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Research Article

Stability and Permanence of a Pest Management Model with Impulsive Releasing and Harvesting

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We formulate a pest management model with periodically releasing infective pests, immature and mature natural enemies, and harvesting pests and crops at two different fixed moments. Sufficient conditions ensuring the locally and globally asymptotical stability of the susceptible pest-eradication period solution are found by means of Floquet theory, small amplitude perturbation techniques, and multicomparison results. Furthermore, the permanence of system is also derived. By numerical analysis, we also show that impulsive releasing and harvesting at two different fixed moments can bring obvious effects on the dynamics of system, which also corroborates our theoretical results.

1. Introduction

As is known to all, pest outbreaks often cause serious ecological and economic problems. Therefore, how to effectively control insects and other arthropods has become an increasingly complex issue. Usually, chemical pesticides were taken as a relatively simple way to solve the pest-related problems, and some mathematical models on pest management with toxin (pesticide) input were studied in [1–4]. However, the overuse of chemical pesticides may create new ecological and sociological harm such as pesticide pollution and pesticide-resistant pest varieties and inflicts harmful effects on humans and so forth. Therefore, nonchemical use instead for pest control has become a hot topic in order to reduce pest density to tolerable levels and minimize the damage caused. For instance, biological control methods by periodically releasing infective pests or their natural enemies are often taken due to their advantage in the aspects of self-sustainable mechanism, lower environmental impact, and cost effectiveness.

Recently, some biocontrol models on pest management described by impulsive differential equation were proposed and the dynamics such as stability, permanence, periodicity, and bifurcation are deeply investigated (see also, e.g., [2–12]). In [5], an impulsive system to model the process of periodic releasing natural enemies and harvesting pest at different fixed time for pest control is considered, and the sufficient conditions on the existence and global stability of the periodic solution are derived for the given model. Georgescu et al. [6, 7] construct an integrated pest management model which relies on the simultaneous periodic release of infective pest individuals and of natural predators with age structure and obtain some sufficient conditions on the local and global stability, permanence, and bifurcation of the systems. However, most of the existing models on pest management scarcely take into account the factor on the relation between pest and its food (e.g., crop). In fact, farmers may harvest crops several times in process of its growth, which should cause a great impact on the density of the pest.

Motived by the above discussion, we construct a model of pest control by periodically releasing infective pests, immature and mature natural enemies, and harvesting pests and crops. To account for the discontinuity of release and harvest at different fixed moments, our model is based on impulsive differential equations. We analyze the dynamical behavior of the system by using the theory of impulsive differential equation introduced in [13–15].

The rest of this paper is organized as follows. A pest management model with impulsive releasing and harvesting is introduced in Section 2 and some useful preliminaries are given in Section 3. Section 4 deals with stability and permanence analysis of system. In this section, two sufficient conditions are deduced including the locally and globally asymptotical stability of the susceptible pest-eradication period solution, the permanence of system is also discussed. A simple example and conclusions are given in Section 5.

2. Model Description

In the following, to establish our pest management model, we rely on the following biological assumptions.

($A1$) The pest population is divided into two classes, the susceptible and infective. The infective pests neither recover nor reproduce and infective pests cannot damage crops. The disease is transmitted from infective pests to susceptible pests and does not propagate to predators.

($A2$) In the absence of susceptible pests, the crops have a logistic growth rate with intrinsic birth rate r and carrying capacity K.

($A3$) The predators (natural enemies) have an age structure, that is, immature and mature. Only the mature predators have the ability to feed on susceptible pests, but do not prey on infective pests and crops.

($A4$) The functional response of the susceptible pest is described by the abstract function P_1, the functional response of the mature predator is described by the abstract function P_2, and the infection rate is described by the abstract function g, where P_1, P_2, and g satisfy certain assumptions outlined below.

On the basis of the above assumptions, we establish the following impulsively controlled system:

$$\begin{aligned}
&x'(t) = rx(t)\left(1-\frac{x(t)}{K}\right) - P_1(x(t))S(t),\\
&S'(t) = \beta P_1(x(t))S(t) - g(I(t))S(t)\\
&\qquad - P_2(S(t))y_M(t) - d_S S(t),\\
&I'(t) = g(I(t))S(t) - d_I I(t),\\
&y_J'(t) = \lambda P_2(S(t))y_M(t) - d_J y_J(t) - m y_J(t),\\
&y_M'(t) = m y_J(t) - d_M y_M(t),\\
&\qquad\qquad t \neq (n+\tau-1)T,\\
&\qquad\qquad t \neq nT,\\
&\Delta x(t) = -\delta x(t),\\
&\Delta S(t) = -P_S S(t),\\
&\Delta I(t) = -P_I I(t),\\
&\Delta y_J(t) = -P_J y_J(t),\\
&\Delta y_M(t) = -P_M y_M(t),\\
&\qquad t = (n+\tau-1)T,\\
&\Delta x(t) = 0,\\
&\Delta S(t) = 0,\\
&\Delta I(t) = \delta_I,\\
&\Delta y_J(t) = \delta_J,\\
&\Delta y_M(t) = \delta_M,\\
&t = nT,\ n \in \mathbb{N},
\end{aligned} \tag{1}$$

where $x(t)$ represents the density of the crop at time t, $S(t)$ represents the density of the susceptible pest at time t, $I(t)$ represents the density of the infective pest at time t, $y_J(t)$ and $y_M(t)$ represent the density of the immature and mature predator at time t, respectively; r is the logistic intrinsic growth rate of the crop in the absence of the susceptible pest, K is its carrying capacity; $0 < \beta, \lambda \le 1$ represent the conversion rate at which ingested preys in excess of what is needed for maintenance is translated into predator population increase; m is the rate at which the immature predators become the mature predators. $d_S, d_I, d_J, d_M > 0$ are the death rates of the susceptible pest population, infective pest population, and of the immature and mature predator population, respectively; $\Delta x(t) = x(t^+) - x(t)$, $\Delta S(t) = S(t^+) - S(t)$, $\Delta I(t) = I(t^+) - I(t)$, $\Delta y_J(t) = y_J(t^+) - y_J(t)$, $\Delta y_M(t) = y_M(t^+) - y_M(t)$; T is the period of the impulsive effect; δ $(0 \le \delta < (1-e^{-rT})/2)$ is the harvesting rate of crop population; $0 \le P_S, P_I, P_J, P_M < 1$ denote the transfer rate of susceptible pest population, infective pest population, immature and mature predator population at every impulsive period $(n+\tau-1)T$ $(n \in \mathbb{N}, 0 < \tau < 1)$, respectively; δ_I, δ_J, $\delta_M > 0$ represents the amount of infective pests, immature and mature predators, respectively, which are released at every impulsive period nT $(n \in \mathbb{N})$, respectively; Also, $P_1(\cdot)$, $P_2(\cdot)$, $g(\cdot) \in H$, here $H = \{f : R \to R \mid f(0) = 0, f'(x) > 0$ and $f''(x) \le 0$ for all $x > 0\}$.

Some familiar examples of functions $f \in H$ in the biological literature include

($F1$) $f_1(x) = ax$, with $a > 0$;

($F2$) $f_2(x) = ax/(1+bx)$, with $a, b > 0$;

($F3$) $f_3(x) = a(1-e^{-bx})$, with $a, b > 0$,

where functions ($F1$) and ($F2$) are known as Holling type functional responses (see, [16–26]), and ($F3$) belongs to Ivlev type functional responses (see, [27–30]).

3. Preliminaries

In this section, we will give some definitions and lemmas, which will be useful for our main results. Let $\mathbb{R}_+ = [0,\infty)$ and $\mathbb{R}_+^5 = \{X = (x(t), S(t), I(t), y_J(t), y_M(t)) \in \mathbb{R}^5 \mid x(t), S(t), I(t), y_J(t), y_M(t) \ge 0\}$. Denote $f = (f_x, f_S, f_I, f_J, f_M)^T$ the map defined by the right hand of the first five equations in system (1). Let $V : \mathbb{R}_+ \times \mathbb{R}_+^5 \to \mathbb{R}_+$, then $V \in V_0$ if

(1) V is continuous in $((n-1)T,(n+\tau-1)T]\times\mathbb{R}_+^5$, $((n+\tau-1)T,nT]\times\mathbb{R}_+^5$ and for each $x\in R_+^5$, $n\in\mathbb{N}$, $\lim_{(t,y)\to((n+\tau-1)T^+,x)}V(t,y)=V((n+\tau-1)T^+,x)$ and $\lim_{(t,y)\to(nT^+,x)}V(t,y)=V(nT^+,x)$ exist.

(2) V is locally Lipschitzian x.

Definition 1. Letting $V\in V_0$, one defines the upper right derivative of V with respect to the impulsive differential system (1) at $(t,x)\in((n-1)T,(n+\tau-1)T]\times\mathbb{R}_+^5$ and $((n+\tau-1)T,nT]\times\mathbb{R}_+^5$ by

$$D^+V(t,x)=\limsup_{h\to0^+}\frac{1}{h}\left[V(t+h,x+hf(t,x))-V(t,x)\right]. \tag{2}$$

Definition 2. The system (1) is said to be permanent if there are positive constants m, $M>0$ and a finite time T_0 such that all solutions of (1) with initial values $x(0^+)$, $S(0^+)$, $I(0^+)$, $y_J(0^+)$, $y_M(0^+)$, $m\le x(t),S(t),I(t),y_J(t),y_M(t)\le M$ hold for all $t\ge T_0$, where m and M are independent of initial value, T_0 may depend on initial value.

Remark 3. The global existence and uniqueness of system (1) is guaranteed by the smoothness properties of f (for details, see [13, 14]).

Lemma 4 (see [15]). *Let $V:\mathbb{R}_+\times\mathbb{R}^n\to\mathbb{R}_+^m$ satisfy $V_i\in V_0$, $i=1,2,\ldots,m$, and assume that*

$$\begin{aligned}
&D^+V(t,x(t))\le(\ge)\,g(t,V(t,x(t))),\quad t\ne(k+\tau-1)T,\ kT,\\
&V(t,x(t^+))\le(\ge)\,\psi_k^\tau(V(t,x(t))),\quad t=(k+\tau-1)T,\\
&V(t,x(t^+))\le(\ge)\,\psi_k(V(t,x(t))),\quad t=kT,\ k\in\mathbb{N},\\
&x(0^+)=x_0,
\end{aligned} \tag{3}$$

where $g:\mathbb{R}_+\times\mathbb{R}_+^m\to\mathbb{R}_+^m$ is continuous in $((k-1)T,(k+\tau-1)T]\times\mathbb{R}^m$ and $((k+\tau-1)T,kT]\times\mathbb{R}^m$, for each $p\in\mathbb{R}^m$, $k=1,2,\ldots$, the limit $\lim_{(t,q)\to((k+\tau-1)T^+,p)}g(t,q)=g((k+\tau-1)T^+,p)$ and $\lim_{(t,q)\to((k-1)T^+,p)}g(t,q)=g((k-1)T^+,p)$ exists. $g(t,q)$ is quasimonotone nondecreasing in q. $\psi_k^\tau,\psi_k:\mathbb{R}_+^m\to\mathbb{R}_+^m$ is nondecreasing for all $k\in\mathbb{N}$. Let $\theta(t)$ be the maximal (minimal) solution of the following impulsive differential equation on $[0,\infty)$:

$$\begin{aligned}
&w'(t)=g(t,w(t)),\quad t\ne(k+\tau-1)T,\ kT,\\
&w(t^+)=\psi_k^\tau(w(t)),\quad t=(k+\tau-1)T,\\
&w(t^+)=\psi_k(w(t)),\quad t=kT,\ k\in\mathbb{N},\\
&w(0^+)=w_0.
\end{aligned} \tag{4}$$

Then for any solution $x(t)$ of the system (3), $V(0^+,x_0)\le(\ge)w_0$ implies that $V(t,x(t))\le(\ge)\theta(t)$ for all $t\ge0$.

Lemma 5 (see [13, 15]). *Consider the following system:*

$$\begin{aligned}
&v'(t)\le(\ge)\,p(t)v(t)+q(t),\quad t\ne t_k,\\
&v(t_k^+)\le(\ge)\,d_kv(t_k)+b_k,\quad t=t_k,\ k\in\mathbb{N},\\
&v(0^+)\le(\ge)\,v_0,
\end{aligned} \tag{5}$$

where $p,q\in PC(\mathbb{R}_+,\mathbb{R})$ and $d_k\ge0$, v_0 and b_k are constants. Suppose that

($A1$) *the sequence t_k satisfies $0\le t_1\le t_2<\cdots$, with $\lim_{t\to\infty}t_k=\infty$;*

($A2$) *$v\in PC'(\mathbb{R}_+,\mathbb{R})$ and $v(t)$ is left-continuous at t_k, $k\in\mathbb{N}$.*

Then, for $t>0$,

$$\begin{aligned}
v(t)\le\ (\ge)\,&v_0e^{\int_0^tp(s)ds}\prod_{0<t_k<t}d_k\\
&+\sum_{0<t_k<t}\left(\prod_{t_k<t_j<t}d_je^{\int_{t_k}^tp(s)ds}\right)b_k\\
&+\int_0^t\left(\prod_{s<t_k<t}d_k\right)e^{\int_s^tp(\tau)d\tau}q(s)\,ds.
\end{aligned} \tag{6}$$

Lemma 6. *There exists a constant $M=\max\{(1/\lambda)((L/d)+(\rho e^{dT}/(e^{dT}-1))),K\}>0$, such that $x(t)$, $S(t)$, $I(t)$, $y_J(t)$, $y_M(t)\le M$ for each solution of (1) with t large enough.*

Proof. Since $x'(t)\le rx(1-(x(t)/K))$, then $x'(t)|_{x(t)=K}\le0$, and $x((n+\tau-1)T^+)\le x((n+\tau-1)T)$ $(0<\delta<1)$, so $x(t)\le K$ for t large enough. Let us define $V(t)\in V_0$ by $V(t)=\lambda\beta x(t)+\lambda S(t)+\lambda I(t)+y_J(t)+y_M(t)$ and denote $d=\min\{d_S,d_I,d_J,d_M\}$. Then, it is obvious that

$$\begin{aligned}
&\frac{dV(t)}{dt}+dV(t)\le\lambda\beta(r+d)x(t)-\frac{\lambda\beta rx^2(t)}{K},\\
&\qquad t\ne(n+\tau-1)T,\ t\ne nT.
\end{aligned} \tag{7}$$

Since the right-hand side (7) is bounded from above by $L=K\lambda\beta(r+d)^2/4r$, it follows that

$$\frac{dV(t)}{dt}+dV(t)\le L,\quad t\ne(n+\tau-1)T,\ t\ne nT. \tag{8}$$

When $t=(n+\tau-1)T$ and $t=nT$, it is easy to obtain that

$$\begin{aligned}
&V((n+\tau-1)T^+)\le V((n+\tau-1)T),\\
&V(nT^+)=V(nT)+(\lambda\delta_I+\delta_J+\delta_M).
\end{aligned} \tag{9}$$

Then, by Lemma 5, we can obtain that

$$V(t) \le V(0) e^{-dt} + \int_0^t L e^{-d(t-s)} ds + \sum_{0<kT<t} \rho e^{-d(t-kT)} \longrightarrow \frac{L}{d} + \frac{\rho e^{dT}}{e^{dT}-1}, \quad t \longrightarrow \infty, \tag{10}$$

where $\rho = \lambda\delta_I + \delta_J + \delta_M$. So it follows that $V(t)$ is uniformly bounded on $[0, \infty)$. The proof is completed. □

Lemma 7 (see [31]). *Let one consider the following impulsive control subsystem:*

$$\begin{aligned} x'(t) &= rx(t)\left(1 - \frac{x(t)}{K}\right), \quad t \ne (n+\tau-1)T, \\ \Delta x(t) &= -\delta x(t), t = (n+\tau-1)T. \end{aligned} \tag{11}$$

Suppose $\delta_0^* = 1 - e^{-rT}$. Then one has the following results.

(1) *If $\delta > \delta_0^*$, then the trivial periodic solution of system* (11) *is locally asymptotically stable.*

(2) *If $\delta < \delta_0^*$, then the system* (11) *has a unique positive periodic solution $x^*(t)$, which is globally asymptotically stable, where*

$$\begin{aligned} x^*(t) &= \frac{K\left(1-\delta-e^{-rT}\right)}{1-\delta-e^{-rT}+\delta e^{-r(t-(n+\tau-1)T)}}, \\ & \quad t \in ((n+\tau-1)T, (n+\tau)T], n \in \mathbb{N}, \\ x^*(0^+) = x^*(nT^+) &= \frac{K\left((1-\delta)e^{rT}-1\right)}{\left(e^{r\tau T}-1\right)+(1-\delta)\left(e^{rT}-e^{r\tau T}\right)}. \end{aligned} \tag{12}$$

Remark 8. From Lemma 7, we have

(1a) if $\delta_0^* > 2\delta$, then $x^*(t) > K/2$ for all $t \ge 0$;

(2a) if $t \in ((n-1)T, nT]$, $n \in \mathbb{N}$, then the periodic solution $x^*(t)$ can be rewritten in the form

$$x^*(t) = \begin{cases} \dfrac{K\left(1-\delta-e^{-rT}\right)}{1-\delta-e^{-rT}+\delta e^{-r(T-\tau T)}e^{-r(t-(n-1)T)}}, \\ \qquad t \in ((n-1)T, (n+\tau-1)T], \\ \dfrac{K\left(1-\delta-e^{-rT}\right)}{1-\delta-e^{-rT}+\delta e^{-r(t-(n+\tau-1)T)}}, \\ \qquad t \in ((n+\tau-1)T, nT], n \in \mathbb{N}. \end{cases} \tag{13}$$

Lemma 9. *Let one consider the following impulsive control subsystem:*

$$\begin{aligned} z'(t) &= a(t) - dz(t), \quad t \ne (n+\tau-1)T,\ t \ne nT, \\ \Delta z(t) &= -pz(t), \quad t = (n+\tau-1)T, \\ \Delta z(t) &= \delta, \quad t = nT,\ n \in \mathbb{N}, \\ z(0^+) &= z_0, \end{aligned} \tag{14}$$

where $a(t)$ is a T-periodic $PC(\mathbb{R}_+, \mathbb{R})$ function. p, d are the positive real constants and $p < 1$. Then system (14) *has a unique T-periodic solution $z^*(t)$, and for each solution $z(t)$ of* (14), *$z(t) \to z^*(t)$ as $t \to \infty$, where*

$$\begin{aligned} z^*(t) &= e^{-d(t-(n-1)T)}\left(z^*(0^+) + \int_0^{t-(n-1)T} a(s)e^{ds}ds\right), \\ & \qquad t \in ((n-1)T, (n+\tau-1)T], \\ z^*(t) &= e^{-d(t-(n-1)T)}\left(z^*(\tau T^+)e^{d\tau T} + \int_{\tau T}^{t-(n-1)T} a(s)e^{ds}ds\right), \\ & \qquad t \in ((n+\tau-1)T, nT], \\ z^*(0^+) &= \frac{\left[(1-p)\int_0^{\tau T} a(s)e^{ds}ds + \int_{\tau T}^{T} a(s)e^{ds}ds\right]e^{-dT} + \delta}{1-(1-p)e^{-dT}}, \\ z^*(\tau T^+) &= \frac{(1-p)\left[\int_0^{\tau T} a(s)e^{ds}ds + e^{-dT}\int_{\tau T}^{T} a(s)e^{ds}ds + \delta\right]e^{-d\tau T}}{1-(1-p)e^{-dT}}. \end{aligned} \tag{15}$$

Proof. First, it is easy to obtain that

$$\begin{aligned} z(t) &= e^{-dt}\left(z(0^+) + \int_0^t a(s)e^{ds}ds\right), \quad t \in (0, \tau T], \\ z(t) &= e^{-d(t-\tau T)}z(\tau T^+) + e^{-dt}\int_{\tau T}^t a(s)e^{ds}ds, \quad t \in (\tau T, T]. \end{aligned} \tag{16}$$

Since the T-periodicity requirement, we have

$$\begin{aligned} z^*(\tau T^+) &= e^{-d\tau T}\left(z^*(0^+) + \int_0^{\tau T} a(s)e^{ds}ds\right)(1-p), \\ z^*(0^+) &= e^{-d(T-\tau T)}z^*(\tau T^+) + e^{-dT}\int_{\tau T}^T a(s)e^{ds}ds + \delta. \end{aligned} \tag{17}$$

By (17), we can obtain that

$$\begin{aligned} z^*(0^+) &= \frac{\left[(1-p)\int_0^{\tau T} a(s)e^{ds}ds + \int_{\tau T}^{T} a(s)e^{ds}ds\right]e^{-dT} + \delta}{1-(1-p)e^{-dT}}, \\ z^*(\tau T^+) &= \frac{(1-p)\left[\int_0^{\tau T} a(s)e^{ds}ds + e^{-dT}\int_{\tau T}^{T} a(s)e^{ds}ds + \delta\right]e^{-d\tau T}}{1-(1-p)e^{-dT}}. \end{aligned} \tag{18}$$

So, we will obtain the T-periodic solution of (14):

$$\begin{aligned} z^*(t) &= e^{-d(t-(n-1)T)}\left(z^*(0^+) + \int_0^{t-(n-1)T} a(s)e^{ds}ds\right), \\ &\qquad t \in ((n-1)T, (n+\tau-1)T], \\ z^*(t) &= e^{-d(t-(n-1)T)}\left(z^*(\tau T^+)e^{d\tau T} + \int_{\tau T}^{t-(n-1)T} a(s)e^{ds}ds\right), \\ &\qquad t \in ((n+\tau-1)T, nT]. \end{aligned} \tag{19}$$

Let $Z(t) = z(t) - z^*(t)$. Substituting $Z(t)$ into (14), we have

$$\begin{aligned} Z'(t) &= -dZ(t), \quad t \neq (n+\tau-1)T,\ t \neq nT, \\ \Delta Z(t) &= -pZ(t), \quad t = (n+\tau-1)T, \\ \Delta Z(t) &= 0, \quad t = nT,\ n \in \mathbb{N}, \\ Z(0^+) &= z_0 - z^*(0^+). \end{aligned} \tag{20}$$

Then, $Z(t) = Z(0^+)e^{-dt}\prod_{0<(n+\tau-1)T<t}(1-p) \to 0$, as $t \to \infty$. The proof is completed. □

4. Main Results

4.1. Local and Global Stability. In this section, we will study the existence and stability of the system (1) susceptible pest-eradication periodic solution $(x^*(t), 0, I^*(t), y_J^*(t), y_M^*(t))$. To this purpose, it is seen first that when $S(t) = 0$, system (1) can be rewritten in the form

$$\begin{aligned} x'(t) &= rx(t)\left(1-\frac{x(t)}{K}\right), \\ I'(t) &= -d_I I(t), \\ y_J'(t) &= -(d_J+m)y_J(t), \\ y_M'(t) &= my_J(t) - d_M y_M(t), \\ &\qquad t \neq (n+\tau-1)T, \\ &\qquad t \neq nT, \\ \Delta x(t) &= -\delta x(t), \\ \Delta I(t) &= -P_I I(t), \\ \Delta y_J(t) &= -P_J y_J(t), \\ \Delta y_M(t) &= -P_M y_M(t), \\ &\qquad t = (n+\tau-1)T, \\ \Delta x(t) &= 0, \\ \Delta I(t) &= \delta_I, \\ \Delta y_J(t) &= \delta_J, \\ \Delta y_M(t) &= \delta_M, \\ &\qquad t = nT, \end{aligned} \tag{21}$$

which describes the dynamics of system (1) in the absence of the susceptible pest population. So, when $t \in ((n-1)T, nT](n \in \mathbb{N})$, we can calculate the T-periodic solution of (21) by Lemmas 7 and 9. It is seen that

$$x^*(t) = \begin{cases} \dfrac{K\left(1-\delta-e^{-rT}\right)}{1-\delta-e^{-rT}+\delta e^{-r(T-\tau T)}e^{-r(t-(n-1)T)}}, \\ \qquad t \in ((n-1)T, (n+\tau-1)T], \\ \dfrac{K\left(1-\delta-e^{-rT}\right)}{1-\delta-e^{-rT}+\delta e^{-r(t-(n+\tau-1)T)}}, \\ \qquad t \in ((n+\tau-1)T, nT], \end{cases} \tag{22}$$

$$I^*(t) = \begin{cases} \dfrac{\delta_I e^{-d_I(t-(n-1)T)}}{1-(1-P_I)e^{-d_I T}}, \\ \qquad t \in ((n-1)T, (n+\tau-1)T], \\ \dfrac{\delta_I(1-P_I)e^{-d_I(t-(n-1)T)}}{1-(1-P_I)e^{-d_I T}}, \\ \qquad t \in ((n+\tau-1)T, nT], \end{cases} \tag{23}$$

$$y_J^*(t) = \begin{cases} \dfrac{\delta_J e^{-(m+d_J)(t-(n-1)T)}}{1-(1-P_J)e^{-(m+d_J)T}}, \\ \qquad t \in ((n-1)T, (n+\tau-1)T], \\ \dfrac{\delta_J(1-P_J)e^{-(m+d_J)(t-(n-1)T)}}{1-(1-P_J)e^{-(m+d_J)T}}, \\ \qquad t \in ((n+\tau-1)T, nT], \end{cases} \tag{24}$$

$$y_M^*(t) = \begin{cases} e^{-d_M(t-(n-1)T)}\left(y_M^*(0^+) + A(t-(n-1)T)\right), \\ \qquad t \in ((n-1)T, (n+\tau-1)T], \\ e^{-d_M(t-(n-1)T)}\left(y_M^*(\tau T^+)e^{d_M\tau T} + B(t-(n-1)T)\right), \\ \qquad t \in ((n+\tau-1)T, nT], \end{cases} \tag{25}$$

$$y_M^*(0^+) = \frac{\left[(1-P_M)A(\tau T) + B(T)\right]e^{-d_M T} + \delta_M}{1-(1-P_M)e^{-d_M T}}, \tag{26}$$

$$y_M^*(\tau T^+) = \frac{(1-P_M)\left[A(\tau T) + e^{-d_M T}B(T) + \delta_M\right]e^{-d_M\tau T}}{1-(1-P_M)e^{-d_M T}}, \tag{27}$$

where

$$\begin{aligned} A(t) &= \frac{m\delta_J\left(e^{(d_M-(m+d_J))t} - 1\right)}{\left(1-(1-P_J)e^{-(m+d_J)T}\right)\left(d_M-(m+d_J)\right)}, \\ &\qquad t \in (0, \tau T], \\ B(t) &= \frac{m\delta_J(1-P_J)\left(e^{(d_M-(m+d_J))t} - e^{(d_M-(m+d_J))\tau T}\right)}{\left(1-(1-P_J)e^{-(m+d_J)T}\right)\left(d_M-(m+d_J)\right)}, \\ &\qquad t \in (\tau T, T]. \end{aligned} \tag{28}$$

To discuss the locally asymptotical stability of the susceptible pest-eradication periodic solution, we now introduce the Floquet theory for a linear impulsive control system:

$$\begin{aligned} \omega'(t) &= A(t)\omega(t), \quad t \neq \tau_k, \\ \Delta\omega(t) &= B_k\omega(t), \quad t = \tau_k,\ k \in \mathbb{N}, \end{aligned} \tag{29}$$

under the following conditions:

*H*1: $A(\cdot) \in \mathrm{PC}(\mathbb{R}, M_n(\mathbb{R}))$ and $A(t+T) = A(t)$ for $t \geq 0$.

*H*2: $B_k \in M_n$, $\det(I_n + B_k) \neq 0$, $\tau_k < \tau_{k+1}$ for $k \in \mathbb{N}$, and I_n denotes the $n \times n$ real identity matrix.

*H*3: There is a $q \in \mathbb{N}$ such that $B_{k+q} = B_k$, $\tau_{k+q} = \tau_k + T$ for $k \in \mathbb{N}$.

Let $\Psi(t)$ be a fundamental matrix of (29), then there is a unique reversible matrix $M \in M_n(\mathbb{R})$ such that $\Psi(t+T) = \Psi(t)M$ for all $t \in \mathbb{R}$, which is called the monodromy matrix of (29) corresponding to Ψ. All monodromy matrices of (29) are similar and they have the same eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$, which are called the Floquet multipliers of (29).

Lemma 10 (see [13] (Floquet theory)). *Let the conditions H1–H3 hold. Then system* (29) *have the following properties*

(1) *stable if and only if all Floquet multipliers λ_i $(1 \leq i \leq n)$ of* (29) *satisfy $|\lambda_i| \leq 1$ and moreover, to those λ_i for which $|\lambda_i| = 1$, there correspond simple elementary divisors;*

(2) *asymptotically stable if and only if all Floquet multipliers λ_i $(1 \leq i \leq n)$ of* (29) *satisfy $|\lambda_i| < 1$;*

(3) *unstable if there is a Floquet multipliers λ_i $(1 \leq i \leq n)$ of* (29) *such that $|\lambda_i| > 1$.*

In the following, we present two main results with the locally and globally asymptotical stability of the susceptible pest-eradication periodic solution $(x^*(t), 0, I^*(t), P_J^*(t), P_M^*(t))$.

Theorem 11. *If*

$$\begin{aligned} &\beta\int_0^T P_1(x^*(t))\,dt - \int_0^T g(I^*(t))\,dt \\ &\quad - P_2'(0)\int_0^T y_M^*(t)\,dt - d_S T < \ln\frac{1}{1-P_S}, \end{aligned} \tag{30}$$

then the susceptible pest-eradication periodic solution $(x^*(t), 0, I^*(t), y_J^*(t), y_M^*(t))$ of system (1) *is locally asymptotically stable.*

Proof. Let $(x(t), S(t), I(t), y_J(t), y_M(t))$ be any solution of system (1). We define error $e_1(t) = x(t) - x^*(t)$, $e_2(t) = S(t)$, $e_3(t) = I(t) - I^*(t)$, $e_4(t) = y_J(t) - y_J^*(t)$, $e_5(t) = y_M(t) - y_M^*(t)$. The linearized system of (1) at $(x^*(t), 0, I^*(t), y_J^*(t), y_M^*(t))$ is

$$\begin{aligned} e_1'(t) &= \left(r - 2r\frac{x^*(t)}{K}\right)e_1(t) - P_1(x^*(t))e_2(t), \\ e_2'(t) &= \left(\beta P_1(x^*(t)) - g(I^*(t)) - P_2'(0)y_M^*(t) - d_S\right)e_2(t), \\ e_3'(t) &= g(I^*(t))e_2(t) - d_I e_3(t), \\ e_4'(t) &= \lambda P_2'(0)y_M^*(t)e_2(t) - (d_J + m)e_4(t), \\ e_5'(t) &= m e_4(t) - d_M e_5(t), \\ &\qquad t \neq (n+\tau-1)T, \\ &\qquad t \neq nT, \\ \Delta e_1(t) &= -\delta e_1(t), \\ \Delta e_2(t) &= -P_S e_2(t), \\ \Delta e_3(t) &= -P_I e_3(t), \\ \Delta e_4(t) &= -P_J e_4(t), \\ \Delta e_5(t) &= -P_M e_5(t), \\ &\qquad t = (n+\tau-1)T, \\ \Delta e_1(t) &= \Delta e_2(t) = \Delta e_3(t) = \Delta e_4(t) = \Delta e_5(t) = 0, \\ &\qquad t = nT. \end{aligned} \tag{31}$$

Let $\Psi(t)$ be the fundamental matrix of (31), then $\Psi(t)$ satisfies

$$\frac{d\Psi(t)}{dt}=\begin{pmatrix} r-2r\frac{x^*(t)}{K} & P_1(x^*(t)) & 0 & 0 & 0\\ 0 & \beta P_1(x^*(t))-g(I^*(t))-P_2'(0)\,y_M^*(t)-d_S & 0 & 0 & 0\\ 0 & g(I^*(t)) & -d_I & 0 & 0\\ 0 & \lambda P_2'(0)\,y_M^*(t) & 0 & -(d_J+m) & 0\\ 0 & 0 & 0 & m & -d_M \end{pmatrix}\Psi(t). \tag{32}$$

Then, a fundamental matrix $\Psi(t)$ $(\Psi(0)=I_4)$ of (31) is

$$\Psi(t)=\begin{pmatrix} e^{\int_0^t(r-2r(x^*(s)/K))ds} & \phi_{12}(t) & 0 & 0 & 0\\ 0 & e^{\int_0^t(\beta P_1(x^*(s))-g(I^*(s))-P_2'(0)y_M^*(s)-d_S)ds} & 0 & 0 & 0\\ 0 & \phi_{32}(t) & e^{-d_It} & 0 & 0\\ 0 & \phi_{42}(t) & 0 & e^{-(d_J+m)t} & 0\\ 0 & \phi_{52}(t) & 0 & \phi_{54}(t) & e^{-d_Mt} \end{pmatrix}, \tag{33}$$

where

$$\begin{aligned}
\phi_{12}(t) &= -e^{-\int_0^t(r-2r(x^*(s)/K))ds}\\
&\quad\times\int_0^t P_1(x^*(s))\,e^{\int_0^s(\beta P_1(x^*(\xi))-g(I^*(\xi))-P_2'(0)y_M^*(\xi)-d_S)d\xi}\\
&\quad\times e^{\int_0^s(r-2r(x^*(\xi)/K))d\xi}ds,\\
\phi_{32}(t)&\\
&= e^{-d_It}\int_0^t g(I^*(s))\,e^{\int_0^s(\beta P_1(x^*(\xi))-g(I^*(\xi))-P_2'(0)y_M^*(\xi)-d_S)d\xi}\\
&\quad\times e^{d_Is}ds,\\
\phi_{42}(t)&\\
&= e^{-d_Jt}\int_0^t \lambda P_2'(0)\,y_M^*(s)\,e^{\int_0^s(\beta P_1(x^*(\xi))-g(I^*(\xi))-P_2'(0)y_M^*(\xi)-d_S)d\xi}\\
&\quad\times e^{d_Js}ds,\\
\phi_{52}(t) &= e^{-d_Mt}\int_0^t m\phi_{42}(s)\,e^{d_Ms}ds,\\
\phi_{54}(t) &= \frac{m\left(e^{-(d_J+m)t}-e^{-d_Mt}\right)}{d_M-(d_J+m)}.
\end{aligned} \tag{34}$$

The resetting impulsive condition of (31) becomes

$$\begin{gathered}
\begin{pmatrix} e_1((n+\tau-1)T^+)\\ e_2((n+\tau-1)T^+)\\ e_3((n+\tau-1)T^+)\\ e_4((n+\tau-1)T^+)\\ e_5((n+\tau-1)T^+) \end{pmatrix}
=\begin{pmatrix} 1-\delta & 0 & 0 & 0 & 0\\ 0 & 1-P_S & 0 & 0 & 0\\ 0 & 0 & 1-P_I & 0 & 0\\ 0 & 0 & 0 & 1-P_J & 0\\ 0 & 0 & 0 & 0 & 1-P_M \end{pmatrix}
\times\begin{pmatrix} e_1((n+\tau-1)T)\\ e_2((n+\tau-1)T)\\ e_3((n+\tau-1)T)\\ e_4((n+\tau-1)T)\\ e_5((n+\tau-1)T) \end{pmatrix},\\
\begin{pmatrix} e_1(nT^+)\\ e_2(nT^+)\\ e_3(nT^+)\\ e_4(nT^+)\\ e_5(nT^+) \end{pmatrix}
=\begin{pmatrix} 1&0&0&0&0\\ 0&1&0&0&0\\ 0&0&1&0&0\\ 0&0&0&1&0\\ 0&0&0&0&1 \end{pmatrix}
\begin{pmatrix} e_1(nT)\\ e_2(nT)\\ e_3(nT)\\ e_4(nT)\\ e_5(nT) \end{pmatrix}.
\end{gathered} \tag{35}$$

Then, it is easy to obtain all eigenvalues of

$$M=\begin{pmatrix} 1-\delta & 0 & 0 & 0 & 0\\ 0 & 1-P_S & 0 & 0 & 0\\ 0 & 0 & 1-P_I & 0 & 0\\ 0 & 0 & 0 & 1-P_J & 0\\ 0 & 0 & 0 & 0 & 1-P_M \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \Psi(T). \tag{36}$$

We have $\lambda_1 = (1-\delta)e^{\int_0^T (r-2r(x^*(s)/K))ds}$, $\lambda_2 = (1-P_S) e^{\int_0^T (\beta P_1(x^*(s))-g(I^*(s))-P_2'(0)y_M^*(s)-d_S)ds}$, $\lambda_3 = (1-P_I)e^{-d_IT} < 1$, $\lambda_4 = (1-P_J)e^{-(d_J+m)T} < 1$ and $\lambda_5 = (1-P_M)e^{-d_MT} < 1$. Since $x^*(t) > (K/2)$, so $\lambda_1 < 1$. By the condition (30), we have $\lambda_2 < 1$. Therefore, according to Lemma 10, the susceptible pest-eradication periodic solution $(x^*(t), 0, I^*(t), y_J^*(t), y_M^*(t))$ of system (1) is locally asymptotically stable. The proof is completed. □

Theorem 12. *If*

$$\beta\int_0^T P_1(x^*(t))\,dt - \int_0^T g(I^*(t))\,dt - \min_{0\le\varpi\le U_S} P_2'(\varpi)\int_0^T y_M^*(t)\,dt - d_ST < \ln\frac{1}{1-P_S}, \tag{37}$$

where U_S is an ultimate boundedness constant for S, then the susceptible pest-eradication periodic solution $(x^*(t), 0, I^*(t), y_J^*(t), y_M^*(t))$ of system (1) is globally asymptotically stable.

Proof. Since

$$\beta\int_0^T P_1(x^*(t))\,dt - \int_0^T g(I^*(t))\,dt - \min_{0\le\varpi\le U_S} P_2'(\varpi)\int_0^T y_M^*(t)\,dt - d_ST < \ln\frac{1}{1-P_S}, \tag{38}$$

we can choose an ε small enough such that

$$\begin{aligned}\beta\int_0^T P_1(x^*(t)+\varepsilon)\,dt - \int_0^T g(I^*(t)-\varepsilon)\,dt \\ - \min_{0\le\varpi\le U_S} P_2'(\varpi)\int_0^T \left(\overline{y_M^*(t)}-\varepsilon\right)dt - d_ST \\ - \ln\frac{1}{1-P_S} = \epsilon < 0,\end{aligned} \tag{39}$$

where $\overline{y_M^*(t)}$ is defined in the following. According to system (1), we have

$$\begin{aligned}
&x'(t) \le rx(t)\left(1-\frac{x(t)}{K}\right),\\
&I'(t) \ge -d_II(t),\\
&y_J'(t) \ge -d_Jy_J(t) - my_J(t),\\
&y_M'(t) = my_J(t) - d_My_M(t),\\
&\qquad t \ne (n+\tau-1)T,\\
&\qquad t \ne nT,\\
&\Delta x(t) = -\delta x(t),\\
&\Delta I(t) = -P_II(t),\\
&\Delta y_J(t) = -P_Jy_J(t),\\
&\Delta y_M(t) = -P_My_M(t),\\
&\qquad t = (n+\tau-1)T,\\
&\Delta x(t) = 0,\\
&\Delta I(t) = \delta_I,\\
&\Delta y_J(t) = \delta_J,\\
&\Delta y_M(t) = \delta_M,\\
&\qquad t = nT,\ n\in\mathbb{N}.
\end{aligned} \tag{40}$$

From the first equation of system (40), we obtain the following comparison system:

$$\begin{aligned}
&v'(t) = rv(t)\left(1-\frac{v(t)}{K}\right), \quad t\ne(n+\tau-1)T,\\
&\Delta v(t) = -\delta v(t), \quad t=(n+\tau-1)T.
\end{aligned} \tag{41}$$

By Lemma 7, system (41) has a positive periodic solution $v^*(t)$, and for any solution $v(t)$ of (41), $v(t) \to v^*(t)$ as t large enough, where $v^*(t) = x^*(t)$. Then, according to Lemmas 4 and 7, there exists a positive constant n^* such that for all $t \ge n^*T$

$$x(t) \le x^*(t) + \varepsilon. \tag{42}$$

Let us define $V(t) = (V_1(t), V_2(t))^T \in C[\mathbb{R}_+\times\mathbb{R}^2, \mathbb{R}_+^2]$ and $V_i(t)\in V_0$, $(i=1,2)$, where $V_1(t) = I(t)$, $V_2(t) = y_J(t)$. Then, we have

$$V'(t) \ge \begin{pmatrix} -d_II(t) \\ -(d_J+m)\,y_J(t)\end{pmatrix} = \begin{pmatrix} -d_I & 0 \\ 0 & -(d_J+m)\end{pmatrix}V(t), \quad t\ne(n+\tau-1)T,\ t\ne nT, \tag{43}$$

$$\begin{aligned}
&V((n+\tau-1)T^+) = \begin{pmatrix} 1-P_I & 0 \\ 0 & 1-P_J\end{pmatrix}V((n+\tau-1)T),\\
&V(nT^+) = \begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix}V(nT) + \begin{pmatrix}\delta_I \\ \delta_J\end{pmatrix},\\
&V(0^+) = (I(0^+), y_J(0^+)).
\end{aligned} \tag{44}$$

Then, the multicomparison system of (43) is

$$\begin{aligned} w'(t) &= Aw(t), \quad t \neq (n+\tau-1)T, nT, \\ w(t^+) &= Bw(t), \quad t = (n+\tau-1)T, \\ w(t^+) &= I_2 w(t) + C, \quad t = nT, \\ w(0^+) &= V(0^+), \end{aligned} \tag{45}$$

where $A = \begin{pmatrix} -d_I & 0 \\ 0 & -(d_J+m) \end{pmatrix}$, $B = \begin{pmatrix} 1-P_I & 0 \\ 0 & 1-P_J \end{pmatrix}$, and $C = \begin{pmatrix} \delta_I \\ \delta_J \end{pmatrix}$.

By Lemma 9, it is easy to obtain a periodic solution $(I^*(t), y_J^*(t))^T$ of system (45). Then, according to Lemmas 4 and 9, one may find n_0^* $(n_0^* > n^*)$ such that for all $t \geq n_0^* T$

$$I(t) \geq I^*(t) - \varepsilon, \qquad y_J(t) \geq y_J^*(t) - \varepsilon. \tag{46}$$

From the fourth equation of system (40), we have $y_M'(t) \geq m(y_J^*(t) - \varepsilon) - d_M y_M(t)$, by Lemmas 4 and 9, there exists n_1^* $(n_1^* > n_0^*)$ such that for all $t \geq n_1^* T$

$$y_M(t) \geq \overline{y_M^*(t)} - \varepsilon, \tag{47}$$

where

$$\overline{y_M^*(t)} = \begin{cases} e^{-d_M(t-(n-1)T)} \\ \quad \times \left(\overline{y_M^*(0^+)} \right. \\ \quad \left. + m \int_0^{t-(n-1)T} \left(y_J^*(t) - \varepsilon\right) e^{d_M s} ds\right), \\ \quad t \in ((n-1)T, (n+\tau-1)T], \\ e^{-d_M(t-(n-1)T)} \\ \quad \times \left(\overline{y_M^*(\tau T^+)} e^{d_M \tau T} \right. \\ \quad \left. + m \int_{\tau T}^{t-(n-1)T} \left(y_J^*(t) - \varepsilon\right) e^{d_M s} ds\right), \\ \quad t \in ((n+\tau-1)T, nT], \end{cases} \tag{48}$$

$$\begin{aligned} \overline{y_M^*(0^+)} &= \left(m\left[(1-P_M)\int_0^{\tau T} \left(y_J^*(t) - \varepsilon\right) e^{d_M s} ds \right.\right. \\ &\quad \left.\left. + \int_{\tau T}^{T} \left(y_J^*(t) - \varepsilon\right) e^{d_M s} ds\right] e^{-d_M T} + \delta_M\right) \\ &\quad \times \left(1 - (1-P_M) e^{-d_M T}\right)^{-1}, \\ \overline{y_M^*(\tau T^+)} & \\ &= \left((1-P_M)\left[m\int_0^{\tau T} \left(y_J^*(t) - \varepsilon\right) e^{d_M s} ds \right.\right. \\ &\quad \left. + m e^{-d_M T} \int_{\tau T}^{T} \left(y_J^*(t) - \varepsilon\right) e^{d_M s} ds + \delta_M\right] \\ &\quad \left. \times e^{-d_M \tau T}\right) \\ &\quad \times \left(1 - (1-P_M) e^{-d_M T}\right)^{-1}. \end{aligned} \tag{49}$$

Therefore,

$$\begin{aligned} S'(t) &\leq \left[\beta P_1\left(x^*(t) + \varepsilon\right) - g\left(I^*(t) - \varepsilon\right)\right. \\ &\quad \left. - \min_{0 \leq \varpi \leq U_S} P_2'(\varpi)\left(\overline{y_M^*(t)} - \varepsilon\right) - d_S\right] S(t), \\ &\qquad t \neq (n+\tau-1)T, \\ &\qquad t \neq nT, \\ \Delta S(t) &= -P_S S(t), \quad t = (n+\tau-1)T, \\ \Delta S(t) &= 0, \quad t = nT, \end{aligned} \tag{50}$$

for $t \geq n_1^* T$. Let $N \in \mathbb{N}$ and $(N + \tau - 1) \geq n_1^*$. Integrating (50) on $((n+\tau-1)T, (n+\tau)T]$, $n \geq N$, we have

$$\begin{aligned} &S((n+\tau)T) \\ &\quad \leq S((n+\tau-1)T)\left(1 - P_S\right) \\ &\quad\quad \times e^{\int_{(n+\tau-1)T}^{(n+\tau)T} (\beta P_1(\widetilde{x^*(t)} - \varepsilon) - g(\widetilde{I^*(t)} + \varepsilon) - \min_{0 \leq \varpi \leq U_S} P_2'(\varpi)(\widetilde{y_M^*(t)} + \varepsilon) - d_S) dt} \\ &\quad = S((n+\tau-1)T) e^{\epsilon}. \end{aligned} \tag{51}$$

Then $S(t) \leq S((n+\tau)T) e^{k\epsilon}$ for $t \in ((n+\tau+k)T, (n+\tau+k+1)T]$. Since $\epsilon < 0$, we can easily get $S(t) \to 0$, as $t \to \infty$. In the following, we prove $x(t) \to x^*(t), I(t) \to I^*(t), y_J(t) \to y_J^*(t), y_M(t) \to y_M^*(t)$, as $t \to \infty$. Give $\varepsilon_0 > 0$ small enough $(\varepsilon_0 < (r/{P'}_1(0)))$, there must exist n_2^* $(n_2^* > n_1^*)$ such that $S(t) < \varepsilon_0$, for $t \geq n_2^* T$. Then, we have

$$\begin{aligned} x'(t) &\geq \left(r - P_1'(0)\varepsilon_0\right) x(t)\left(1 - \frac{rx(t)}{K\left(r - P_1'(0)\varepsilon_0\right)}\right), \\ I'(t) &\leq -\left(d_I - g'(0)\varepsilon_0\right) I(t), \\ y_J'(t) &\leq \lambda P_2(\varepsilon_0) M - (d_J + m) y_J(t), \\ y_M'(t) &= m y_J(t) - d_M y_M(t), \\ &\qquad t \neq (n+\tau-1)T, \\ &\qquad t \neq nT, \\ \Delta x(t) &= -\delta x(t), \\ \Delta I(t) &= -P_I I(t), \\ \Delta y_J(t) &= -P_J y_J(t), \\ \Delta y_M(t) &= -P_M y_M(t), \\ &\qquad t = (n+\tau-1)T, \\ \Delta x(t) &= 0, \\ \Delta I(t) &= \delta_I, \end{aligned}$$

$$\begin{aligned}
&\Delta y_J(t) = \delta_J,\\
&\Delta y_M(t) = \delta_M,\\
&\qquad t = nT.
\end{aligned} \tag{52}$$

Analyzing (52) with similarity as (40), there exists n_3^* ($n_3^* > n_2^*$) such that for all $t \geq n_3^* T$

$$\begin{aligned}
&x(t) \geq \widehat{x^*(t)} - \varepsilon, \qquad I(t) \leq \widehat{I^*(t)} + \varepsilon,\\
&y_J(t) \leq \widehat{y_J^*(t)} + \varepsilon, \qquad y_M(t) \leq \widehat{y_M^*(t)} + \varepsilon,
\end{aligned} \tag{53}$$

where

$$\widehat{x^*(t)} = \begin{cases} K\left(r - P_1'(0)\varepsilon_0\right)\left(1 - \delta - e^{-(r-P_1'(0)\varepsilon_0)T}\right) \\ \quad \times \left(r\left[1 - \delta - e^{-(r-P_1'(0)\varepsilon_0)T} + \delta e^{-(r-P_1'(0)\varepsilon_0)(T-\tau T)} e^{-(r-P_1'(0)\varepsilon_0)(t-(n-1)T)}\right]\right)^{-1}, \\ \qquad t \in ((n-1)T, (n+\tau-1)T], \\ \dfrac{K\left(r - P_1'(0)\varepsilon_0\right)\left(1 - \delta - e^{-(r-P_1'(0)\varepsilon_0)T}\right)}{r\left[1 - \delta - e^{-(r-P_1'(0)\varepsilon_0)T} + \delta e^{-(r-P_1'(0)\varepsilon_0)(t-(n+\tau-1)T)}\right]}, \\ \qquad t \in ((n+\tau-1)T, nT], \end{cases}$$

$$\widehat{I^*(t)} = \begin{cases} \dfrac{\delta_I e^{-(d_I - g'(0)\varepsilon_0)(t-(n-1)T)}}{1 - (1-P_I)e^{-(d_I-g'(0)\varepsilon_0)T}}, \\ \qquad t \in ((n-1)T, (n+\tau-1)T], \\ \dfrac{\delta_I (1-P_I) e^{-(d_I - g'(0)\varepsilon_0)(t-(n-1)T)}}{1 - (1-P_I)e^{-(d_I-g'(0)\varepsilon_0)T}}, \\ \qquad t \in ((n+\tau-1)T, nT], \end{cases}$$

$$\widehat{y_J^*(t)} = \begin{cases} e^{-(m+d_J)(t-(n-1)T)} \\ \quad \times \left(\widehat{y_J^*(0^+)} + \lambda P_2(\varepsilon_0) M \int_0^{t-(n-1)T} e^{(m+d_J)s} ds\right), \\ \qquad t \in ((n-1)T, (n+\tau-1)T], \\ e^{-(m+d_J)(t-(n-1)T)} \\ \quad \times \left(\widehat{y_J^*(\tau T^+)} e^{(m+d_J)\tau T} + \lambda P_2(\varepsilon_0) M \int_{\tau T}^{t-(n-1)T} e^{(m+d_J)s} ds\right), \\ \qquad t \in ((n+\tau-1)T, nT], \end{cases}$$

$$\widehat{y_J^*(0^+)} = \left(\lambda P_2(\varepsilon_0) M\left[(1-P_J)\int_0^{\tau T} e^{(m+d_J)s} ds + \int_{\tau T}^{T} e^{(m+d_J)s} ds\right] \times e^{-(m+d_J)T} + \delta_J\right)\left(1 - (1-P_J)e^{-(m+d_J)T}\right)^{-1},$$

$$\widehat{y_J^*(\tau T^+)} = \left((1-P_J)\left[\lambda P_2(\varepsilon_0) M \int_0^{\tau T} e^{(m+d_J)s} ds + e^{-(m+d_J)T}\lambda P_2(\varepsilon_0) M \int_{\tau T}^{T} e^{(m+d_J)s} ds + \delta_J\right] \times e^{-(m+d_J)\tau T}\right)\left(1 - (1-P_J)e^{-(m+d_J)T}\right)^{-1},$$

$$\widehat{y_M^*(t)} = \begin{cases} e^{-d_M(t-(n-1)T)} \\ \quad \times \left(\widehat{y_M^*(0^+)} + m\int_0^{t-(n-1)T}\left(\widehat{y_J^*(t)} + \varepsilon\right) e^{d_M s} ds\right), \\ \qquad t \in ((n-1)T, (n+\tau-1)T], \\ e^{-d_M(t-(n-1)T)} \\ \quad \times \left(\widehat{y_M^*(\tau T^+)} e^{d_M \tau T} + m\int_{\tau T}^{t-(n-1)T}\left(\widehat{y_J^*(t)} + \varepsilon\right) e^{d_M s} ds\right), \\ \qquad t \in ((n+\tau-1)T, nT], \end{cases}$$

$$\widehat{y_M^*(0^+)} = \left(m\left[(1-P_M)\int_0^{\tau T}\left(\widehat{y_J^*(t)} + \varepsilon\right) e^{d_M s} ds + \int_{\tau T}^{T}\left(\widehat{y_J^*(t)} + \varepsilon\right) e^{d_M s} ds\right] e^{-d_M T} + \delta_M\right) \times \left(1 - (1-P_M)e^{-d_M T}\right)^{-1},$$

$$\widehat{y_M^*(\tau T^+)} = \left((1-P_M) \times \left[m\int_0^{\tau T}\left(\widehat{y_J^*(t)} + \varepsilon\right) e^{d_M s} ds + m e^{-d_M T} \times \int_{\tau T}^{T}\left(\widehat{y_J^*(t)} + \varepsilon\right) e^{d_M s} ds + \delta_M\right] e^{-d_M \tau T}\right) \times \left(1 - (1-P_M)e^{-d_M T}\right)^{-1}. \tag{54}$$

Letting $\varepsilon, \varepsilon_0 \to 0$, we have $\widehat{y_M^*(t)} \to y_M^*(t)$, $\widehat{x^*(t)} \to x^*(t)$, $\widehat{I^*(t)} \to I^*(t)$, $\widehat{y_J^*(t)} \to y_J^*(t)$, $\widehat{y_M^*(t)} \to y_M^*(t)$. Together with (42), (46), (47), and (53), we get $x(t) \to x^*(t)$, $I(t) \to I^*(t)$, $y_J(t) \to y_J^*(t)$, $y_M(t) \to y_M^*(t)$ as $t \to \infty$. Therefore, the susceptible pest-eradication periodic solution $(x^*(t), 0, I^*(t), y_J^*(t), y_M^*(t))$ is globally attractive. Since (37) implies (30), it follows from Theorem 11 that $(x^*(t), 0, I^*(t),$

$y_J^*(t)$) is locally asymptotically stable. So, the susceptible pest-eradication periodic solution $(x^*(t), 0, I^*(t), y_J^*(t), y_M^*(t))$ of system (1) is globally asymptotically stable. The proof is completed. □

4.2. Permanence. Next, we will discuss the permanence of system (1). In order to facilitate discussion, we give one lemma.

Lemma 13. *There exists a constant $m_4 > 0$, such that $x(t)$, $I(t), y_J(t), y_M(t) \geq m_4$ for each solution of* (1) *with t large enough.*

Proof. First, we discuss $x(t)$. Since $S(t) \leq M$, by the first equation of system (1), we have

$$\begin{aligned} x'(t) &\geq \left(r - P_1'(0)M\right)x(t)\left(1 - \frac{rx(t)}{K\left(r - P_1'(0)M\right)}\right), \\ &\qquad t \neq (n+\tau-1)T,\ t \neq nT, \\ \Delta x(t) &= -\delta x(t), \quad t = (n+\tau-1)T, \\ \Delta x(t) &= 0, \quad t = nT. \end{aligned} \tag{55}$$

Then, we obtain the following comparison system:

$$\begin{aligned} \chi'(t) &= \left(r - P_1'(0)M\right)\chi(t)\left(1 - \frac{r\chi(t)}{K\left(r - P_1'(0)M\right)}\right), \\ &\qquad t \neq (n+\tau-1)T,\ t \neq nT, \\ \Delta \chi(t) &= -\delta \chi(t), \quad t = (n+\tau-1)T, \\ \Delta \chi(t) &= 0, \quad t = nT. \end{aligned} \tag{56}$$

Letting $r > P_1'(0)M$ and $(1-\delta)e^{(r-P_1'(0)M)T} > 1$, by Lemma 7, the system (56) has a positive periodic solution $\chi^*(t)$, and for any solution $\chi(t)$ of (56), $\chi(t) \to \chi^*(t)$ as t large enough, where

$$\chi^*(t) = \begin{cases} K\left(r - P_1'(0)M\right)\left(1 - \delta - e^{-(r-P_1'(0)M)T}\right) \\ \quad \times \left(r\left[1 - \delta - e^{-(r-P_1'(0)M)T} \right.\right. \\ \qquad + \delta e^{-(r-P_1'(0)M)(T-\tau T)} \\ \qquad \left.\left. \times e^{-(r-P_1'(0)M)(t-(n-1)T)}\right]\right)^{-1}, \\ \qquad t \in ((n-1)T, (n+\tau-1)T], \\ K\left(r - P_1'(0)M\right)\left(1 - \delta - e^{-(r-P_1'(0)M)T}\right) \\ \quad \times \left(r\left[1 - \delta - e^{-(r-P_1'(0)M)T} \right.\right. \\ \qquad \left.\left. + \delta e^{-(r-P_1'(0)M)(t-(n+\tau-1)T)}\right]\right)^{-1}, \\ \qquad t \in ((n+\tau-1)T, nT]. \end{cases} \tag{57}$$

According to Lemmas 4 and 7, one may find $n_4^* \in \mathbb{N}$ such that $x(t) \geq \chi^*(t) - \varepsilon$ for $t \geq n_4^* T$. Since $\chi^*(t) - \varepsilon \geq (K(r - P_1'(0)M)(1-\delta-e^{-(r-P_1'(0)M)T})/r(1-e^{-(r-P_1'(0)M)T})) - \varepsilon = m_0 > 0$, so $x(t) \geq m_0$ for $t \geq n_4^* T$. Next, we will discuss the rest of parts.

From (46) and (47), we know that there exists n_5^* ($n_5^* > \max\{n_1^*, n_4^*\}$) such that $I(t) \geq I^*(t) - \varepsilon$, $y_J(t) \geq y_J^*(t) - \varepsilon$, and $y_M(t) \geq \overline{y_M^*(t)} - \varepsilon$ for all $t \geq n_5^* T$. By (22), (23), and (48), we have $I(t) \geq (\delta_I(1-P_I)e^{-d_I T}/(1-(1-P_I)e^{-d_I T})) - \varepsilon = m_1 > 0$, $y_J(t) \geq (\delta_J(1-P_J)e^{-(m+d_J)T}/(1-(1-P_J)e^{-(m+d_J)T})) - \varepsilon = m_2 > 0$, and $y_M(t) \geq m_M - \varepsilon = m_3 > 0$, where $m_M = \min_{0<t\leq T} \overline{y_M^*(t)}$. Let $m_4 = \min\{m_0, m_1, m_2, m_3\}$, then $x(t), I(t), y_J(t), y_M(t) \geq m_4$ for $t \geq n_5^* T$. The proof is completed. □

Theorem 14. *If*

$$\begin{aligned} &\beta \int_0^T P_1(x^*(t))\,dt - \int_0^T g(I^*(t))\,dt - P_2'(0) \\ &\quad \times \int_0^T y_M^*(t)\,dt - d_S T > \ln \frac{1}{1-P_S}, \end{aligned} \tag{58}$$

then system (1) *is permanent.*

Proof. By Lemmas 6 and 13, we have already known that there exist two constants m_4, $M > 0$, such that $x(t), I(t), y_J(t), y_M(t) \geq m_4$ and $x(t), S(t), I(t), y_J(t), y_M(t) \leq M$ for t large enough. Thus, we only need to find $m^* > 0$ such that $S(t) \geq m^*$ for t large enough. We will do this in the following two steps.

Step 1. Let $m_5 > 0$ and $\varepsilon_1 > 0$ small enough, so that $m_5 < \min\{(r/P_1'(0)), (d_I/g'(0)), M\}$ and

$$\begin{aligned} &\beta \int_0^T P_1\left(\widetilde{x^*(t)} - \varepsilon_1\right)dt - \int_0^T g\left(\widetilde{I^*(t)} + \varepsilon_1\right)dt - P_2'(0) \\ &\quad \times \int_0^T \left(\widetilde{y_M^*(t)} + \varepsilon_1\right)dt - d_S T - \ln \frac{1}{1-P_S} = \eta > 0, \end{aligned} \tag{59}$$

where

$$\widetilde{x^*(t)} = \begin{cases} K\left(r - P_1'(0)m_5\right)\left(1 - \delta - e^{-(r-P_1'(0)m_5)T}\right) \\ \quad \times \left(r\left[1 - \delta - e^{-(r-P'_1(0)m_5)T} \right.\right. \\ \qquad + \delta e^{-(r-P_1'(0)m_5)(T-\tau T)} \\ \qquad \left.\left. \times e^{-(r-P_1'(0)m_5)(t-(n-1)T)}\right]\right)^{-1}, \\ \qquad t \in ((n-1)T, (n+\tau-1)T], \\ K\left(r - P_1'(0)m_5\right)\left(1 - \delta - e^{-(r-P_1'(0)m_5)T}\right) \\ \quad \times \left(r\left[1 - \delta - e^{-(r-P_1'(0)m_5)T} \right.\right. \\ \qquad \left.\left. + \delta e^{-(r-P_1'(0)m_5)(t-(n+\tau-1)T)}\right]\right)^{-1}, \\ \qquad t \in ((n+\tau-1)T, nT], \end{cases}$$

$$\widetilde{I^*(t)} = \begin{cases} \dfrac{\delta_I e^{-(d_I - g'(0)m_5)(t-(n-1)T)}}{1 - (1-P_I)e^{-(d_I - g'(0)m_5)T}}, \\ \qquad t \in ((n-1)T, (n+\tau-1)T], \\ \dfrac{\delta_I(1-P_I)e^{-(d_I - g'(0)m_5)(t-(n-1)T)}}{1 - (1-P_I)e^{-(d_I - g'(0)m_5)T}}, \\ \qquad t \in ((n+\tau-1)T, nT], \end{cases}$$

$$\widetilde{y_J^*(t)} = \begin{cases} e^{-(m+d_J)(t-(n-1)T)} \\ \quad \times\left(\widetilde{y_J^*(0^+)} + \lambda P_2(m_5) M \int_0^{t-(n-1)T} e^{(m+d_J)s} ds\right), \\ \qquad t \in ((n-1)T, (n+\tau-1)T], \\ e^{-(m+d_J)(t-(n-1)T)} \\ \quad \times\left(\widetilde{y_J^*(\tau T^+)} e^{(m+d_J)\tau T} \right. \\ \qquad \left. + \lambda P_2(m_5) M \int_{\tau T}^{t-(n-1)T} e^{(m+d_J)s} ds\right), \\ \qquad t \in ((n+\tau-1)T, nT], \end{cases}$$

$$\widetilde{y_J^*(0^+)} = \left(\lambda P_2(m_5) M\left[(1-P_J)\int_0^{\tau T} e^{(m+d_J)s} ds + \int_{\tau T}^{T} e^{(m+d_J)s} ds\right] \times e^{-(m+d_J)T} + \delta_J\right)\left(1-(1-P_J)e^{-(m+d_J)T}\right)^{-1},$$

$$\widetilde{y_J^*(\tau T^+)} = \left((1-P_J) \times\left[\lambda P_2(m_5) M \int_0^{\tau T} e^{(m+d_J)s} ds + e^{-(m+d_J)T} \times \lambda P_2(m_5) M \int_{\tau T}^{T} e^{(m+d_J)s} ds + \delta_J\right] \times e^{-(m+d_J)\tau T}\right)\left(1-(1-P_J)e^{-(m+d_J)T}\right)^{-1},$$

$$\widetilde{y_M^*(t)} = \begin{cases} e^{-d_M(t-(n-1)T)} \\ \quad \times\left(\widetilde{y_M^*(0^+)} + m\int_0^{t-(n-1)T} \left(\widetilde{y_J^*(t)} + \varepsilon_1\right) e^{d_M s} ds\right), \\ \qquad t \in ((n-1)T, (n+\tau-1)T], \\ e^{-d_M(t-(n-1)T)} \\ \quad \times\left(\widetilde{y_M^*(\tau T^+)} e^{d_M\tau T} + m\int_{\tau T}^{t-(n-1)T} \left(\widetilde{y_J^*(t)} + \varepsilon_1\right) e^{d_M s} ds\right), \\ \qquad t \in ((n+\tau-1)T, nT], \end{cases}$$

$$\widetilde{y_M^*(0^+)} = \left(m\left[(1-P_M)\int_0^{\tau T}\left(\widetilde{y_J^*(t)} + \varepsilon_1\right) e^{d_M s} ds + \int_{\tau T}^{T}\left(\widetilde{y_J^*(t)} + \varepsilon_1\right) e^{d_M s} ds\right] \times e^{-d_M T} + \delta_M\right)\left(1-(1-P_M)e^{-d_M T}\right)^{-1},$$

$$\widetilde{y_M^*(\tau T^+)} = \left((1-P_M) \times\left[m\int_0^{\tau T}\left(\widetilde{y_J^*(t)} + \varepsilon_1\right) e^{d_M s} ds + m e^{-d_M T}\int_{\tau T}^{T}\left(\widetilde{y_J^*(t)} + \varepsilon_1\right) e^{d_M s} ds + \delta_M\right] \times e^{-d_M\tau T}\right)\left(1-(1-P_M)e^{-d_M T}\right)^{-1}. \tag{60}$$

We shall prove that one cannot have $S(t) < m_5$ for all $t > 0$, otherwise

$$\begin{aligned} & x'(t) \geq \left(r - P_1'(0) m_5\right) x(t)\left(1 - \frac{r x(t)}{K\left(r - P_1'(0) m_5\right)}\right), \\ & I'(t) \leq -\left(d_I - g'(0) m_5\right) I(t), \\ & y_J'(t) \leq \lambda P_2(m_5) M - \left(d_J + m\right) y_J(t), \\ & y_M'(t) = m y_J(t) - d_M y_M(t), \\ & \qquad t \neq (n+\tau-1)T, \\ & \qquad t \neq nT, \\ & \Delta x(t) = -\delta x(t), \\ & \Delta I(t) = -P_I I(t), \\ & \Delta y_J(t) = -P_J y_J(t), \\ & \Delta y_M(t) = -P_M y_M(t), \\ & \qquad t = (n+\tau-1)T, \\ & \Delta x(t) = 0, \\ & \Delta I(t) = \delta_I, \\ & \Delta y_J(t) = \delta_J, \\ & \Delta y_M(t) = \delta_M, \\ & \qquad t = nT. \end{aligned} \tag{61}$$

Analyzing (61) with similarity as (40), it is easy to obtain that there exists a positive constant n_6^*, such that $x(t) \geq \widetilde{x^*(t)} - \varepsilon_1$, $I(t) \leq \widetilde{I^*(t)} + \varepsilon_1$, $y_J(t) \leq \widetilde{y_J^*(t)} + \varepsilon_1$, $y_M(t) \leq \widetilde{y_M^*(t)} + \varepsilon_1$ for $t \geq n_6^* T$. Therefore,

$$\begin{aligned} S'(t) \geq{} & \beta P_1\left(\widetilde{x^*(t)} - \varepsilon_1\right) S(t) - g\left(\widetilde{I^*(t)} + \varepsilon_1\right) S(t) \\ & - P_2'(0)\left(\widetilde{y_M^*(t)} + \varepsilon_1\right) S(t) - d_S S(t), \end{aligned}$$

$$\begin{gathered} t \neq (n+\tau-1)T,\ t \neq nT, \\ \Delta S(t) = -P_S S(t), \quad t = (n+\tau-1)T, \\ \Delta S(t) = 0, \quad t = nT, \end{gathered} \tag{62}$$

for $t \geq n_6^* T$. Let $N_0 \in \mathbb{N}$ and $(N_0 + \tau - 1) \geq n_6^*$. Integrating (62) on $((n+\tau-1)T, (n+\tau)T], n \geq N_0$, we have

$$\begin{aligned} & S((n+\tau)T) \\ & \quad \geq S((n+\tau-1)T)(1-P_S) \\ & \quad \times e^{\int_{(n+\tau-1)T}^{(n+\tau)T} (\beta P_1(\widetilde{x^*(t)}-\varepsilon_1) - g(\widetilde{I^*(t)}+\varepsilon_1) - P_2'(0)(\widetilde{y_M^*(t)}+\varepsilon_1) - d_S) dt} \\ & \quad = S((n+\tau-1)T) e^{\eta}. \end{aligned} \tag{63}$$

Then $S((N_0+\tau+k)T) \geq S((N_0+\tau)T)e^{k\eta} \to \infty$ as $k \to \infty$, which is a contradiction. So there exists a t_1 $(t_1 > n_6^* T)$ such that $S(t_1) \geq m_5$.

Step 2. If $S(t) \geq m_5$ for all $t \geq t_1$, then Our purpose is obtained. If not, let $t_2 = \inf\{t > t_1 \mid S(t) < m_5\}$. Then $S(t) \geq m_5$ for $t \in [t_1, t_2)$ and $S(t_2) = m_5$. In this step, we consider two possible cases for t_2.

Case 1. $t_2 = (n_1 + \tau - 1)T, n_1 \in \mathbb{N}$. Then $S(t_2^+) = (1 - P_S)S(t_2) < m_5$. Select $n_2, n_3 \in \mathbb{N}$ such that $(n_2 - 1) \geq n_6^*$ and $(1-P_S)^{n_2} e^{n_3\eta + n_2\sigma T} > (1-P_S)^{n_2} e^{n_3\eta + (n_2+1)\sigma T} > 1$, where $\sigma = \beta P_1(m_4) - g(M) - P_2'(0)M - d_S < 0$. Let $\widetilde{T} = (n_2 + n_3)T$, then we have the claim: there exists $t_3 \in (t_2, t_2 + \widetilde{T}]$ such that $S(t_3) \geq m_5$. If the claim is false, we will get a contradiction in the following.

According to Step 1, we have $x(t) \geq \widetilde{x^*(t)} - \varepsilon_1, I(t) \leq \widetilde{I^*(t)} + \varepsilon_1, y_J(t) \leq \widetilde{y_J^*(t)} + \varepsilon_1, y_M(t) \leq \widetilde{y_M^*(t)} + \varepsilon_1$ for $t \geq (n_1 + n_2 - 1)T$. Then, we have

$$\begin{gathered} S'(t) \geq \beta P_1\left(\widetilde{x^*(t)} - \varepsilon_1\right) S(t) - g\left(\widetilde{I^*(t)} + \varepsilon_1\right) S(t) \\ - P_2'(0)\left(\widetilde{y_M^*(t)} + \varepsilon_1\right) S(t) - d_S S(t), \\ t \neq (n+\tau-1)T,\ t \neq nT, \\ \Delta S(t) = -P_S S(t), \quad t = (n+\tau-1)T, \\ \Delta S(t) = 0, \quad t = nT, \end{gathered} \tag{64}$$

for $t \in [t_2 + n_2 T, t_2 + \widetilde{T}]$. As in Step 1, we have

$$S\left(t_2 + \widetilde{T}\right) \geq S\left(t_2 + n_2 T\right) e^{n_3 \eta}. \tag{65}$$

Since $x(t) \geq m_4, I(t) \leq M, y_M(t) \leq M$ and $P_2(S(t)) < P_2'(0)S(t)$, we have

$$\begin{gathered} S'(t) \\ \geq \left(\beta P_1(m_4) - g(M) - P_2'(0)M - d_S\right) S(t) = \sigma S(t), \\ t \neq (n+\tau-1)T, \quad t \neq nT, \\ \Delta S(t) = -P_S S(t), \quad t = (n+\tau-1)T, \\ \Delta S(t) = 0, \quad t = nT, \end{gathered} \tag{66}$$

for $t \in [t_2, t_2 + n_2 T]$. Integrating (66) on $[t_2, t_2 + n_2 T]$, we have

$$\begin{aligned} S(t_2 + n_2 T) & \geq S(t_2^+) e^{n_2 \sigma T} \\ & = (1-P_S) m_5 e^{n_2 \sigma T} \\ & \geq (1-P_S)^{n_2} m_5 e^{n_2 \sigma T}. \end{aligned} \tag{67}$$

Thus, by (65) and (67), we have $S(t_2 + \widetilde{T}) \geq (1 - P_S)^{n_2} m_5 e^{n_3\eta + n_2\sigma T} > m_5$, which is a contradiction. Let $t_4 = \inf\{t > t_2 \mid S(t) \geq m_5\}$, then for $t \in [t_2, t_4), S(t) < m_5$ and $S(t_4) = m_5$. So, $S(t) \geq S(t_2^+)e^{\sigma(t-t_2)} = (1-P_S)m_5 e^{\sigma(t-t_2)} \geq (1-P_S)^{n_2+n_3} m_5 e^{\sigma(n_2+n_3)T} = \widetilde{m_1}$ for $t \in [t_2, t_4)$.

Case 2 $(t_2 \neq (n_1 + \tau - 1)T, n_1 \in \mathbb{N})$. Suppose that $t_2 \in ((n_1' + \tau - 1)T, (n_1' + \tau)T), n_1' \in \mathbb{N}$. $S(t) \geq m_5$ for $t \in [t_1, t_2)$ and $S(t_2) = m_5$. There are two possible cases for $t \in (t_2, (n_1' + \tau)T)$.

Case 2a. If $S(t) \leq m_5$ for all $t \in (t_2, (n_1' + \tau)T)$, similar to Case 1, we can prove there exists a $t_3' \in ((n_1' + \tau)T, (n_1' + \tau)T + \widetilde{T}]$ such that $S(t_3') \geq m_5$. Let $t_4' = \inf\{t > t_2 \mid S(t) \geq m_5\}$, then for $t \in [t_2, t_4'), S(t) < m_5$ and $S(t_4') = m_5$. So, $S(t) \geq S(t_2)e^{\sigma(t-t_2)} = m_5 e^{\sigma(t-t_2)} \geq (1-P_S)^{n_2+n_3} m_5 e^{\sigma(n_2+n_3+1)T} = m^* < \widetilde{m_1}$ for all $t \in [t_2, t_4')$.

Case 2b. If there exists a $t \in (t_2, (n_1' + \tau)T)$ such that $S(t) \geq m_5$. Let $\overline{t_4'} = \inf\{t > t_2 \mid S(t) \geq m_5\}$, then for $t \in [t_2, \overline{t_4'}), S(t) < m_5$ and $S(\overline{t_4'}) = m_5$. So, $S(t) \geq S(t_2)e^{\sigma(t-t_2)} = m_5 e^{\sigma(t-t_2)} \geq m_5 e^{\sigma T} > m^*$ for all $t \in [t_2, \overline{t_4'})$.

Since $S(t) \geq m_5$ for some $t \geq t_1$, in both cases a similar discussion can be continued. The proof is completed. □

5. Numerical Simulations and Conclusions

In this section, we will give an example and its simulations to show the efficiency of the criteria derived in Section 4.

In system (1), let $P_1(x(t)) = ax(t)$, $g(I(t)) = bI(t)$, and $P_2(S(t)) = h(1 - e^{-cS(t)})$, $a, b, c, h > 0$. Namely, $P_1(x(t))$ describes an Holling type-I functional response of the pest, $P_2(S(t))$ describes a Ivlev-type functional response of the pest 's natural predator. Therefore, we consider the pest management model with impulsive releasing and harvesting at two different fixed moments:

$$\begin{aligned} x'(t) &= rx(t)\left(1 - \frac{x(t)}{K}\right) - ax(t)S(t), \\ S'(t) &= \beta ax(t)S(t) - bI(t)S(t) \\ &\quad - h\left(1 - e^{-cS(t)}\right) y_M(t) - d_S S(t), \\ I'(t) &= bI(t)S(t) - d_I I(t), \\ y_J'(t) &= \lambda h\left(1 - e^{-cS(t)}\right) y_M(t) - d_J y_J(t) - m y_J(t), \\ y_M'(t) &= m y_J(t) - d_M y_M(t), \\ &\qquad t \neq (n+\tau-1)T, \\ &\qquad t \neq nT, \end{aligned}$$

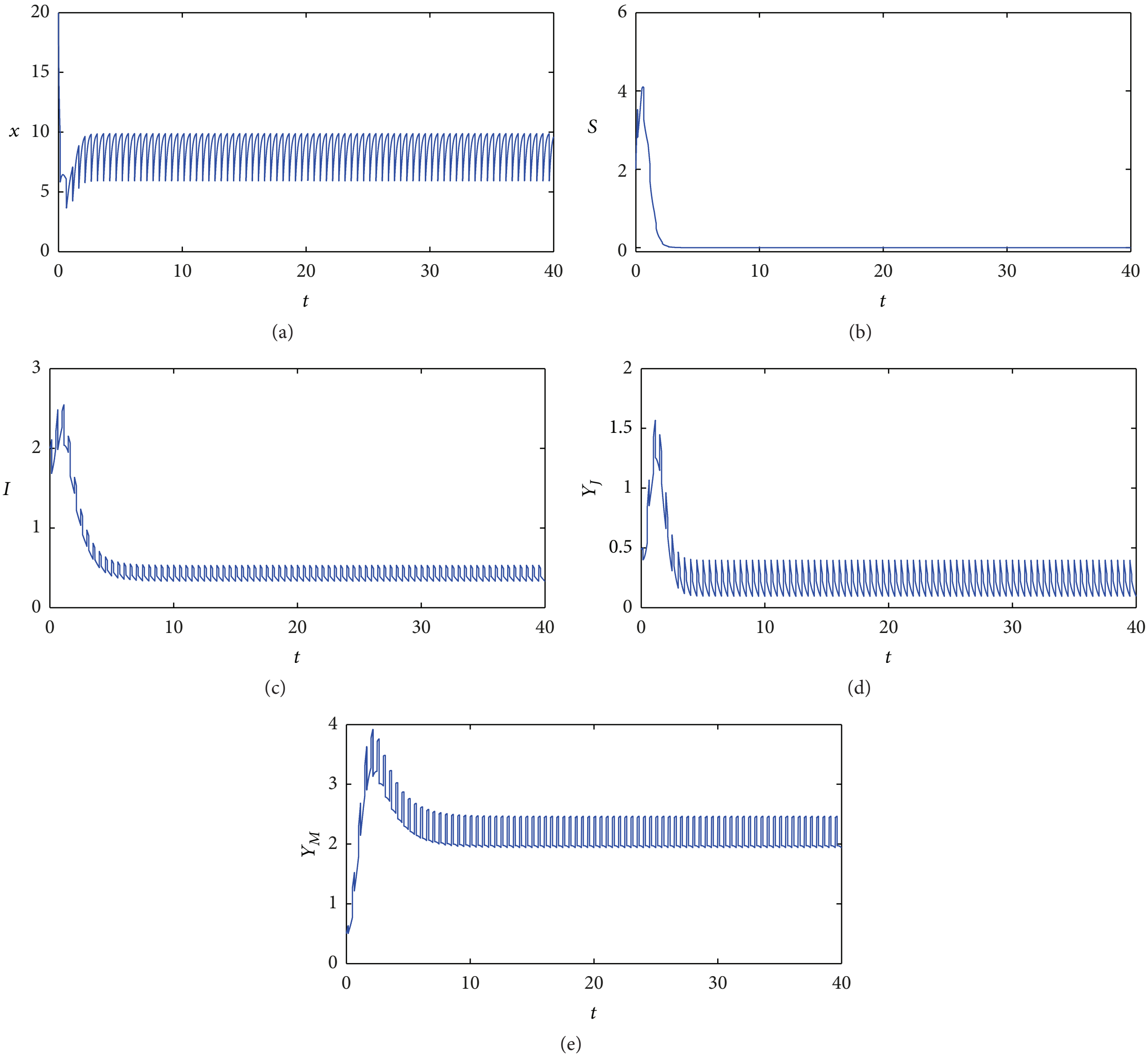


Figure 1: Time series of the system (68) with $r = 8$, $K = 10$, $a = 0.8$, $\beta = 0.5$, $b = 0.3$, $h = 8$, $c = 0.2$, $\lambda = 0.6$, $m = 2$, $d_S = 0.2$, $d_I = 0.5$, $d_J = 0.4$, $d_M = 0.2$, $\delta = 0.4$, $P_S = P_I = P_J = P_M = 0.2$, $\delta_I = 0.2$, $\delta_J = 0.3$, $\delta_M = 0.5$, $\tau = 0.3$, $T = 0.5$, $x(0) = 20$, $S(0) = 2$, $I(0) = 2$, $y_J(0) = 0.5$, $y_M(0) = 0.5$.

$$\begin{aligned}
&\Delta x(t) = -\delta x(t), \\
&\Delta S(t) = -P_S S(t), \\
&\Delta I(t) = -P_I I(t), \\
&\Delta y_J(t) = -P_J y_J(t), \\
&\Delta y_M(t) = -P_M y_M(t), \\
&\qquad t = (n+\tau-1)T, \\
&\Delta x(t) = 0, \\
&\Delta S(t) = 0, \\
&\Delta I(t) = \delta_I, \\
&\Delta y_J(t) = \delta_J, \\
&\Delta y_M(t) = \delta_M, \\
&\qquad t = nT.
\end{aligned} \tag{68}$$

So, by (22), (23), and (25), we have

$$\begin{aligned}
&\beta \int_0^T P_1(x^*(t))\,dt = \frac{\beta a K}{r}(rT + \ln(1-\delta)) = \theta_1, \\
&\int_0^T g(I^*(t))\,dt \\
&\quad = \frac{b\delta_I\left[1 - e^{-d_I\tau T} + (1-P_I)\left(e^{-d_I\tau T} - e^{-d_I T}\right)\right]}{d_I\left(1-(1-P_I)e^{-d_I T}\right)} \\
&\quad = \theta_2,
\end{aligned}$$

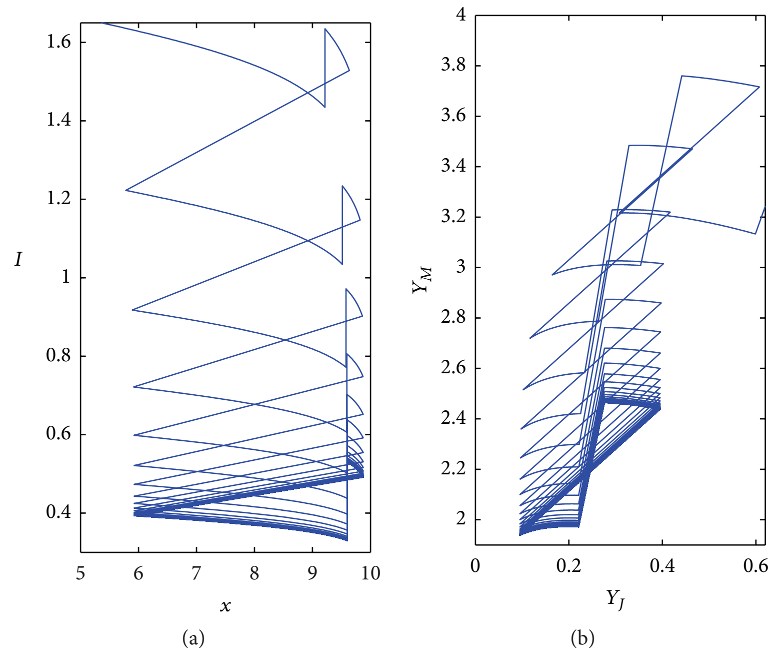


FIGURE 2: Phase portrait of the system (68) with $r = 8$, $K = 10$, $a = 0.8$, $\beta = 0.5$, $b = 0.3$, $h = 8$, $c = 0.2$, $\lambda = 0.6$, $m = 2$, $d_S = 0.2$, $d_I = 0.5$, $d_J = 0.4$, $d_M = 0.2$, $\delta = 0.4$, $P_S = P_I = P_J = P_M = 0.2$, $\delta_I = 0.2$, $\delta_J = 0.3$, $\delta_M = 0.5$, $\tau = 0.3$, $T = 0.5$, $x(0) = 20$, $S(0) = 2$, $I(0) = 2$, $y_J(0) = 0.5$, $y_M(0) = 0.5$.

$$\begin{aligned}
&\int_0^T y_M^*(t)\,dt \\
&= \frac{y_M^*(0^+)}{d_M}\left(1 - e^{-d_M\tau T}\right) \\
&\quad + \frac{m\delta_J}{\left(1-(1-P_J)e^{-(m+d_J)T}\right)\left(d_M-(m+d_J)\right)} \\
&\quad \times \left(\frac{1-e^{-(m+d_J)\tau T}}{m+d_J} - \frac{1-e^{-d_M\tau T}}{d_M}\right) \\
&\quad + \frac{y_M^*(\tau T^+)e^{d_M\tau T}}{d_M}\left(e^{-d_M\tau T} - e^{d_M T}\right) \\
&\quad + \frac{m\delta_J(1-P_J)}{\left(1-(1-P_J)e^{-(m+d_J)T}\right)\left(d_M-(m+d_J)\right)} \\
&\quad \times \left(\frac{e^{-(m+d_J)\tau T} - e^{-(m+d_J)T}}{m+d_J}\right. \\
&\qquad \left. - \frac{e^{-d_M\tau T} - e^{-d_M T}}{d_M} e^{(d_M-(m+d_J))\tau T}\right) \\
&= \theta_3,
\end{aligned} \tag{69}$$

where

$$y_M^*(0^+) = \frac{\left[(1-P_M)A(\tau T) + B(T)\right]e^{-d_M T} + \delta_M}{1-(1-P_M)e^{-d_M T}},$$

$$y_M^*(\tau T^+) = \frac{(1-P_M)\left[A(\tau T) + e^{-d_M T}B(T) + \delta_M\right]e^{-d_M\tau T}}{1-(1-P_M)e^{-d_M T}},$$

$$A(\tau T) = \frac{m\delta_J\left(e^{(d_M-(m+d_J))\tau T} - 1\right)}{\left(1-(1-P_J)e^{-(m+d_J)T}\right)\left(d_M-(m+d_J)\right)},$$

$$B(T) = \frac{m\delta_J(1-P_J)\left(e^{(d_M-(m+d_J))T} - e^{(d_M-(m+d_J))\tau T}\right)}{\left(1-(1-P_J)e^{-(m+d_J)T}\right)\left(d_M-(m+d_J)\right)}.$$

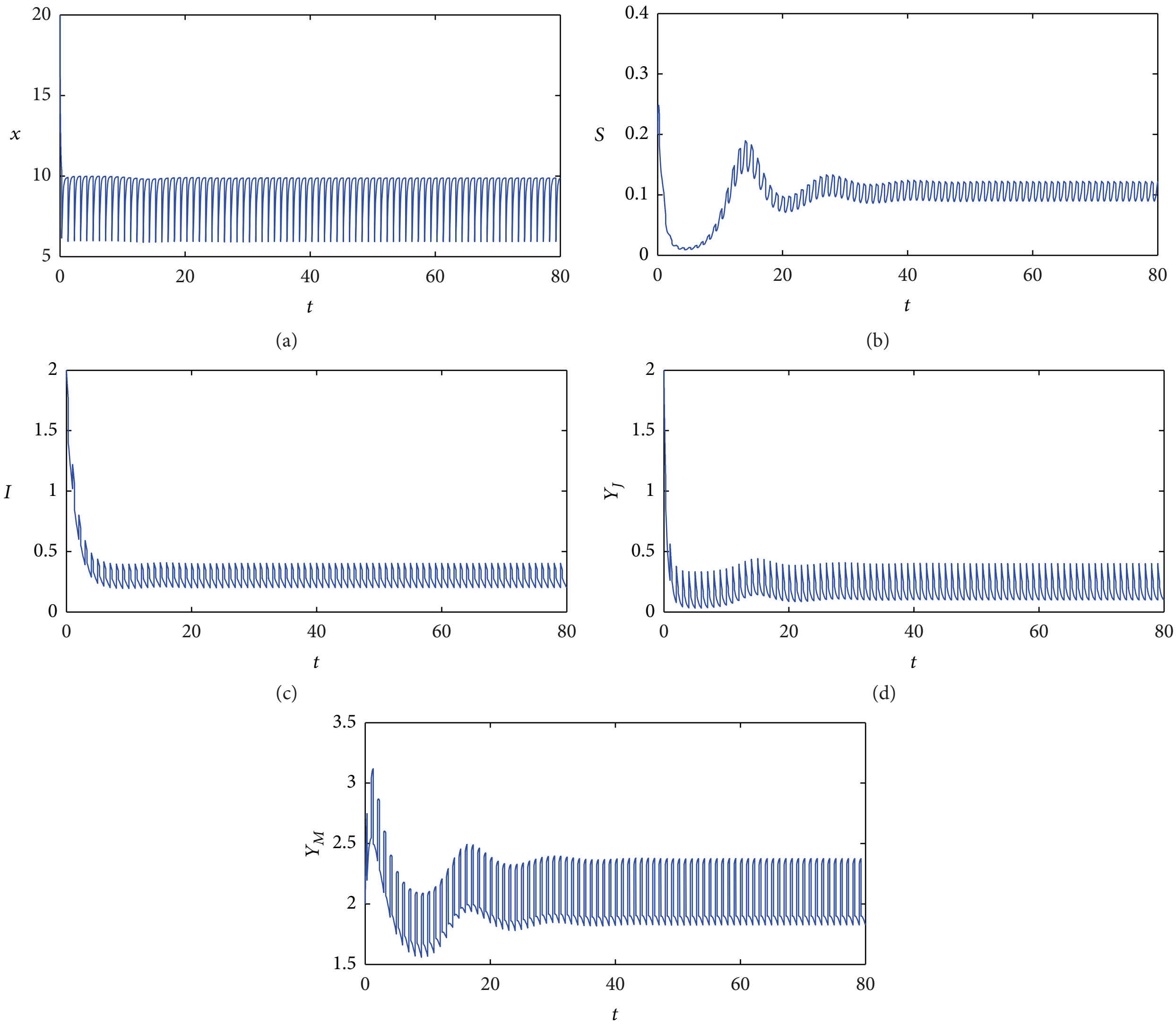
Then, by Theorems 11 and 14, we have the following.

($T1$) If $\theta_1 - \theta_2 - \theta_3 - d_S T < \ln(1/(1-P_S))$, then the susceptible pest-eradication periodic solution $(x^*(t), 0, I^*(t), y_J^*(t), y_M^*(t))$ of system (68) is locally asymptotically stable.

($T2$) If $\theta_1 - \theta_2 - hce^{-cM}\theta_3 - d_S T < \ln(1/(1-P_S))$, then the susceptible pest-eradication periodic solution $(x^*(t), 0, I^*(t), y_J^*(t), y_M^*(t))$ of system (68) is globally asymptotically stable, where M is defined in Lemma 6.

($T3$) If $\theta_1 - \theta_2 - hc\theta_3 - d_S T > \ln(1/(1-P_S))$, then system (68) is permanent.

In the following, we analyze the locally asymptotical stability of the susceptible pest-eradication periodic solution and permanence of system (68).



(e)

Figure 3: Time series of the system (68) with $r = 8$, $K = 10$, $a = 0.8$, $\beta = 0.5$, $b = 0.3$, $h = 8$, $c = 0.2$, $\lambda = 0.6$, $m = 2$, $d_S = 0.2$, $d_I = 0.5$, $d_J = 0.4$, $d_M = 0.2$, $\delta = 0.4$, $P_S = P_I = P_J = P_M = 0.2$, $\delta_I = 0.2$, $\delta_J = 0.3$, $\delta_M = 0.5$, $\tau = 0.3$, $T = 1$, $x(0) = 20$, $S(0) = 0.2$, $I(0) = 2$, $y_J(0) = 2$, $y_M(0) = 2$.

Assume that $x(0) = 20$, $S(0) = 2$, $I(0) = 2$, $y_J(0) = 0.5$, $y_M(0) = 0.5$, $r = 8$, $K = 10$, $a = 0.8$, $\beta = 0.5$, $b = 0.3$, $h = 8$, $c = 0.2$, $\lambda = 0.6$, $m = 2$, $d_S = 0.2$, $d_I = 0.5$, $d_J = 0.4$, $d_M = 0.2$, $\delta = 0.4$, $P_S = P_I = P_J = P_M = 0.2$, $\delta_I = 0.2$, $\delta_J = 0.3$, $\delta_M = 0.5$, $\tau = 0.3$, $T = 0.5$. Obviously, the condition of $(T1)$ is satisfied, then the susceptible pest-eradication periodic solution of system (68) is locally asymptotically stable, which can be seen from the numerical simulation in Figures 1 and 2.

Assume that $x(0) = 20$, $S(0) = 0.2$, $I(0) = 2$, $y_J(0) = 2$, $y_M(0) = 2$, $r = 8$, $K = 10$, $a = 0.8$, $\beta = 0.5$, $b = 0.3$, $h = 8$, $c = 0.2$, $\lambda = 0.6$, $m = 2$, $d_S = 0.2$, $d_I = 0.5$, $d_J = 0.4$, $d_M = 0.2$, $\delta = 0.4$, $P_S = P_I = P_J = P_M = 0.2$, $\delta_I = 0.2$, $\delta_J = 0.3$, $\delta_M = 0.5$, $\tau = 0.3$, $T = 1$. Obviously, the condition of $(T3)$ is satisfied. Then, system (68) is permanent, which can also be seen from Figures 3 and 4.

From results of the numerical simulation, we know that there exists an impulsive harvesting(or releasing) periodic threshold T^*, which satisfies $0.5 < T^* < 1$. If $T < T^*$ and the other parameters are fixed ($r = 8$, $K = 10$, $a = 0.8$, $\beta = 0.5$, $b = 0.3$, $h = 8$, $c = 0.2$, $\lambda = 0.6$, $m = 2$, $d_S = 0.2$, $d_I = 0.5$, $d_J = 0.4$, $d_M = 0.2$, $\delta = 0.4$, $P_S = P_I = P_J = P_M = 0.2$, $\delta_I = 0.2$, $\delta_J = 0.3$, $\delta_M = 0.5$, $\tau = 0.3$, $T = 0.5$.), then the susceptible pest-eradication periodic solution $(x^*(t), 0, I^*(t), y_J^*(t), y_M^*(t))$ of system (68) is locally asymptotically stable. If $T > T^*$ and the other parameters are fixed ($r = 8$, $K = 10$, $a = 0.8$, $\beta = 0.5$, $b = 0.3$, $h = 8$, $c = 0.2$, $\lambda = 0.6$, $m = 2$, $d_S = 0.2$, $d_I = 0.5$, $d_J = 0.4$, $d_M = 0.2$, $\delta = 0.4$, $P_S = P_I = P_J = P_M = 0.2$, $\delta_I = 0.2$, $\delta_J = 0.3$, $\delta_M = 0.5$, $\tau = 0.3$, $T = 0.5$.), then system (68) is permanent. The same discussion can be applied to other parameters.

In this paper, we proposed a pest management model with impulsive releasing (periodic infective pests, immature and mature natural enemies releasing) and harvesting (periodic crops harvesting) at two different fixed moments. By means

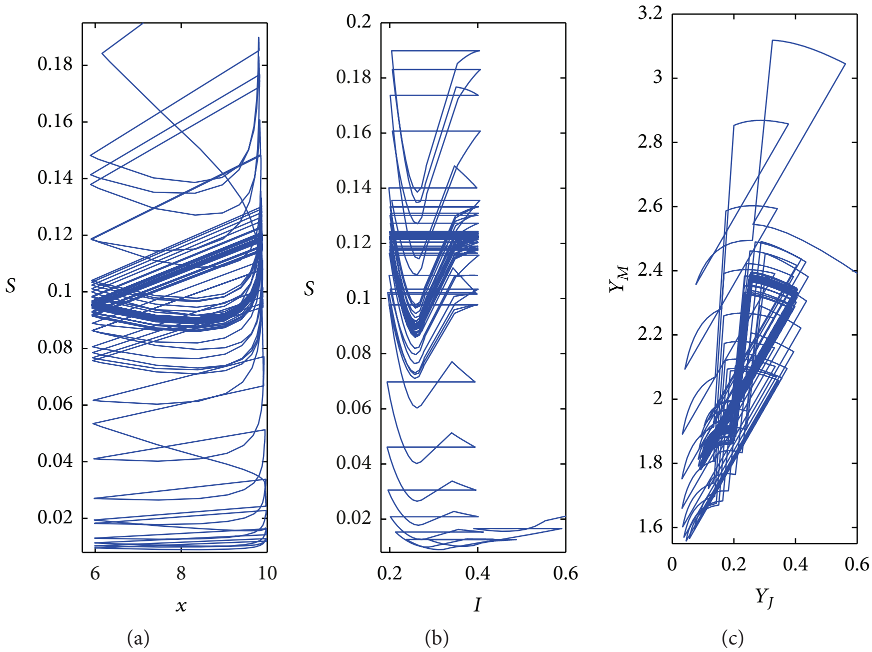


FIGURE 4: Phase portrait of the system (68) with $r = 8$, $K = 10$, $a = 0.8$, $\beta = 0.5$, $b = 0.3$, $h = 8$, $c = 0.2$, $\lambda = 0.6$, $m = 2$, $d_S = 0.2$, $d_I = 0.5$, $d_J = 0.4$, $d_M = 0.2$, $\delta = 0.4$, $P_S = P_I = P_J = P_M = 0.2$, $\delta_I = 0.2$, $\delta_J = 0.3$, $\delta_M = 0.5$, $\tau = 0.3$, $T = 1$, $x(0) = 20$, $S(0) = 0.2$, $I(0) = 2$, $y_J(0) = 2$, $y_M(0) = 2$.

of Floquet theory and multicomparison results for impulsive differential equations, two sufficient conditions ensuring the locally and globally asymptotical stability of the susceptible pest-eradication period solution and permanence of the system are derived.

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