Research Article

Robust H_∞ Switching Rule Design for Boost Converters with Uncertain Parameters and Disturbances

Yang Yang,[1] Hamid Reza Karimi,[2] and Zhengrong Xiang[1]

[1] *School of Automation, Nanjing University of Science and Technology, Nanjing 210094, China*
[2] *Department of Engineering, Faculty of Engineering and Science, University of Agder, 4898 Grimstad, Norway*

Correspondence should be addressed to Zhengrong Xiang; xiangzr@mail.njust.edu.cn

This paper is concerned with the design problem of robust H_∞ switching rule for Boost converters with uncertain parameters and disturbances. Firstly, the Boost converter is modeled as a switched affine linear system with uncertain parameters and disturbances. Then, using common Lyapunov function approach and linear matrix inequality (LMI) technique, a novel switching rule is proposed such that the H_∞ model reference tracking performance is satisfied. Finally, a simulation result is provided to show the validity of the proposed method.

1. Introduction

The last years have witnessed the crescent interest of the scientific community in the study of hybrid systems because of their wide applications in many fields, such as chemical processing, communication networks, traffic control, automotive engine control, and aircraft control [1–4]. The stochastic hybrid systems, also called Markov jump systems, represent an important class of hybrid systems that are popular in modeling many practical systems, such as manufacturing systems, power systems, aerospace systems, and networked control systems that may experience random abrupt changes in their structures and parameters [5–8]. Recently, the problem of the adaptive tracking for a class of stochastic nonlinear systems with stationary Markovian switching was considered in [9]. The problem of robust mode-dependent delayed state feedback H_∞ control for a class of uncertain time-delay systems with Markovian switching parameters and mixed discrete, neutral, and distributed delays was investigated in [10].

As a specific type of hybrid systems, switched systems have received a great attention in the last decades. Many examples of such systems can be found in real world, such as power electronics, networked control systems [11]. The switching rule design is one of the key issues in the study of switched systems. Based on the switching rule design, many important problems, such as stability, model reduction, H_∞ control or filtering problem, and tracking control, have been extensively investigated in [12–22]. For example, the problems of stability and stabilization of impulsive switched systems with time delays were considered in [12]. The problems of finite-time stability analysis and stabilization for switched nonlinear discrete-time systems were addressed in [13]. The model reduction of switched systems was investigated in [14, 15]. In [16–22], the H_∞ control and filtering problems of switched systems were studied. In [23], the tracking control for switched linear systems with time delay was investigated, and a tracking control law was designed such that the H_∞ model reference tracking performance is satisfied. There are also many papers discussing the switching rule design for switched affine systems; see, for instance, [24–26] and the references therein.

On the other hand, the switch-mode DC-DC converters are widely applied in DC motor drives and regulated DC power supplies where the object is to convert the unregulated DC input into a desired DC output voltage level [27]. Due to the fact that the DC-DC converters operate in a switch mode where an interaction between continuous and discrete dynamics exists, they can be modeled as hybrid systems. Recently, a unified method for fast modeling of DC-DC switching converters in the continuous conduction mode and discontinuous conduction mode was proposed in [28]. A tristate converter was modeled as a switched affine system, and
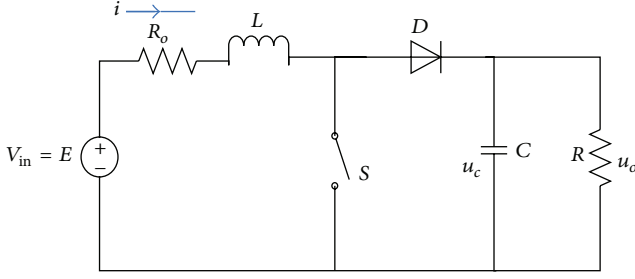
FIGURE 1: A Boost converter with inductor ESR.

a controller was designed to ensure that the corresponding closed-loop system is always stable in the safe set [29]. In [27], a hybrid feedback switching rule based on the common Lyapunov function approach was applied to a Buck converter. The method dealing with switched affine systems was used to study DC-DC converters, and a set of equilibrium points and a class of switching rules were determined in [30, 31]. In order to improve the switching strategy, the clock delay control was discussed in [32]. In addition, the method of state feedback H_∞ controller design for Monrovian switching systems with mixed delays was applied to DC-DC converters in [33]. However, DC-DC converters with uncertain parameters and disturbances have not yet gained sufficient research attention, which motivates the present study.

In this paper, we are interested in investigating the robust H_∞ switching rule design for Boost converters with uncertain parameters and disturbances. Firstly, the Boost converter under consideration is modeled as a switched affine system. Then the common quadratic Lyapunov function approach is utilized for the design of switching rule. By determining a set of attainable equilibrium points, a class of switching rules and the activation region of each subsystem are developed. A sufficient condition for the existence of such switching rule guaranteeing the H_∞ model reference tracking performance is formulated in terms of linear matrix inequality (LMI).

The rest of this paper is organized as follows. Section 2 presents the model of switched affine linear system for a Boost converter with uncertain parameters and disturbances. In Section 3, a switching rule is designed such that the H_∞ model reference tracking performance is satisfied. The simulation result is illustrated through a numerical example in Section 4. Brief conclusion is discussed in Section 5.

Notations. R^n denotes the n-dimensional Euclidean space, and $R^{n \times n}$ is the set of $n \times n$ real matrices. For real matrices or vectors, the superscript "T" stands for the transpose. The convex combination is given by $A_\lambda = \sum_{i=1}^{m} \lambda_i A_i$, where λ belongs to the set of Λ composed by all nonnegative vectors such that $\sum_{i=1}^{m} \lambda_i = 1$. The space of square integrable functions on $[0, \infty)$ is denoted by $L_2[0, \infty)$.

2. Model of Boost Converter

A Boost converter with the inductor equivalent series resistance (ESR), uncertain parameters, and disturbance is shown in Figure 1.

Suppose that the converter operates in a continuous current conduction mode. Let the inductor current i and the capacitance voltage u_c be the state variables, and denote that $x = [x_1 \ x_2]^T = [i \ u_c]^T$. From Figure 1, it can be seen that the converter operates between two modes. Considering that the output load resistance R and its variation are bounded, when the switch S is closed, the system equation is

$$\begin{bmatrix} \dot{i} \\ \dot{u}_c \end{bmatrix} = \begin{bmatrix} -\dfrac{R_0}{L} & 0 \\ 0 & -\dfrac{1}{(R + \Delta R_1)C} \end{bmatrix} \begin{bmatrix} i \\ u_c \end{bmatrix} + \begin{bmatrix} \dfrac{E}{L} \\ 0 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} \omega,$$

$$u_0 = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ u_c \end{bmatrix},$$

(1)

when the switch S is open, the system equation is

$$\begin{bmatrix} \dot{i} \\ \dot{u}_c \end{bmatrix} = \begin{bmatrix} -\dfrac{R_0}{L} & -\dfrac{1}{L} \\ \dfrac{1}{C} & -\dfrac{1}{(R + \Delta R_2)C} \end{bmatrix} \begin{bmatrix} i \\ u_c \end{bmatrix} + \begin{bmatrix} \dfrac{E}{L} \\ 0 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} \omega,$$

$$u_0 = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ u_c \end{bmatrix},$$

(2)

where ω is the disturbance of the input voltage and ΔR_1 and ΔR_2 are the variations of the load resistance.

In order to separate the uncertainties from the system equation, we can use the first-order Taylor series expansion, and the system equation can be written as when the switch S is closed,

$$\dot{x}(t) = (A_1 + \Delta A_1(t)) x(t) + b_1 + D_1 \omega(t),$$

$$y(t) = C_1 x(t),$$

(3)

when the switch S is open,

$$\dot{x}(t) = (A_2 + \Delta A_2(t)) x(t) + b_2 + D_2 \omega(t),$$

$$y(t) = C_2 x(t),$$

(4)

where

$$A_1 = \begin{bmatrix} -\dfrac{R_0}{L} & 0 \\ 0 & -\dfrac{1}{RC} \end{bmatrix}, \qquad b_1 = \begin{bmatrix} \dfrac{E}{L} \\ 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -\dfrac{R_0}{L} & -\dfrac{1}{L} \\ \dfrac{1}{C} & -\dfrac{1}{RC} \end{bmatrix}, \qquad b_2 = \begin{bmatrix} \dfrac{E}{L} \\ 0 \end{bmatrix},$$

(5)

$$D_1 = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

The uncertainties can be written as

$$\Delta A_1(t) = M_1 F_1(t) N_1, \qquad \Delta A_2(t) = M_2 F_2(t) N_2, \quad (6)$$

where $F_i^T(t) F_i(t) \le I$, $M_1 = M_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $N_1 = N_2 = \begin{bmatrix} 0 & 0 \\ 0 & \Delta R_{\max}/R^2 C \end{bmatrix}$, and $\Delta R_{\max}$ is the upper bound of the variation of the load resistance.

3. Switching Rule Design

Because the hybrid system model of the converter described in Section 2 involves uncertain parameters and disturbance, the model can be described by the following switched affine systems:

$$\dot{x}(t) = \overline{A}_{\sigma(t)} x(t) + b_{\sigma(t)} + D_{\sigma(t)} \omega(t),$$
$$y(t) = C_{\sigma(t)} x(t), \qquad (7)$$

where $x(t) \in R^n$ is the system state, $y(t) \in R^q$ is the output, $\omega \in R^p$ is the bounded exogenous disturbance of the system, $\sigma(t) : [0, \infty) \to \underline{m} = \{1, 2, \ldots, m\}$ is a piecewise constant function of time, so-called switching rule, and m is the number of system modes. The ith system mode (A_i, b_i, D_i, C_i) is active at time t when $\sigma(t) = i$. $\overline{A}_i = A_i + \Delta A_i$, ΔA_i represents the uncertainty, and A_i, b_i, D_i, and C_i are constant matrices of appropriate dimensions.

The problem concerned here is to design a switching rule $\sigma(t)$ and determine an equilibrium point $x_r \in R^n$ that is attainable under such switching rule, that is, $x(t) \to x_r$ as $t \to \infty$. Furthermore, the switching rule should guarantee the H_∞ model reference tracking performance.

Given the desired equilibrium point x_r and system (7), the tracking error system is

$$\dot{e}_r(t) = \overline{A}_{\sigma(t)} e_r(t) + \overline{k}_{\sigma(t)} + D_{\sigma(t)} \omega(t),$$
$$\overline{k}_i = b_i + \overline{A}_i x_r, \qquad e_r(t) = x(t) - x_r. \qquad (8)$$

Consider the measurement vector $y(t)$ of system (7), and define the output tracking error as follows:

$$\varepsilon(t) = y(t) - C_{\sigma(t)} x_r = C_{\sigma(t)} e_r(t). \qquad (9)$$

Given a performance index

$$J = \int_{t_0}^{\infty} \varepsilon^T(t) \varepsilon(t) \, dt - \gamma^2 \int_{t_0}^{\infty} \omega^T(t) \omega(t) \, dt, \qquad (10)$$

then the problem under consideration can be transformed to design a switching rule $\sigma(t)$ such that error system (8) is stabilizable with an H_∞ disturbance attenuation γ.

Assumption 1. The uncertainty ΔA_i is given as follow:

$$\Delta A_i(t) = M_i F_i(t) N_i, \qquad (11)$$

where M_i and N_i are known constant matrices of the appropriate dimensions and $F_i(t)$ is an unknown bounded function which satisfies

$$F_i^T(t) F_i(t) \le I. \qquad (12)$$

Definition 2. Given a constant $\gamma > 0$, error switched system (8) is said to be stabilizable with an H_∞ disturbance attenuation γ if there exists a switching rule $\sigma(t)$ such that the following conditions are satisfied:

(1) system (8) with $\omega(t) \equiv 0$ is asymptotically stable for all admissible uncertainties;

(2) under zero-initial condition, that is, $x(0) = 0$,

$$J = \int_{t_0}^{\infty} \varepsilon^T(t) \varepsilon(t) \, dt - \gamma^2 \int_{t_0}^{\infty} \omega^T(t) \omega(t) \, dt < 0 \qquad (13)$$

holds for all admissible uncertainties and all nonzero $\omega(t) \in L_2[0, \infty)$.

Lemma 3 (see [31]). *Consider switched system (7) with $b_{\sigma(t)} \equiv 0$, if there exist $\lambda \in \Lambda$ and a symmetric positive-definite matrix $P \in R^{n \times n}$ such that*

$$A_\lambda^T P + P A_\lambda < 0. \qquad (14)$$

Then the switching rule

$$\sigma(x) = \arg \min_{i \in \underline{m}} x^T P A_i x \qquad (15)$$

makes the equilibrium point $x_r = 0$ globally asymptotically stable.

Lemma 4 (see [24]). *Consider switched affine linear system (7) with $m = 2$; the point $x_r = 0$ is a quadratic stabilizable switched equilibrium if and only if there exists $\alpha \in (0, 1)$ such that*

$$A_\lambda = \alpha A_1 + (1 - \alpha) A_2 \text{ is } Hurwitz,$$
$$b_\lambda = \alpha b_1 + (1 - \alpha) b_2 = 0. \qquad (16)$$

Lemma 5 (see [34]). *If X and Y are constant matrices of appropriate dimensions, then*

$$X^T Y + Y^T X \le \xi^{-1} X^T X + \xi Y^T Y, \qquad (17)$$

where ξ is a positive scalar.

The following theorem provides an LMI condition for designing the switching rule $\sigma(t)$ such that error system (8) is stabilizable with an H_∞ disturbance attenuation γ.

Theorem 6. *Let x_r be a given constant vector representing the desired equilibrium point of the system (7), and consider error switched system (8). Given constants $\gamma > 0$ and $\varepsilon > 0$, if there exist $\lambda \in \Lambda$ and a symmetric positive definite matrix $P \in R^{n \times n}$ such that*

$$\begin{bmatrix} A_i^T P + P A_i & \varepsilon^{-1} N_i^T & \varepsilon P M_i & \gamma^{-1} P D_i & C_i^T \\ \varepsilon^{-1} N_i & -I & 0 & 0 & 0 \\ \varepsilon M_i^T P & 0 & -I & 0 & 0 \\ \gamma^{-1} D_i^T P & 0 & 0 & -I & 0 \\ C_i & 0 & 0 & 0 & -I \end{bmatrix} < 0, \qquad (18)$$

$$A_\lambda x_r + b_\lambda = 0, \qquad (19)$$

for all $i \in \underline{m}$, then the following switching rule

$$\sigma(t)$$
$$= \arg \min_{i \in \underline{m}} \left\{ e_r^T(t) \left(A_i^T P + P A_i + \varepsilon^{-2} N_i^T N_i + \varepsilon^2 P M_i M_i^T P \right.\right.$$
$$\left.\left. + \gamma^{-2} P D_i D_i^T P + C_i^T C_i \right) e_r(t) \right\} \tag{20}$$

can make the error switched system (8) stabilizable with an H_∞ disturbance attenuation γ.

Proof. Choose the following Lyapunov function:

$$V(e_r) = e_r^T(t) P e_r(t). \tag{21}$$

Along the trajectory of system (8), we obtain from (20) that

$$\dot{V}(e_r(t))$$
$$= \dot{x}^T(t) P e_r(t) + e_r^T(t) P \dot{x}(t)$$
$$= 2 e_r^T(t) P \dot{x} = 2 e_r^T(t) P \left(A_{\sigma(t)} + \Delta A_{\sigma(t)}(t) \right) x(t)$$
$$\quad + 2 e_r^T(t) P b_{\sigma(t)} + 2 e_r^T(t) P D_{\sigma(t)} \omega(t)$$
$$= \min_{i \in \underline{m}} \left(2 e_r^T(t) P \left(A_i x(t) + \Delta A_i(t) x(t) \right.\right.$$
$$\left.\left. + b_i + D_i \omega(t) \right) \right)$$
$$= \min_{i \in \underline{m}} \left(e_r^T(t) \left(A_i^T P + P A_i + \Delta A_i^T(t) P + P \Delta A_i(t) \right) e_r(t) \right.$$
$$\left. + 2 e_r^T(t) P \left((A_i + \Delta A_i(t)) x_r + b_i + D_i \omega(t) \right) \right)$$
$$= \min_{i \in \underline{m}} \left(e_r^T(t) \left(A_i^T P + P A_i + N_i^T F_i^T(t) M_i^T P \right.\right.$$
$$\left.\left. + P M_i F_i(t) N_i \right) e_r(t) + 2 e_r^T(t) P D_i \omega(t) \right)$$
$$+ \min_{\lambda \in \Lambda} \left(2 e_r^T(t) P \left((A_\lambda + \Delta A_\lambda(t)) x_r + b_\lambda \right) \right). \tag{22}$$

By Lemma 5, one obtains

$$N_i^T F_i^T(t) M_i^T P + P M_i F_i(t) N_i$$
$$\quad \leq \varepsilon^{-2} N_i^T F_i^T(t) F_i(t) N_i + \varepsilon^2 P M_i M_i^T P$$
$$\quad \leq \varepsilon^{-2} N_i^T N_i + \varepsilon^2 P M_i M_i^T P,$$
$$2 e_r^T(t) P D_i \omega(t) \leq \gamma^{-2} e_r^T(t) P D_i D_i^T P e_r(t) + \gamma^2 \omega^T(t) \omega(t). \tag{23}$$

It follows from (19) and (23) that

$$\dot{V}(e_r(t))$$
$$\leq \min_{i \in \underline{m}} \left(e_r^T(t) \left(A_i^T P + P A_i + \varepsilon^{-2} N_i^T N_i + \varepsilon^2 P M_i M_i^T P \right.\right.$$
$$\left.\left. + \gamma^{-2} P D_i D_i^T P \right) e_r(t) + \gamma^2 \omega^T(t) \omega(t) \right). \tag{24}$$

For any $t \geq 0$, the feasibility of (18) implies $\dot{V}(e_r(t)) < 0$, which ensures the asymptotic stability of system (8) with $\omega(t) \equiv 0$.

On the other hand, denote that $\{(t_k, \sigma(t_k)) \mid k = 0, 1, \ldots, s; \ 0 = t_0 \leq t_1 \leq \cdots \leq t_s < t_f\}$ as the switching sequence of $\sigma(t)$ on the interval $[0, t_f)$. Under zero condition, we have

$$\int_{t_0}^{t_f} \left[\varepsilon^T(t) \varepsilon(t) - \gamma^2 \omega^T(t) \omega(t) \right] dt$$
$$= \sum_{k=0}^{s-1} \left\{ \int_{t_k}^{t_{k+1}} \left[\varepsilon^T(t) \varepsilon(t) - \gamma^2 \omega^T(t) \omega(t) + \dot{V}(e_r(t)) \right] dt \right.$$
$$\left. - \left[V(e_r(t_{k+1})) - V(e_r(t_k)) \right] \right\}$$
$$+ \int_{t_s}^{t_f} \left[\varepsilon^T(t) \varepsilon(t) - \gamma^2 \omega^T(t) \omega(t) + \dot{V}(e_r(t)) \right] dt$$
$$- \left[V(e_r(t_f)) - V(e_r(t_s)) \right]$$
$$= \sum_{k=0}^{s-1} \int_{t_k}^{t_{k+1}} \left[e_r^T(t) C_i^T C_i e_r(t) - \gamma^2 \omega^T(t) \omega(t) + \dot{V}(e_r(t)) \right] dt$$
$$+ \int_{t_s}^{t_f} \left[\varepsilon^T(t) \varepsilon(t) - \gamma^2 \omega^T(t) \omega(t) + \dot{V}(e_r(t)) \right] dt$$
$$- V(e_r(t_f))$$
$$\leq \sum_{k=0}^{s-1} \int_{t_k}^{t_{k+1}} \left[e_r^T(t) \left(A_i^T P + P A_i + \varepsilon^{-2} N_i^T N_i + \varepsilon^2 P M_i M_i^T P \right.\right.$$
$$\left.\left. + \gamma^{-2} P D_i D_i^T P + C_i^T C_i \right) e_r(t) \right] dt$$
$$+ \int_{t_s}^{t_f} \left[e_r^T(t) \left(A_i^T P + P A_i + \varepsilon^{-2} N_i^T N_i + \varepsilon^2 P M_i M_i^T P \right.\right.$$
$$\left.\left. + \gamma^{-2} P D_i D_i^T P + C_i^T C_i \right) e_r(t) \right] dt. \tag{25}$$

According to Schur's complement and the inequality (25), it is easy to get

$$\int_{t_0}^{t_f} \varepsilon^T(t) \varepsilon(t) \, dt \leq \gamma^2 \int_{t_0}^{t_f} \omega^T(t) \omega(t) \, dt. \tag{26}$$

Letting $t_f = \infty$, we get $J < 0$.

This completes the proof. □

From Theorem 6, it can be seen that (18) is a LMI; thus we can first get the matrix P through LMI toolbox of MATLAB [35]. Then the desired switching rule can be obtained from (20) for any selected equilibrium point x_r.

According to the switched affine system model of the Boost converter described in Section 3, if the converter operates in a continuous current conduction mode, we can get $m = 2$. In the case of two subsystems, if we ignore the uncertainties in the switching point average system, then we

can obtain the convex combination as in Lemma 4 and in [8], and the convex combination is given by

$$\dot{x}(t) = A_\lambda x(t) + b_\lambda, \tag{27}$$

where $A_\lambda = \sum_{i=1}^{2} \lambda_i A_i$, $b_\lambda = \sum_{i=1}^{2} \lambda_i b_i$ with $0 < \lambda_i < 1$ and $\sum_{i=1}^{2} \lambda_i = 1$. If the dwell time of the ith subsystem is proportional to λ_i, as the concept of "duty ratio" in DC-DC converters, then (27) is an average model of switched system (7). However, unlike time average control, the quadratic stability of the switched equilibrium takes the state feedback form. A necessary and sufficient condition for the quadratic stability of the switched equilibrium is shown in Lemma 4. In addition, the proposed switching rule depends on state variables; hence rapid switching can be avoided.

Condition (19) is an expression of the desired equilibrium point x_r for DC-DC converters. Notice that the inequality (18) imposes that $A_\lambda = \sum_{i=1}^{2} \lambda_i A_i$ is asymptotically stable, where λ_i can be regard as the concept of "duty ratio" in DC-DC converters. Then we can calculate the desired equilibrium point x_r for DC-DC converters by the following equation:

$$x_r = -A_\lambda^{-1} b_\lambda, \tag{28}$$

where $A_\lambda = \sum_{i=1}^{2} \lambda_i A_i$, $b_\lambda = \sum_{i=1}^{2} \lambda_i b_i$, $0 < \lambda_i < 1$, and $\sum_{i=1}^{2} \lambda_i = 1$.

The switching rule (20) can be expressed as

$$\sigma(t) = \arg\min_{i \in \underline{m}} \left\{ e_r^T(t) T_i e_r(t) \right\}, \tag{29}$$

where

$$T_i = A_i^T P + P A_i + \varepsilon^{-2} N_i^T N_i + \varepsilon^2 P M_i M_i^T P \\ + \gamma^{-2} P D_i D_i^T P + C_i^T C_i. \tag{30}$$

From (18), we can easily obtain

$$T_i = A_i^T P + P A_i + \varepsilon^{-2} N_i^T N_i + \varepsilon^2 P M_i M_i^T P \\ + \gamma^{-2} P D_i D_i^T P + C_i^T C_i \leq 0. \tag{31}$$

Denote

$$S = \left\{ x \in R^n \mid (x - x_r)^T (T_1 - T_2)(x - x_r) = 0 \right\}. \tag{32}$$

Then the activation regions of subsystems 1 and 2 can be defined as follow:

$$\Psi_1 = \left\{ x \in R^n \mid (x - x_r)^T T_1 (x - x_r) < (x - x_r)^T T_2 (x - x_r) \right\}, \\ \Psi_2 = \left\{ x \in R^n \mid (x - x_r)^T T_1 (x - x_r) > (x - x_r)^T T_2 (x - x_r) \right\}. \tag{33}$$

Therefore, the switching rule (20) in the case of two subsystems can be rewritten as

$$\sigma(x) = \begin{cases} 1, & \text{if } (x - x_r)^T (T_1 - T_2)(x - x_r) < 0, \\ 2, & \text{if } (x - x_r)^T (T_1 - T_2)(x - x_r) > 0, \end{cases} \tag{34}$$

where

$$T_1 = A_1^T P + P A_1 + \varepsilon^{-2} N_1^T N_1 + \varepsilon^2 P M_1 M_1^T P \\ + \gamma^{-2} P D_1 D_1^T P + C_1^T C_1, \\ T_2 = A_2^T P + P A_2 + \varepsilon^{-2} N_2^T N_2 + \varepsilon^2 P M_2 M_2^T P \\ + \gamma^{-2} P D_2 D_2^T P + C_2^T C_2, \tag{35}$$

with A_i, M_i, N_i, C_i, and D_i, $i = 1, 2$, have been given in (3), (4), and (6).

Remark 7. In Theorem 6, a method of robust H_∞ switching rule design is proposed for Boost converters. It is worth pointing out that the proposed design method can also be applied to other types of converters that can be described by switched affine system models, such as Buck converters and Buck-Boost converters.

Remark 8. It can be seen from (34) that the matrix P is necessary to the implementation of the switching rule (34); because the matrix P defined in Theorem 6 does not depend on each $\lambda \in \Lambda$ associated with x_r. Hence, for any equilibrium point $x_r = [i_r \ u_{cr}]^T$ selected by the designer, the matrix P can be derived from (18), and consequently, the switching rule (34) can be implemented in practice.

4. Simulation Results

To illustrate the obtained result, consider the Boost converter described in Section 2. The parameters of the Boost converter are $E = 12\,\text{V}$, $L = 10\,\text{mH}$, $C = 100\,\mu\text{F}$, $R = 30\,\Omega$, $R_0 = 0.1\,\Omega$, $\Delta R_1 = \sin(\theta t)\Omega$, $\Delta R_2 = \cos(\theta t)\Omega$, $\theta = 100$, and $\omega = 0.001 e^{-100t}$, and the steady-state duty ratio is 0.5. From (3), (4), and (6), we can obtain

$$A_1 = \begin{bmatrix} -10 & 0 \\ 0 & -331.3 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -10 & -100 \\ 10000 & -331.3 \end{bmatrix},$$

$$b_1 = \begin{bmatrix} 1200 \\ 0 \end{bmatrix}, \qquad b_2 = \begin{bmatrix} 1200 \\ 0 \end{bmatrix},$$

$$\Delta A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \sin(100t) \begin{bmatrix} 0 & 0 \\ 0 & 11.11 \end{bmatrix}, \tag{36}$$

$$\Delta A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cos(100t) \begin{bmatrix} 0 & 0 \\ 0 & 11.11 \end{bmatrix},$$

$$C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \qquad D_1 = D_2 = \begin{bmatrix} 100 \\ 0 \end{bmatrix}.$$

Let $\lambda_1 = 0.5$, $\lambda_2 = 0.5$, $\varepsilon = 1$, and $\gamma = 1000$; then we can get $x_r = \begin{bmatrix} 1.5788 \\ 23.684 \end{bmatrix}$ by (28). Solving (18), the matrix P can be obtained as follows:

$$P = \begin{bmatrix} 19.2121 & -0.0031 \\ -0.0031 & 0.1922 \end{bmatrix}. \tag{37}$$

From (34), the desired switching rule can be derived. Figures 2–4 shows the response of the previous Boost converter, where the initial state is given by $x_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$.
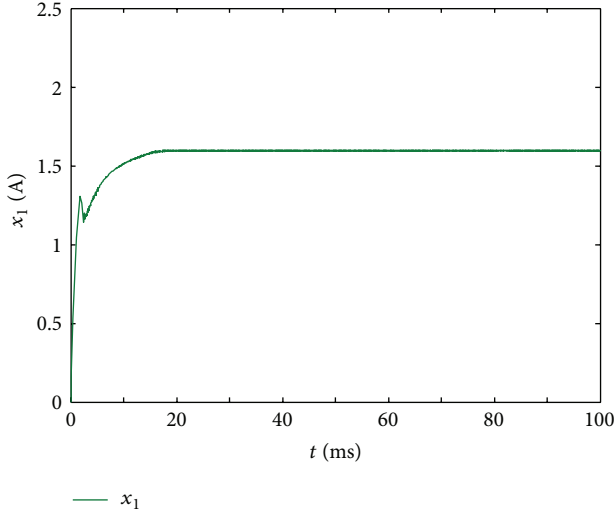
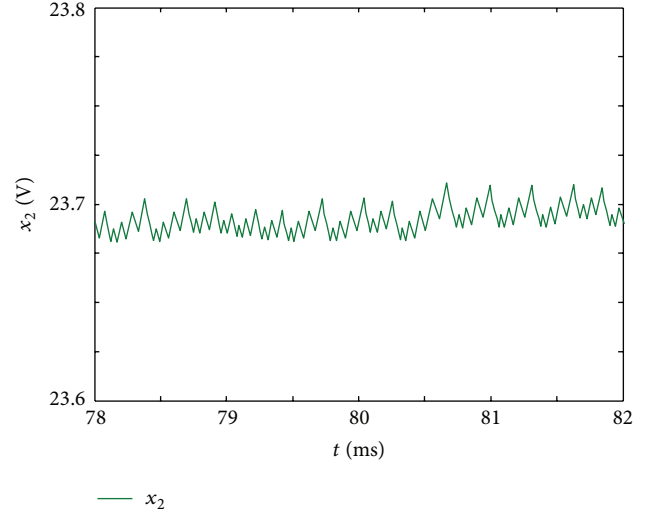FIGURE 2: Response of inductor current.

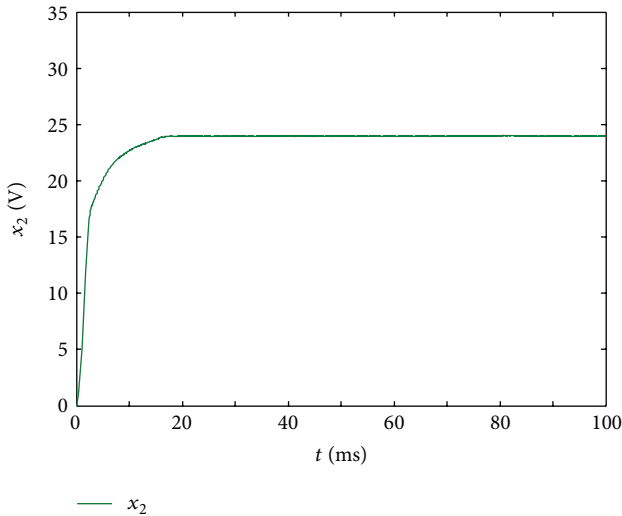FIGURE 4: Response of output voltage in steady state.

FIGURE 3: Response of output voltage.

From Figures 2 and 3, it can be observed that the inductor current and capacitance voltage u_c can track the reference current and the reference voltage rapidly, respectively. Therefore, the converter with the switching rule (34) has an H_∞ model reference tracking performance. This demonstrates the effectiveness of the proposed result.

In addition, for this example, when the value of γ is decreased to 393, the feasible solution of (18) cannot be found. Thus, it can be obtained that a larger γ is favorable for the feasibility of matrix inequality (18). On the other hand, we can get from Remark 8 that the solution of (18) does not dependent on the equilibrium point x_r. But the proposed switching rule depends on the equilibrium point x_r, which can be seen from (34).

5. Conclusions

The problem of robust H_∞ switching rule design for Boost converters with uncertain parameters and disturbance has been discussed in this paper. Firstly, a switched linear affine system model for the Boost converter under consideration is built. Then, the state-dependent switching rule is designed for the switched linear affine systems by using common Lyapunov function technique. The simulation result is given to demonstrate that the proposed switching rule can guarantee the H_∞ model reference tracking performance. Our further work will focus on extending the proposed design method to other types of converters. We will also consider the robust H_∞ switching rule design based on min-projection strategy.

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