-LC)e_x(k)+\left(\alpha_k-\alpha\right)\widetilde{A}x(k)+\alpha\widetilde{A}x(k)+A_d\sum_{i=1}^{N}e_x(k-i),\\ e_y(k)&=Ce_x(k).\end{aligned} \tag{6}$$

Next, we define the system residual as $r(k)=e_y(k)$. Note that if there is no fault in the system, the residual is close to zero. We set up the residual evaluation function J and the fault threshold J_{th} as

$$J(k)=\sqrt{\sum_{k=1}^{N}r^T(k)r(k)}, \quad J_{\text{th}}=\sup J(k). \tag{7}$$

By comparing J and J_{th}, it can be decided whether the fault happens or not:

$$\begin{aligned}&J\le J_{\text{th}} \quad \text{No fault happens},\\ &J>J_{\text{th}} \quad \text{Fault happens}.\end{aligned} \tag{8}$$

Lemma 1 (Schur complement). *For matrices A, B, and C, $A+B^TCB<0$ equals*

$$\begin{bmatrix} A & B^T \\ B & -C^{-1}\end{bmatrix}<0 \quad \text{or} \quad \begin{bmatrix} -C^{-1} & B \\ B^T & A\end{bmatrix}<0. \tag{9}$$

3. Main Results

In this section, a fault detection observer will be designed based on the Lyapunov stability theory. A sufficient condition for the stability of the system is obtained by the linear matrix inequalities.

Theorem 2. *The error system* (6) *is quadratically stable if there exist matrices $P > 0$, $Q > 0$, and $R > 0$ with appropriate dimensions satisfying the following inequality:*

$$W' = \begin{pmatrix} W'_{11} & W_{12} & W_{13} & W_{14} & W_{15} \\ * & W'_{22} & 0 & W_{24} & W_{25} \\ * & * & W'_{33} & 0 & W_{35} \\ * & * & * & W'_{44} & W_{45} \\ * & * & * & * & W_{55} \end{pmatrix} < 0, \quad (10)$$

where

$$\begin{aligned}
W'_{11} &= \alpha(A+BK)^T P\widetilde{A} + \alpha\widetilde{A}^T P(A+BK) - P + NQ, \\
W_{12} &= \alpha\widetilde{A}^T P(A+BK) - \alpha\widetilde{A}^T RC, \\
W_{13} &= (A+BK)^T PA_d + \alpha\widetilde{A}^T PA_d, \\
W_{14} &= \alpha\widetilde{A}^T PA_d, \\
W_{15} &= \begin{bmatrix}(A+BK)^T P & \sqrt{2}\overline{\alpha}\widetilde{A}^T P & \sqrt{2}\overline{\alpha}A^T P & 0 & 0 & 0\end{bmatrix}, \\
W'_{22} &= NQ - P, \\
W_{24} &= (A+BK)^T PA_d - (RC)^T A_d, \\
W_{25} &= \begin{bmatrix}0 & 0 & 0 & (A+BK)^T P - (RC)^T & 0 & 0\end{bmatrix}, \\
W'_{33} &= -\frac{2}{(1+N)N}Q, \\
W_{35} &= \begin{bmatrix}0 & 0 & 0 & 0 & A_d^T P & 0\end{bmatrix}, \\
W'_{44} &= -\frac{2}{(1+N)N}Q, \\
W_{45} &= \begin{bmatrix}0 & 0 & 0 & 0 & 0 & A_d^T P\end{bmatrix}, \\
W_{55} &= \operatorname{diag}\{-P,-P,-P,-P,-P,-P\}, \\
\overline{\alpha} &= \sqrt{\alpha(1-\alpha)}, \\
R &= L^T P.
\end{aligned} \quad (11)$$

Proof. We choose the following Lyapunov function:

$$V(k) = V_1(k) + V_2(k). \quad (12)$$

where

$$\begin{aligned}
V_1(k) &= e_x^T(k)Pe_x(k) + x^T(k)Px(k), \\
V_2(k) &= \sum_{i=1}^{N}\sum_{l=k-i}^{k-1}\left\{e_x^T(k)Qe_x(k) + x^T(k)Qx(k)\right\}.
\end{aligned} \quad (13)$$

Calculate the difference of (12) along system (6); we have

$$\begin{aligned}
E\Delta V_1(k) &= e_x^T(k+1)Pe_x(k+1) + x^T(k+1)Px(x+1) \\
&\quad - e_x^T(k)Pe_x(k) - x^T(k)Px(k) \\
&= e_x^T(k)(A-LC+BK)^T P(A-LC+BK)e_x(k) \\
&\quad + \alpha e_x^T(k)(A-LC+BK)^T P\widetilde{A}x(k) \\
&\quad + e_x^T(k)(A-LC+BK)^T PA_d\sum_{i=1}^{N}e_x(k-i) \\
&\quad + \alpha(1-\alpha)x^T(k)\widetilde{A}^T P\widetilde{A}x(k) \\
&\quad + \alpha x^T(k)\widetilde{A}^T P(A-LC+BK)e_x(k) \\
&\quad + \alpha^2 x^T(k)\widetilde{A}^T P\widetilde{A}x(k) \\
&\quad + \alpha x^T(k)\widetilde{A}^T PA_d\sum_{i=1}^{N}e_x(k-i) \\
&\quad + \sum_{i=1}^{N}e_x^T(k-i)A_d^T P(A-LC+BK)e_x(k) \\
&\quad + \alpha\sum_{i=1}^{N}e_x^T(k-i)A_d^T P\widetilde{A}x(k) \\
&\quad + \sum_{i=1}^{N}e_x^T(k-i)A_d^T PA_d\sum_{i=1}^{N}e_x(k-i) \\
&\quad - e_x^T(k)Pe_x(k) \\
&\quad + x^T(k)(A+BK)^T P(A+BK)x(k) \\
&\quad + \alpha x^T(k)(A+BK)^T P\widetilde{A}x(k) \\
&\quad + x^T(k)(A+BK)^T PA_d\sum_{i=1}^{N}x(k-i) \\
&\quad + \alpha(1-\alpha)x^T(k)\widetilde{A}^T P\widetilde{A}x(k) \\
&\quad + \alpha x^T(k)\widetilde{A}^T P(A+BK)x(k) \\
&\quad + \alpha^2 x^T(k)\widetilde{A}^T P\widetilde{A}x(k) \\
&\quad + \alpha x^T(k)\widetilde{A}^T PA_d\sum_{i=1}^{N}x(k-i) \\
&\quad + \sum_{i=1}^{N}x^T(k-i)A_d^T P(A+BK)x(k) \\
&\quad + \alpha\sum_{i=1}^{N}x^T(k-i)A_d^T P\widetilde{A}x(k)
\end{aligned}$$

$$+\sum_{i=1}^{N}x^{T}(k-i)A_{d}^{T}PA_{d}\sum_{i=1}^{N}x(k-i)$$
$$-x^{T}(k)Px(k). \tag{14}$$

According to Jensen's inequality, we have

$$\begin{aligned}E\Delta V_{2}(k)=&\sum_{i=1}^{N}\left\{\sum_{l=k+1-i}^{k}e_{x}^{T}(l)Qe_{x}(l)\right.\\&-\sum_{l=k-i}^{k-1}e_{x}^{T}(l)Qe_{x}(l)\\&\left.+\sum_{l=k+1-i}^{k}x^{T}(l)Qx(l)-\sum_{l=k-i}^{k-1}x^{T}(l)Qx(l)\right\}\\=&\sum_{i=1}^{N}\left\{e_{x}^{T}(k)Qe_{x}(k)-e_{x}^{T}(k-i)Qe_{x}(k-i)\right.\\&\left.+x^{T}(k)Qx(k)-x^{T}(k-i)Qx(k-i)\right\}\\\le& Ne_{x}^{T}(k)Qe_{x}(k)+Nx^{T}(k)Qx(k)\\&-\frac{2}{(1+N)N}\sum_{i=1}^{N}e_{x}^{T}(k-i)Q\sum_{i=1}^{N}e_{x}(k-i)\\&-\frac{2}{(1+N)N}\sum_{i=1}^{N}x^{T}(k-i)Q\sum_{i=1}^{N}x(k-i).\end{aligned}\tag{15}$$

Substituting $E\Delta V_1(k)$ and $E\Delta V_2(k)$ into (12), we have

$$\begin{aligned}E\Delta V(k)&=E\Delta V_{1}(k)+E\Delta_{2}V(k)\\&=Z^{T}(k)WZ(k),\end{aligned}\tag{16}$$

where

$$Z(k)=\left[x^{T}(k)\;\; e_{x}^{T}(k)\;\;\sum_{i=1}^{N}x^{T}(k-i)\;\;\sum_{i=1}^{N}e_{x}^{T}(k-i)\right]^{T},$$
$$W=\begin{pmatrix}W_{11}&W_{12}&W_{13}&W_{14}\\ *&W_{22}&0&W_{24}\\ *&*&W_{33}&0\\ *&*&*&W_{44}\end{pmatrix},$$
$$\begin{aligned}W_{11}=&(A+BK)^{T}P(A+BK)\\&+\alpha(A+BK)^{T}P\widetilde{A}+2\alpha(1-\alpha)\widetilde{A}^{T}P\widetilde{A},\\&+\alpha\widetilde{A}^{T}P(A+BK)+2\alpha^{2}\widetilde{A}^{T}P\widetilde{A}-P+NQ,\\W_{22}=&(A-LC+BK)^{T}P(A-LC+BK)-P+NQ,\\W_{33}=&A_{d}^{T}PA_{d}-\frac{2}{(1+N)N}Q,\\W_{44}=&A_{d}^{T}PA_{d}-\frac{2}{(1+N)N}Q,\end{aligned}\tag{17}$$

and W_{12}, W_{13}, W_{14}, and W_{24} are the same as in (10).

According to the Lyapunov stability theory, the error system is stable if

$$W<0. \tag{18}$$

According to Lemma 1, (18) is equivalent to

$$W'=\begin{pmatrix}W'_{11}&W_{12}&W_{13}&W_{14}&W'_{15}\\ *&W'_{22}&0&W_{24}&W'_{25}\\ *&*&W'_{33}&0&W'_{35}\\ *&*&*&W'_{44}&W'_{45}\\ *&*&*&*&W'_{55}\end{pmatrix}<0, \tag{19}$$

where

$$\begin{aligned}W'_{15}&=\left[(A+BK)^{T}\;\;\sqrt{2\overline{\alpha}}\widetilde{A}^{T}\;\;\sqrt{2\overline{\alpha}}A^{T}\;\;0\;\;0\;\;0\right],\\W'_{25}&=\left[0\;\;0\;\;0\;\;(A-LC+BK)^{T}\;\;0\;\;0\right],\\W'_{35}&=\left[0\;\;0\;\;0\;\;0\;\;A_{d}^{T}\;\;0\right],\\W'_{45}&=\left[0\;\;0\;\;0\;\;0\;\;0\;\;A_{d}^{T}\right],\\W'_{55}&=\operatorname{diag}\left\{-P^{-1},-P^{-1},-P^{-1},-P^{-1},-P^{-1},-P^{-1}\right\}.\end{aligned}\tag{20}$$

We set $R=L^{T}P$ and multiply $\operatorname{diag}\{I,I,I,I,P,P,P,P,P\}$ on both sides of (19). Then we can get inequality (10). The proof is completed. □

Remark 3. As is well known, the randomly occurring phenomena have been extensively investigated in recent years. In this paper, we consider the case where the communication environment is affected by some factors in a probabilistic way described by Bernoulli random variable α_k.

Remark 4. In practice, many systems have stochastic Markovian jumping dynamics [11–15]. Future research efforts will be devoted to the fault detection for wireless sensor networks with stochastic Markovian jumping dynamics.

4. An Illustrative Example

In this section, we will provide a numerical example to illustrate the effectiveness of the theoreticalresults.

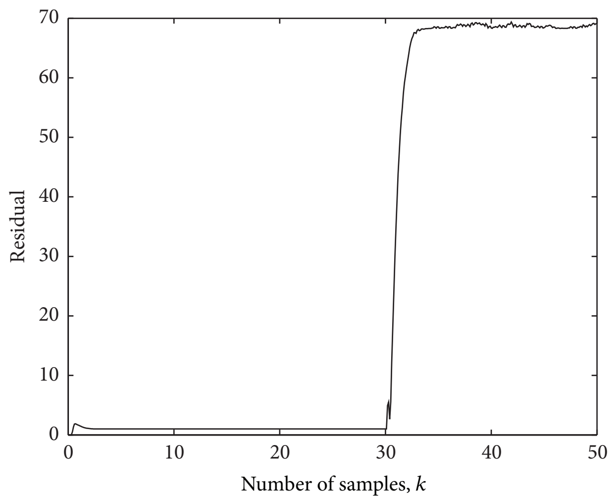

Figure 1: System residual signal with fault.

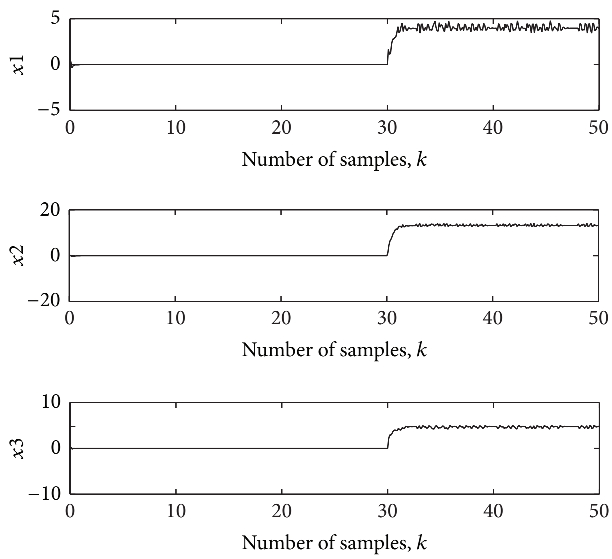

Figure 2: System fault in different states.

Example 1. Consider system (1), where

$$A = \begin{pmatrix} 0.4985 & 0.4849 & -0.7260 \\ 0.4550 & 0.4441 & 1.1011 \\ 0.2396 & 0 & 0.2662 \end{pmatrix},$$

$$A_d = \begin{pmatrix} -0.2807 & 0.0121 & 0 \\ -0.0230 & 0.0750 & 0.0387 \\ -0.0448 & 0.0944 & 0.1803 \end{pmatrix},$$

$$B = \begin{pmatrix} 0.0883 \\ 0.0544 \\ 0.0835 \end{pmatrix}, \tag{21}$$

$$C = \begin{pmatrix} 2.3232 & 0.8912 & 0.0121 \\ 1.0563 & 0.6195 & 0.0157 \\ 0.0750 & 0.0254 & 0.8712 \end{pmatrix},$$

$$\widetilde{A} = \begin{pmatrix} -0.0102 & 0.0239 & 0.0101 \\ 0 & -0.1018 & 0.1201 \\ 0.2120 & -0.0349 & 0.0002 \end{pmatrix},$$

$$K = \begin{pmatrix} 275.7123 & 245.9043 & -350.6332 \end{pmatrix}.$$

We set $\alpha = 0.2$, and P, Q, and R can be obtained as

$$P = \begin{pmatrix} 131.2345 & 28.2728 & 7.8883 \\ 28.2728 & 110.5035 & -5.9995 \\ 7.8883 & -5.9995 & 196.6286 \end{pmatrix},$$

$$Q = \begin{pmatrix} 15.7873 & -0.0697 & -0.0521 \\ -0.0697 & 15.2671 & 0.1465 \\ -0.0521 & 0.1465 & 15.3377 \end{pmatrix}, \tag{22}$$

$$R = \begin{pmatrix} -46.0191 & -51.8770 & 71.2995 \\ 170.4173 & 158.7889 & -128.9558 \\ -89.4728 & 102.9384 & 27.9875 \end{pmatrix}.$$

Moreover, we have

$$L = \begin{pmatrix} -0.2917 & 1.0973 & -0.9504 \\ -0.3751 & 1.1201 & 1.1865 \\ 0.3629 & -0.6657 & 0.2167 \end{pmatrix}. \tag{23}$$

When fault happens at sampling time 30, we can see that residual signal rises quickly, which indicates that fault occurs, as shown in Figure 1. In addition, we can clearly see different fault happens in every state from Figure 2.

5. Conclusion

In this paper, we discussed the fault detection problem for WiNCS with both stochastic uncertainties and multiple time delays. By adopting the Lyapunov method, a sufficient condition for the stability of the system is provided, and the gain of observer is also acquired. Finally, simulation results show the effectiveness of theoretical results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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