Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 230871, 3 pages
http://dx.doi.org/10.1155/2013/230871

Research Article

Analysis of Stability of Traveling Wave for Kadomtsev-Petviashvili Equation

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Received 31 January 2013; Accepted 4 February 2013

Academic Editor: de Dai

This paper presents the boundedness and uniform boundedness of traveling wave solutions for the Kadomtsev-Petviashvili (KP) equation. They are discussed by means of a traveling wave transformation and Lyapunov function.

1. Introduction

We consider the Kadomtsev-Petviashvili (KP) equation:

$$u_{tx} + 6u_x u_{xx} + u_{xxxx} + u_{yy} + cu = 0. \tag{1}$$

It is well known that Kadomtsev-Petviashvili equation arises in a number of remarkable nonlinear problems both in physics and mathematics. By using various methods and techniques, exact traveling wave solutions, solitary wave solutions, doubly periodic solutions, and some numerical solutions have been obtained in [1–6].

In this paper, (1) can be changed into an ordinary differential equation by using traveling wave transformation; the boundedness and uniform boundedness of solution for the resulting ordinary differential equation are discussed using the method of Lyapunov function.

2. The Boundedness

Taking a traveling wave transformation $\xi = \alpha x + \beta y + \gamma t$ in (1), then (1) can be transformed into the following form:

$$u^{(4)} + \left(\frac{\gamma}{\alpha^3} + \frac{\beta^2}{\alpha^4} + \frac{6}{\alpha^2}u\right)u'' + \frac{6}{\alpha^2}u'^2 + \frac{c}{\alpha^4}u = 0. \tag{2}$$

In general, we use the following system, which is equivalent to (2):

$$\begin{aligned} u^{(4)} + au''' + f\left(t,u,u''\right) + g\left(u'\right) + du \\ = p\left(t,u,u',u'',u'''\right), \end{aligned} \tag{3}$$

where

$$f\left(t,u,u'\right) = \left(\frac{\gamma}{\alpha^3} + \frac{\beta^2}{\alpha^4} + \frac{6}{\alpha^2}u\right)u'', \qquad g\left(u'\right) = \frac{6}{\alpha^2}u'^2,$$
$$p\left(t,u,u',u'',u'''\right) = -au''', \qquad d = \frac{c}{\alpha^4}. \tag{4}$$

We consider the following system, which is equivalent to (3):

$$\begin{aligned} x_1' = x_2, \qquad x_2' = x_3, \qquad x_3' = x_4, \\ x_4' = -ax_4 - f\left(t,x_1,x_2,x_3\right) - g\left(x_2\right) - dx_1 \\ + p\left(t,x_1,x_2,x_3,x_4\right). \end{aligned} \tag{5}$$

Theorem 1. *If the following conditions hold for the system* (5)*:*

(i) *there are positive constants a, b, d, δ, k, and λ such that*

$$k \le b^2\lambda, \quad ab\frac{g\left(x_2\right)}{x_2} - \left[\frac{g\left(x_2\right)}{x_2}\right]^2 - a^2 d \ge \delta, \quad \left(x_2 \ne 0\right). \tag{6}$$

(ii) $f(t, x_1, x_2, 0) = 0$, $0 \le f(t, x_1, x_2, x_3)/x_3 - b \le 2\delta\lambda/k$ $(x_2 \ne 0)$.

(iii) $x_3 f_t'(t, x_1, x_2, x_3) + x_2 x_3 f_{x_1}'(t, x_1, x_2, x_3) + x_3^2 f_{x_2}'(t, x_1, x_2, x_3) \le 0$.

(iv) $|p(t, x_1, x_2, x_3, x_4)| \le q(t)(x_1^2 + x_2^2 + x_3^2 + x_4^2)^{1/2}$, *where* $q(t)$ *is a nonnegative continuous function and* $\int_0^\infty q(t)dt < \infty$.

Then, all the solutions of system (5) *are bounded.*

Proof. We first construct the Lyapunov function $V = V(t, x_1, x_2, x_3, x_4)$ defined by

$$\begin{aligned} V &= b^2(2x_4 + ax_3 + bx_2)^2 + 2bd(2x_3 + ax_2 + bx_1)^2 \\ &\quad + \left(b^2 - 4d\right)(ax_4 + bx_2)^2 + 4ab^2 \\ &\quad \times \int_0^{x_2} \left[\frac{g(x_2)}{x_2} - \frac{ad}{b}\right] x_2 dx_2 \\ &\quad + \left[2b\left(b^2 - 4d\right) + 4a^2 d\right] x_3^2 \\ &\quad + 8b^2 \int_0^{x_3} \left[\frac{f(t, x_1, x_2, x_3)}{x_3} - b\right] x_3 dx_3. \end{aligned} \tag{7}$$

It follows from conditions (i) and (ii) that

$$\begin{gathered} b^2 - 4d \ge 0, \\ 0 \le \int_0^{x_2} \left[\frac{g(x_2)}{x_2} - \frac{ad}{b}\right] x_2 dx_2 \le \frac{a\left(b^2 - d\right)}{2b} x_2^2, \\ 0 \le \int_0^{x_3} \left[\frac{f(t, x_1, x_2, x_3)}{x_3} - b\right] x_3 dx_3 \le \frac{\delta\lambda}{k} x_3^2. \end{gathered} \tag{8}$$

Summing up the above discussions, we get

$$V \ge 2b\left(b^2 - 4d\right) x_3^2 + 4a^2 d x_3^2. \tag{9}$$

Thus, we deduce that the function $V(t, x_1, x_2, x_3, x_4)$ defined in (7) is a positive definite function which has infinite inferior limit and infinitesimal upper limit. Hence, there exsits a positive constant $\varepsilon(>0)$ such that

$$V(t, x_1, x_2, x_3, x_4) \ge \varepsilon\left(x_1^2 + x_2^2 + x_3^2 + x_4^2\right). \tag{10}$$

Taking the total derivative of (7) with respect to t along the trajectory of (5), we obtain

$$\begin{aligned} \frac{dV}{dt} &= -2ab^2\left[x_4 + \frac{1}{a}g(x_2)\right]^2 \\ &\quad - 2b^3 x_2 x_3 \left[\frac{f(t, x_1, x_2, x_3)}{x_3} - b\right] \\ &\quad - 2ab^2 \left[\frac{f(t, x_1, x_2, x_3)}{x_3} - b\right] x_3^2 - \frac{2b^2}{a} \\ &\quad \times \left[ab\frac{g(x_2)}{x_2} - \frac{g^2(x_2)}{x_2^2} - a^2 d\right] x_2^2 \\ &\quad + 4b^2 \int_0^{x_3} f_t'(t, x_1, x_2, x_3) dx_3 \\ &\quad + 4b^2 x_2 \int_0^{x_3} f_{x_1}'(t, x_1, x_2, x_3) dx_3 \\ &\quad + 4b^2 x_3 \int_0^{x_3} f_{x_2}'(t, x_1, x_2, x_3) dx_3 \\ &\quad + 2b^2 (bx_2 + ax_3 + 2x_4) p(t, x, x_2, x_3, x_4). \end{aligned} \tag{11}$$

By using conditions (i) and (iii), it follows that

$$\begin{aligned} \frac{dV}{dt} &\le -\frac{2b^2\delta}{a} x_2^2 - 2b^3 x_2 x_3 \left[\frac{f(t, x_1, x_2, x_3)}{x_3} - b\right] \\ &\quad - 2ab^2 \left[\frac{f(t, x_1, x_2, x_3)}{x_3} - b\right] x_3^2 \\ &\quad + 2b^2 (bx_2 + ax_3 + 2x_4) p(t, x, x_2, x_3, x_4). \end{aligned} \tag{12}$$

According to (ii), we have

$$\begin{aligned} & 2b^3 x_2 x_3 \left[\frac{f(t, x_1, x_2, x_3)}{x_3} - b\right] \\ &\quad + 2ab^2 \left[\frac{f(t, x_1, x_2, x_3)}{x_3} - b\right] x_3^2 \\ &= -\frac{b^4}{2a} \left[\frac{f(t, x_1, x_2, x_3)}{x_3} - b\right] x_2^2 \\ &\quad + \frac{2b^2}{a} \left[\frac{f(t, x_1, x_2, x_3)}{x_3} - b\right] \cdot \left(ax_3 + \frac{b}{2}x_2\right)^2 \\ &\ge -\frac{b^4}{2a} \left[\frac{f(t, x_1, x_2, x_3)}{x_3} - b\right] x_2^2 \\ &= -\frac{b^4}{2a} \cdot \frac{2\delta\lambda}{k} x_2^2 = -\frac{b^4\delta\lambda}{ak} x_2^2. \end{aligned} \tag{13}$$

Hence,

$$\begin{aligned}\frac{dV}{dt} \le &-\frac{2b^2\delta}{a}x_2^2+\frac{b^4\delta\lambda}{ak}x_2^2\\&+2b^2\left(bx_2+ax_3+2x_4\right)p\left(t,x,x_2,x_3,x_4\right)\\=&-\frac{b^2\delta}{a}x_2^2+\frac{b^2\delta}{ak}\left(b^2\lambda-k\right)x_2^2\\&+2b^2\left(4+a^2+b^2\right)^{1/2}\left(x_2^2+x_3^2+x_4^2\right)^{1/2}\\&\times\left(x_2^2+x_3^2+x_4^2\right)^{1/2}q\left(t\right)\\\le&-\frac{b^2\delta}{a}x_2^2+2b^2\left(4+a^2+b^2\right)^{1/2}\left(x_2^2+x_3^2+x_4^2\right)q\left(t\right)\\\le&-\frac{b^2\delta}{a}x_2^2+2b^2\left(4+a^2+b^2\right)^{1/2}\cdot q\left(t\right)\cdot\frac{V}{\varepsilon}\\\le&\ 2b^2\left(4+a^2+b^2\right)^{1/2}\cdot\frac{q\left(t\right)}{\varepsilon}\cdot V\equiv\varphi\left(V,t\right).\end{aligned} \tag{14}$$

Thus, all the solutions of system (5) are bounded. □

Theorem 2. *Let conditions (i)–(iv) of Theorem 1 be satisfied for the system* (5)*, and let the following condition hold:*

$$\left(4+a^2+b^2\right)^{1/2}\cdot\frac{q\left(t\right)}{\varepsilon}\cdot V-\frac{\delta}{a}x_2^2\le 0. \tag{15}$$

Then, all the solutions of system (5) *are uniformly bounded.*

Proof. It is clear that the function $V(t,x_1,x_2,x_3,x_4)$ defined in (7) satisfies the conditions (15), therefore, all the solutions of system (5); are uniformly bounded [7]. □

Acknowledgments

This work was financially supported by the Chinese Natural Science Foundation (11061028) and Yunnan Natural Science Foundation (2010CD086).

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