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Research Article

Dynamical Analysis of Delayed Plant Disease Models with Continuous or Impulsive Cultural Control Strategies

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Delayed plant disease mathematical models including continuous cultural control strategy and impulsive cultural control strategy are presented and investigated. Firstly, we consider continuous cultural control strategy in which continuous replanting of healthy plants is taken. The existence and local stability of disease-free equilibrium and positive equilibrium are studied by analyzing the associated characteristic transcendental equation. And then, plant disease model with impulsive replanting of healthy plants is also considered; the sufficient condition under which the infected plant-free periodic solution is globally attritive is obtained. Moreover, permanence of the system is studied. Some numerical simulations are also given to illustrate our results.

1. Introduction

Plant viruses or pathogens are an important constraint to crop production worldwide and cause major production and economic losses in agriculture and forestry. For example, soybean rust (a fungal disease in soybeans) has caused a significant economic loss, and just by removing 20% of the infection, the farmers may benefit with an approximately 11 million-dollar profit [1]. Several plant diseases caused by plant viruses in cassava (Manihot esculenta) and sweet potato (Ipomoea batatas) are among the main constraints to sustainable production of these vegetatively propagated staple food crops in lesser-developed countries [2–4]. A strain of the virus causing cassava mosaic disease gives rise to losses in Africa [5]. Therefore, people have turned more attention to plant diseases. Several conferences have been held to discuss how to control or prevent plant virus. Therefore, farmers have been evolving practices for controlling plant diseases, which involves a number of dynamic processes such as the growth of plants and the spread of diseases. Recently, the integrated disease management (IDM) which combines biological, cultural, and chemical tactics and so on to reduce the numbers of infected individuals to a tolerable level and aims to minimize losses and maximize returns [6, 7] has been developed gradually. IDM includes four main control strategies for vegetatively propagated plant diseases, which are containing transmission vectors, improving the production of planting material, controlling the crop sanitation through removal of infected plants, and breeding plants for resistance to the virus. Breeding plants for resistance to the virus as an cultural strategy has been widely used into practice [8–11]. In the system of IDM, mathematical modeling has shown its unique value on describing, analyzing, and predicting plant epidemics [12–16]. Meng and Li have investigated vegetatively propagated plant diseases and developed a mathematical model with continuous control strategies and impulsive cultural control strategies [17], which leads to

$$\begin{aligned} S'(t) &= \rho - \frac{\beta S(t)I(t)}{1+\alpha S(t)} - \mu S(t) + \omega I(t), \\ I'(t) &= \frac{\beta S(t)I(t)}{1+\alpha S(t)} - (d+r+\omega)I(t), \end{aligned} \tag{1.1}$$

where $S(t)$ and $I(t)$ denote the number of susceptible and infected plants, respectively. β is the transmission rate, α denotes potentially density dependent, μ either denotes harvest time or the end of reproductive life time of plants, ρ represents the total rate at which the susceptible plants enter the system, r is the removal rate for the infected plants, ω is the recovery rate of the cured diseased plants, and the infected plants suffer an extra disease-related death with constant rate d. In system (1.1), the authors refer to two-control strategy: one is continuous control and the other is impulsive control by implementing periodic replanting of healthy plants or removing infected plants at a critical time. A model for the spread of an infectious disease (involving only susceptibles and infective individuals) transmitted by a vector after an incubation time was proposed by Cooke [18]. This is called the phenomena of time delay. Many authors have directly incorporated time delays in modeling equations, and, as a result, the models take the form of delay differential equations [19–23]. Motivated by Meng, we get the following reasonable plant disease models by introducing time delay:

$$\begin{aligned} S'(t) &= \rho - \beta e^{-\mu\tau}\frac{S(t)I(t-\tau)}{1+\alpha S(t)} - \mu S(t) + \omega I(t), \\ I'(t) &= \beta e^{-\mu\tau}\frac{S(t)I(t-\tau)}{1+\alpha S(t)} - (d+r+\omega)I(t), \end{aligned} \tag{1.2}$$

$$\begin{aligned} &\left.\begin{aligned} S'(t) &= -\beta e^{-\mu\tau}\frac{S(t)I(t-\tau)}{1+\alpha S(t)} - \mu S(t) + \omega I(t), \\ I'(t) &= \beta e^{-\mu\tau}\frac{S(t)I(t-\tau)}{1+\alpha S(t)} - (d+r+\omega)I(t), \end{aligned}\right\} \quad t \neq nT, \\ &\left.\begin{aligned} S(t^+) &= S(t) + \rho, \\ I(t^+) &= I(t), \end{aligned}\right\} \quad t = nT. \end{aligned} \tag{1.3}$$

From the point of biology, we only consider system (1.2) and (1.3) in the biological meaning region: $D = \{(S, I\ ) \mid S, I \geq 0\}$. Let

$$C_+ = \{\phi = (\phi_1(s), \phi_2(s) \in C : \phi_i(0) > 0\ (i = 1, 2)\}, \tag{1.4}$$

where $\phi_i(s)$ is positive, bounded, and continuous function for $s \in [-\tau, 0]$. Motivated by the application of systems (1.2) and (1.3) to population dynamics (refer to [24]), we assume that solutions of systems (1.1) satisfy the following initial conditions:

$$\phi \in C_+, \qquad \phi(0) > 0. \tag{1.5}$$

2. Plant Disease Continuous Control for System (1.2)

In this section, we consider system (1.2) with continuous replanting and removing and without impulsive effect. By Smith [25, Theorem 5.2.1] or Zhao and Zou [26], for any $\phi \in C_+$, there is a unique solution $(S(t, \phi), I(t, \phi))$ of system (1.2) with $(S(\zeta, \phi), I(\zeta, \phi)) = \phi(\zeta)$, for any $\zeta \in [-\tau, 0]$ and $S(\zeta, \phi) \geq 0$, $I(\zeta, \phi) \geq 0$ for all $t \geq 0$ in its maximal interval of existence.

Define $N(t) = S(t) + I(t)$; then we have

$$\frac{dN(t)}{dt} + LN(t) = \rho + (L - \mu)S(t) + (L - d - r)I(t). \tag{2.1}$$

Let $L = \min\{\mu, d + r\}$; we have

$$\frac{dN(t)}{dt} + LN(t) \leq \rho. \tag{2.2}$$

Then

$$N(t) \leq N(0)e^{-Lt} + \frac{\rho}{L}\left(1 - e^{-Lt}\right) \longrightarrow \frac{\rho}{L} = M \tag{2.3}$$

as $t \to +\infty$. Hence, system (1.2) is uniformly bounded.

Since $S(t)$ and $I(t)$ denote the number of susceptible and infected plants, respectively, it is easy to observe that system (1.2) has a disease-free of the form $E_1(\rho/\mu, 0)$, and a unique infection equilibrium $E_2(S^*, I^*)$ provided that we have the following condition:

(H)

$$\beta e^{-\mu\tau} > \left(\alpha + \frac{\mu}{\rho}\right)(d + r + \omega), \tag{2.4}$$

where

$$S^* = \frac{d + r + \omega}{\beta e^{-\mu\tau} - \alpha(d + r + \omega)}, \qquad I^* = \frac{\rho - \mu S^*}{d + r}. \tag{2.5}$$

2.1. The Stability of the Disease-Free Equilibrium $E_1(\rho/\mu, 0)$

We may firstly consider the stability of the disease-free equilibria $E_1(\rho/\mu, 0)$. Let $x(t) = S(t) - \rho/\mu$, $y(t) = I(t)$; then system (1.2) can be rewritten as the following equivalent system:

$$\begin{aligned} x'(t) &= \rho - \beta e^{-\mu\tau}\frac{(x(t)+\rho/\mu)y(t-\tau)}{1+\alpha(x(t)+\rho/\mu)} - \mu(x(t)+S^*) + \omega y(t), \\ y'(t) &= \beta e^{-\mu\tau}\frac{(x(t)+\rho/\mu)y(t-\tau)}{1+\alpha(x(t)+\rho/\mu)} - (d+r+\omega)y(t). \end{aligned} \tag{2.6}$$

Thus, the disease-free equilibrium $E_1(\rho/\mu, 0)$ of system (1.2) is transformed into zero equilibrium of system (2.6). Linearizing system (2.6) about the equilibrium (0, 0) yields the following linear system:

$$\begin{aligned} x'(t) &= -\mu x(t) - \frac{\beta\rho e^{-\mu\tau}}{\mu+\alpha\rho}y(t-\tau) + \omega y(t), \\ y'(t) &= \frac{\beta\rho e^{-\mu\tau}}{\mu+\alpha\rho}y(t-\tau) - (d+r+\omega)y(t), \end{aligned} \tag{2.7}$$

with characteristic equation:

$$\det\begin{pmatrix} -\mu-\lambda & \omega - \dfrac{\beta\rho e^{-\mu\tau}}{\mu+\alpha\rho}e^{-\lambda\tau} \\ 0 & \dfrac{\beta\rho e^{-\mu\tau}}{\mu+\alpha\rho}e^{-\lambda\tau} - (d+r+\omega) - \lambda \end{pmatrix} = 0, \tag{2.8}$$

that is,

$$(-\mu-\lambda)\left(\frac{\beta\rho e^{-\mu\tau}}{\mu+\alpha\rho}e^{-\lambda\tau} - (d+r+\omega) - \lambda\right) = 0. \tag{2.9}$$

The stability of trivial solution of system (1.2) depends on the locations of roots of characteristic equation (2.9). When all roots of (2.9) locate in the left half-plane of complex plane, the trivial solution of system (1.2) is stable; otherwise, it is unstable. In the following, we will investigate the distribution of roots of (2.9). Obviously, $\lambda_1 = -\mu < 0$. Let

$$\frac{\beta\rho e^{-\mu\tau}}{\mu+\alpha\rho}e^{-\lambda\tau} - (d+r+\omega) - \lambda = 0. \tag{2.10}$$

For (2.10), the root of (2.10) with $\tau = 0$ always has negative real part provided that $\beta\rho e^{-\mu\tau}/(\mu+\alpha\rho) < d+r+\omega$.

In addition, $i\varpi(\varpi > 0)$ is a root of (2.10) if and only if ϖ satisfies the following equation:

$$i\varpi = \frac{\beta\rho e^{-\mu\tau}}{\mu + \alpha\rho}(\cos\varpi\tau - i\sin\varpi\tau) - (d + r + \omega). \tag{2.11}$$

Separating the real and imaginary parts of (2.11) gives the following equations:

$$\begin{aligned} \frac{\beta\rho e^{-\mu\tau}}{\mu + \alpha\rho}\cos\varpi\tau &= d + r + \omega, \\ \frac{\beta\rho e^{-\mu\tau}}{\mu + \alpha\rho}\sin\varpi\tau &= -\varpi. \end{aligned} \tag{2.12}$$

Thus we can have

$$\varpi^2 = \left(\frac{\beta\rho e^{-\mu\tau}}{\mu + \alpha\rho}\right)^2 - (d + r + \omega)^2. \tag{2.13}$$

Then if $\beta\rho e^{-\mu\tau}/(\mu + \alpha\rho) < d + r + \omega$, (2.13) has not positive real root, which leads to (2.10) that has not purely imaginary root. By the Rouche Theory, we know that all the roots of (2.9) have always negative real parts. So the equilibrium $E_1(\rho/\mu, 0)$ of system (1.2) is locally asymptotically stable.

Define

$$\tau_1 = \frac{1}{\mu}\ln\frac{\beta}{(\alpha + \mu/\rho)(d + r + \omega)}. \tag{2.14}$$

For system (1.2), we have the following result on stability of the disease-free equilibrium $E_1(\rho/\mu, 0)$.

Theorem 2.1. *For system* (1.2), *the following statements are true.*

(i) *If* $\tau \in [0, \tau_1)$, *then the disease-free equilibrium* $E_1(\rho/\mu, 0)$ *of system* (1.2) *is unstable.*

(ii) *If* $\tau \in (\tau_1, +\infty)$, *then the disease-free equilibrium* $E_1(\rho/\mu, 0)$ *of system* (1.2) *is locally asymptotically stable.*

2.2. The Stability of the Positive Equilibrium $E_2(S^*, I^*)$ of System (1.2)

In this section, we show that the disease equilibrium is asymptotically stable in the case that time delay τ is less than the unity $(\tau < \tau_1)$; then we have the following theorem.

Theorem 2.2. *For system* (1.2), *if* $\tau \in [0, \tau_1)$, *then the positive equilibrium* $E_2(S^*, I^*)$ *of system* (1.2) *is asymptotically stable.*

Proof. Under the hypothesis (H), let $x(t) = S(t) - S^*$, $y(t) = I(\mathrm{t}) - I^*$, then system (1.2) can be rewritten as the following equivalent system:

$$\begin{aligned} x'(t) &= \rho - \beta e^{-\mu\tau} \frac{(x(t) + S^*)(y(t-\tau) + I^*)}{1 + \alpha(x(t) + S^*)} - \mu(x(t) + S^*) + \omega(y(t) + I^*), \\ y'(t) &= \beta e^{-\mu\tau} \frac{(x(t) + S^*)(y(t-\tau) + I^*)}{1 + \alpha(x(t) + S^*)} - (d + r + \omega)(y(t) + I^*). \end{aligned} \tag{2.15}$$

Thus, the positive equilibrium $E^*(S^*, I^*)$ of system (1.2) is transformed into zero equilibrium of system (2.15). Linearizing system (2.15) about the equilibrium (0, 0) yields the following linear system:

$$\begin{aligned} x'(t) &= -\left(\frac{\beta e^{-\mu\tau} I^*}{(1 + \alpha S^*)^2} + \mu\right) x(t) - \frac{\beta e^{-\mu\tau} S^*}{1 + \alpha S^*} y(t-\tau) + \omega y(t), \\ y'(t) &= \left(\frac{\beta e^{-\mu\tau} I^*}{(1 + \alpha S^*)^2}\right) x(t) + \frac{\beta e^{-\mu\tau} S^*}{1 + \alpha S^*} y(t-\tau) - (d + r + \omega) y(t), \end{aligned} \tag{2.16}$$

with characteristic equation:

$$\det \begin{pmatrix} -\left(\dfrac{\beta e^{-\mu\tau} I^*}{(1 + \alpha S^*)^2} + \mu\right) - \lambda & \omega - \dfrac{\beta e^{-\mu\tau} S^*}{1 + \alpha S^*} e^{-\lambda\tau} \\ \dfrac{\beta e^{-\mu\tau} I^*}{(1 + \alpha S^*)^2} & \dfrac{\beta e^{-\mu\tau} S^*}{1 + \alpha S^*} e^{-\lambda\tau} - (d + r + \omega) - \lambda \end{pmatrix} = 0. \tag{2.17}$$

The stability of trivial solution of system (1.2) depends on the locations of roots of characteristic equation (2.17). For the sake of simplicity, let

$$\begin{aligned} p &= \frac{\beta e^{-\mu\tau} I^*}{(1 + \alpha S^*)^2} + \mu + d + r + \omega > 0, \\ q &= -\frac{\mu \beta e^{-\mu\tau} S^*}{1 + \alpha S^*} = -\mu(d + r + \omega) < 0, \\ g &= -\frac{\beta e^{-\mu\tau} S^*}{1 + \alpha S^*} = -(d + r + \omega) < 0, \\ s &= (d + r)\left(\frac{\beta e^{-\mu\tau} I^*}{(1 + \alpha S^*)^2} + \mu\right) + \mu\omega > 0. \end{aligned} \tag{2.18}$$

Then (2.17) can be briefly denoted as the following equation:

$$\lambda^2 + p\lambda + q e^{-\lambda\tau} + g\lambda e^{-\lambda\tau} + s = 0. \tag{2.19}$$

For (2.19), we can claim that the two roots of (2.19) have always negative real parts. We will prove it in the following two steps.

Step 1. If $\tau = 0$, (2.19) can be simplified as

$$\lambda^2 + (p+g)\lambda + q + s = 0. \tag{2.20}$$

Note that

$$\begin{aligned} p + g &= \frac{\beta e^{-\mu\tau} I^*}{(1+\alpha S^*)^2} + \mu + d + \omega - (d+\omega) = \frac{\beta e^{-\mu\tau} I^*}{(1+\alpha S^*)^2} + \mu > 0, \\ q + s &= -\mu(d+\omega) + d\left(\frac{\beta e^{-\mu\tau} I^*}{(1+\alpha S^*)^2} + \mu\right) + \mu\omega = d\left(\frac{\beta e^{-\mu\tau} I^*}{(1+\alpha S^*)^2}\right) > 0. \end{aligned} \tag{2.21}$$

Therefore, the two roots of (2.19) with $\tau = 0$ have always negative real parts.

Step 2. If $\tau > 0$, $i\omega$ $(\omega > 0)$ is a root of (2.15) if and only if ω satisfies the following equation:

$$-\omega^2 + p\omega i + q(\cos\omega\tau - i\sin\omega\tau) + r\omega i(\cos\omega\tau - i\sin\omega\tau) + s = 0. \tag{2.22}$$

Separating the real and imaginary parts of (2.22) gives the following equations:

$$\begin{aligned} s - \omega^2 + q\cos\omega\tau + r\omega\sin\omega\tau &= 0, \\ p\omega - q\sin\omega\tau + r\omega\cos\omega\tau &= 0. \end{aligned} \tag{2.23}$$

One can obtain

$$\omega^4 + \left(p^2 - r^2 - 2s\right)\omega^2 + s^2 - q^2 = 0. \tag{2.24}$$

We can easily see that

$$\begin{aligned} p^2 - r^2 - 2s &= \left(\frac{\beta e^{-\mu\tau} I^*}{(1+\alpha S^*)^2} + \mu + d + \omega\right)^2 - (d+\omega)^2 - 2\left(d\left(\frac{\beta e^{-\mu\tau} I^*}{(1+\alpha S^*)^2} + \mu\right) + \mu\omega\right) \\ &= \frac{2\omega\beta e^{-\mu\tau} I^*}{(1+\alpha S^*)^2} \\ &> 0, \\ s^2 - q^2 &= \left(d\left(\frac{\beta e^{-\mu\tau} I^*}{(1+\alpha S^*)^2} + \mu\right) + \mu\omega\right)^2 - \left(\mu(d+\omega)\right)^2 \\ &= d\frac{\beta e^{-\mu\tau} I^*}{(1+\alpha S^*)^2}\left(d\frac{\beta e^{-\mu\tau} I^*}{(1+\alpha S^*)^2} + 2\mu(d+\omega)\right) \\ &> 0. \end{aligned} \tag{2.25}$$

So (2.24) has not positive real root, which leads to (2.19) that has not purely imaginary root. By the Rouche Theory, we know that all the roots of (2.19) have always negative real parts. So the equilibrium $E_2(S^*, I^*)$ of system (1.2) is asymptotically stable. The proof is complete. □

3. Plant Disease Impulsive Control for System (1.3)

3.1. Boundedness

Let the initial data be $S(0) > 0$, $I(0) > 0$. Then, one can easily prove that the solutions $(S(t), I(t))$ of system (1.3) are positive for all $t > 0$. Now, let $N(t) = S(t) + I(t)$. We calculate the upper right derivative of $N(t)$ along with a solution of system (1.3) with $t \neq nT$:

$$\frac{dN(t)}{dt} = -\mu S(t) - (d + r)I(t). \tag{3.1}$$

Since $\mu, d, r > 0$, one can deduce that

$$\frac{dN(t)}{dt} \leq -LN(t), \tag{3.2}$$

where $L = \min\{\mu, d + r\}$. We consider the following impulse differential inequalities:

$$\begin{aligned} &\frac{dN(t)}{dt} \leq -LN(t), \quad t \neq nT, \\ &N(nT^+) = N(nT) + \rho, \quad t = nT. \end{aligned} \tag{3.3}$$

According to impulse differential inequalities theory, we get

$$N(t) \leq N(0)e^{-Lt} + \sum_{0<nT<t} \left(\rho e^{-L(t-nT)}\right) \longrightarrow \frac{\rho e^{LT}}{e^{LT} - 1} \tag{3.4}$$

as $t \longrightarrow +\infty$.

So $N(t)$ is uniformly ultimately bounded. Hence, by the definition of $N(t)$, for any $\varepsilon > 0$, there exists a constant $M' = \rho e^{LT}/(e^{LT} - 1) + \varepsilon$ such that $S(t) < M'$ and $I(t) < M'$ for each solution of (1.3) with t being large enough.

3.2. Global Attractivity of the Disease-Free Periodic Solution of System (1.3)

In the following, we shall prove that the disease-free periodic is stable if it exists. For this purpose, we give firstly some basic properties of the following subsystem:

$$\begin{aligned} &S'(t) = -\mu S(t), \quad t \neq nT, \\ &S(t^+) = S(t) + \rho, \quad t = nT. \end{aligned} \tag{3.5}$$

We can find a unique positive periodic solution $S^*(t) = \rho e^{-\mu(t-nT)}/(1-e^{-\mu T})$, $nT < t \le (n+1)T$, which is globally asymptotically stable by using stroboscopic map. As a consequence, system (3.5) always has a disease-free periodic solution $(S^*(t), 0)$. Now, we give the conditions which assure the global attractivity of disease-free periodic solution of the system (1.3).

Denote

$$A = \frac{\omega M'}{\mu} + \frac{\rho}{1-e^{-\mu T}}, \qquad \Re_1 = \frac{A\beta e^{-\mu\tau}}{(d+r+\omega)(1+\alpha A)}. \tag{3.6}$$

Theorem 3.1. *The disease-free periodic solution* $(S^*(t), 0)$ *of system* (1.3) *is globally attractive provided that*

$$\Re_1 < 1. \tag{3.7}$$

Proof. Let $(S(t), I(t))$ be any solution of system (1.3). From the first equation of system (1.3), it follows that $\dot{S}(t) < -\mu S(t) + \omega M'$, $S(t^+) = S(t) + \rho$, for $nT < t \le (n+1)T$; then we consider the following impulse differential system:

$$\begin{aligned} \dot{z}(t) &= -\mu z(t) + \omega M', \quad t \ne nT, \\ z(t^+) &= z(t) + \rho, \quad t = nT. \end{aligned} \tag{3.8}$$

Obviously, system (3.8) has a globally asymptotically stable positive periodic solution:

$$z^*(t) = \frac{\omega M'}{\mu} + \frac{\rho e^{-\mu(t-nT)}}{1-e^{-\mu T}}, \quad t \in (nT, (n+1)T]. \tag{3.9}$$

By the comparison theorem in impulsive differential equation, for any sufficiently small positive ε, there exists an integer n_1 such that

$$S(t) < z^*(t) + \varepsilon < \frac{\omega M'}{\mu} + \frac{\rho}{1-e^{-\mu T}} + \varepsilon = A + \varepsilon, \quad n > n_1. \tag{3.10}$$

Therefore, from the second equation of system (1.3), we have

$$\dot{I}(t) \le \frac{\beta e^{-\mu\tau}(A+\varepsilon)}{1+\alpha(A+\varepsilon)} I(t-\tau) - (d+r+\omega)I(t), \quad t > n_1T + \tau. \tag{3.11}$$

Now we consider the following comparison equation:

$$\dot{R}(t) = \frac{\beta e^{-\mu\tau}(A+\varepsilon)}{1+\alpha(A+\varepsilon)} R(t-\tau) - (d+r+\omega)R(t), \quad t > n_1T + \tau. \tag{3.12}$$

Since $\Re_1 < 1$, we have

$$\frac{\beta e^{-\mu\tau} A}{1 + \alpha A} < d + r + \omega. \tag{3.13}$$

We may choose three sufficiently small positive constants ε such that

$$\frac{\beta e^{-\mu\tau} (A + \varepsilon)}{1 + \alpha(A + \varepsilon)} < d + r + \omega. \tag{3.14}$$

According to the theory of delay differential equation [24], we obtain that $\lim_{t\to+\infty} R(t) = 0$. By impulsive comparison theorem, we have $I(t) < R(t)$ with t being large enough. Therefore, we obtain that $\lim_{t\to+\infty} I(t) = 0$.

Then for a sufficiently small $\varepsilon_1 > 0$ and all t being large enough, we have $0 < I(t) < \varepsilon_1$. Without loss of generality, we may assume $0 < I(t) < \varepsilon_1$ as $t \geq 0$. From the first equation of system (1.3), we have

$$\begin{aligned} \dot{S}(t) &> -\left(\beta e^{-\mu\tau} \varepsilon_1 + \mu\right) S(t), \quad t \neq nT, \\ S(t^+) &= S(t) + \rho, \quad t = nT. \end{aligned} \tag{3.15}$$

Consider the following comparison system:

$$\begin{aligned} \dot{y}(t) &= -\left(\beta e^{-\mu\tau} \varepsilon_1 + \mu\right) y(t), \quad t \neq nT, \\ y(t^+) &= y(t) + \rho, \quad t = nT. \end{aligned} \tag{3.16}$$

Then, system (3.16) has a positive periodic solution:

$$y^*(t) = \frac{\rho e^{-(\beta e^{-\mu\tau}\varepsilon_1 + \mu)(t - nT)}}{1 - e^{-(\beta e^{-\mu\tau}\varepsilon_1 + \mu)T}}, \tag{3.17}$$

which is globally asymptotically stable. Thus, for a sufficiently small $\varepsilon > 0$, when t is large enough, we have

$$S(t) > y(t) > y^*(t) - \varepsilon. \tag{3.18}$$

From the first equation of system (1.3), we also have

$$\begin{aligned} \dot{S}(t) &< -\mu S(t) + \varepsilon_1 \omega, \quad t \neq nT, \\ S(t^+) &= S(t) + \rho, \quad t = nT. \end{aligned} \tag{3.19}$$

Consider the following comparison system:

$$\begin{aligned} \dot{x}(t) &= -\mu x(t) + \varepsilon_1 \omega, \quad t \neq nT, \\ x(t^+) &= x(t) + \rho, \quad t = nT. \end{aligned} \tag{3.20}$$

System (3.20) has a globally asymptotically stable positive periodic solution:

$$x^*(t) = \frac{\varepsilon_1 \omega}{\mu} + \frac{\rho e^{-\mu(t-nT)}}{1 - e^{-\mu T}}, \quad t \in (nT, (n+1)T]. \tag{3.21}$$

Thus, for a sufficiently small $\varepsilon > 0$, when t is large enough, we have

$$S(t) < x(t) < x^*(t) + \varepsilon. \tag{3.22}$$

Combining (3.18) with (3.22), we obtain

$$y^*(t) - \varepsilon < S(t) < x^*(t) + \varepsilon. \tag{3.23}$$

Let $\varepsilon_1 \to 0$; we have $S^*(t) - \varepsilon < S(t) < S^*(t) + \varepsilon$, which implies $\lim_{t \to +\infty} S(t) = S^*(t)$. The proof is completed. □

3.3. *Permanence of the System* (1.3)

Definition 3.2. System (1.3) is said to be permanent if there exist constants $k, K > 0$ (independent of initial value) and a finite time T_0 such that for every positive solution $(S(t), I(t))$ with initial conditions (1.3) satisfies $k < S(t) < K$, $k < I(t) < K$ for all $t > T_0$. Here T_0 may depend on the initial condition.

Denote

$$\begin{aligned} \Re_2 &= \frac{\rho\left(\beta e^{-\mu\tau} - \alpha(d + r + \omega)\right)}{(d + r + \omega)\left(e^{\mu T} - 1\right)}, \\ S^* &= \frac{d + r + \omega}{\beta e^{-\mu\tau} - \alpha(d + r + \omega)}, \\ I^* &= \frac{(1/T)\ln\left(\left(e^{\mu T} - 1\right)\Re_2 + 2\right) - \mu}{\beta e^{-\mu\tau}}. \end{aligned} \tag{3.24}$$

Lemma 3.3. *If $\Re_2 > 1$, then there exists a positive constant m_2 such that* $\lim_{t \to \infty} \inf I(t) \geq m_2$.

Proof. Define

$$U(t) = I(t) + \frac{\beta e^{-\mu\tau} S^*}{1 + \alpha S^*} \int_{t-\tau}^{t} I(\varrho)\, d\varrho. \tag{3.25}$$

Calculating the derivative of $U(t)$ along with the solution of (1.3), we can get

$$\dot{U}(t) = \dot{I} + \frac{\beta e^{-\mu\tau} S^*}{1+S^*}(I(t) - I(t-\tau)) = \beta e^{-\mu\tau}\left(\frac{S(t)}{1+\alpha S(t)} - \frac{S^*}{1+\alpha S^*}\right) I(t-\tau) \tag{3.26}$$

for $t \geq 0$.

Since $\Re_2 > 1$, then $I^* = ((1/T)\ln((e^{\mu T} - 1)\Re_2 + 2) - \mu)/\beta e^{-\mu\tau} > ((1/T)\ln(e^{\mu T} + 1) - \mu)/\beta e^{-\mu\tau} > ((1/T)\ln(e^{\mu T}) - \mu)/\beta e^{-\mu\tau} = 0$. Note that $\Re_2 = \rho/S^*(e^{\mu T} - 1)$ and $I^* = (1/T)(\ln((e^{\mu T}-1)\Re_2 + 2) - \mu)/\beta e^{-\mu\tau} > ((1/T)\ln((e^{\mu T}-1)\Re_2 + 1) - \mu)/\beta e^{-\mu\tau} = ((1/T)\ln(\rho/S^* + 1) - \mu)/\beta e^{-\mu\tau}$.

Solving the aforementioned inequality, we can have that

$$S^* < \frac{\rho e^{-(\beta e^{-\mu\tau} I^* + \mu)T}}{1 - e^{-(\beta e^{-\mu\tau} I^* + \mu)T}}. \tag{3.27}$$

We can choose $\varepsilon > 0$ being small enough such that

$$S^* < \frac{\rho e^{-(\beta e^{-\mu\tau} I^* + \mu)T}}{1 - e^{-(\beta e^{-\mu\tau} I^* + \mu)T}} - \varepsilon = S_\Delta. \tag{3.28}$$

For any positive constant t_0, we claim that the inequality $I(t) < I^*$ cannot hold for all $t \geq t_0$. Otherwise, there exists a positive constant t_0 such that $I(t) < I^*$ for all $t \geq t_0$. From the first equation of (1.3), we have

$$\begin{aligned} \dot{S}(t) &> -\left(\beta e^{-\mu\tau} I^* + \mu\right) S(t), \quad t \neq nT, \\ S(t^+) &= S(t) + \rho, \quad t = nT. \end{aligned} \tag{3.29}$$

Similarly, we know that there exists such $T_1 > t_0 + \tau$, for $t > T_1$ that

$$S(t) > \frac{\rho e^{-(\beta e^{-\mu\tau} I^* + \mu)T}}{1 - e^{-(\beta e^{-\mu\tau} I^* + \mu)T}} - \varepsilon = S_\Delta. \tag{3.30}$$

Then, by (3.30), we have that, for $t > T_1$,

$$\dot{U}(t) = \dot{I} + \frac{\beta e^{-\mu\tau} S^*}{1+S^*}(I(t) - I(t-\tau)) > \beta e^{-\mu\tau}\left(\frac{S_\Delta}{1+\alpha S_\Delta} - \frac{S^*}{1+\alpha S^*}\right) I(t-\tau). \tag{3.31}$$

Let

$$I^l = \min_{t\in[T_1, T_1+\tau]} I(t). \tag{3.32}$$

We show that $I(t) \geq I^l$ for all $t \geq T_1$. Otherwise, there exists a nonnegative constant T_2 such that $I(t) \geq I^l$ for $t \in [T_1, T_1 + \tau + T_2]$, $I(T_1 + \tau + T_2) = I^l$ and $\dot{I}(T_1 + \tau + T_2) \leq 0$. Thus from the second equation of (1.3) and (3.30), we easily see that

$$\begin{aligned}\dot{I}(T_1 + \tau + T_2) &> \left(\beta e^{-\mu\tau}\frac{S(t)}{1+\alpha S(t)} - (d+r+\omega)\right)I^l \\ &= (d+r+\omega)\left(\frac{\beta e^{-\mu\tau}}{d+r+\omega}\frac{S(t)}{1+\alpha S(t)} - 1\right)I^l \\ &> (d+r+\omega)\left(\frac{S_\Delta/(1+\alpha S_\Delta)}{S^*/(1+\alpha S^*)} - 1\right)I^l > 0,\end{aligned} \tag{3.33}$$

which is a contradiction. Hence we get that $I(t) \geq I^l > 0$ for all $t \geq T_1$. Therefore, for all $t > T_1 + \tau$, we have

$$\dot{U}(t) > \beta e^{-\mu\tau}\left(\frac{S_\Delta}{1+\alpha S_\Delta} - \frac{S^*}{1+\alpha S^*}\right)I_l > 0, \tag{3.34}$$

which implies $U(t) \to +\infty$ as $t \to +\infty$. This is a contradiction to $U(t) \leq M(1 + \beta\tau e^{-\mu\tau}(S^*/1 + \alpha S^*))$ for being t large enough. Therefore, for any positive constant t_0, the inequality $I(t) < I^*$ cannot hold for all $t \geq t_0$.

On the one hand, if $I(t) \geq I^*$ holds true for all t being large enough, then our aim is obtained. Otherwise, $I(t)$ is oscillatory about I^*.

Let

$$m_2 = \min\left\{\frac{I^*}{2}, I^* e^{-(d+r+\omega)\tau}\right\}. \tag{3.35}$$

In the following, we will show that $I(t) \geq m_2$ for t being large enough. There exist two positive constants $\bar{t}, \psi$ such that

$$I(\bar{t}) = I(\bar{t} + \psi) = I^*, \tag{3.36}$$

$$I(t) < I^*, \quad \text{for } \bar{t} < t < \bar{t} + \psi. \tag{3.37}$$

Since $I(t)$ is continuous and bounded and is not effected by impulses, we conclude that $I(t)$ is uniformly continuous. Hence there exists a constant T_3 (with $0 < T_3 < \tau$ and T_3 is independent of the choice of $\bar{t}$) such that $I(t) > I^*/2$ for all $\bar{t} \leq t \leq \bar{t} + T_3$.

If $\psi \leq T_3$, our aim is obtained.

If $T_3 < \psi \leq \tau$, from the second equation of (1.3), we have that $\dot{I}(t) \geq -(d+r+\omega)I(t)$ for $\bar{t} < t \leq \bar{t} + \psi$. Then we have $I(t) \geq I^* e^{-(d+r+\omega)\tau}$ for $\bar{t} < t \leq \bar{t} + \psi \leq \bar{t} + \tau$ since $I(\bar{t}) = I^*$. It is clear that $I(t) \geq m_2$ for $\bar{t} < t \leq \bar{t} + \psi$.

If $\psi \geq \tau$, then we have that $I(t) \geq I^* e^{-(d+r+\omega)\tau}$ for $\bar{t} < t \leq \bar{t} + \tau$. Next, we will show that $I(t) \geq I^* e^{-(d+r+\omega)\tau}$ for $\bar{t} + \tau < t \leq \bar{t} + \psi$. In fact, if not, there exists a $T_4 > 0$ such that

$I(t) \geq I^* e^{-(d+r+\omega)\tau}$ for $\bar{t} < t \leq \bar{t} + \tau + T_4$. $I(\bar{t} + \tau + T_4) = I^* e^{-(d+r+\omega)\tau}$ and $\dot{I}(\bar{t} + \tau + T_4) \leq 0$. When $\bar{t}$ is large enough, from (3.37) and the first equation of (1.3), we have

$$\begin{aligned} \dot{S}(t) &> -\left(\beta e^{-\mu\tau} I^* + \mu\right) S(t), \quad t \neq nT, \\ S(t^+) &= S(t) + \rho, \quad t = nT. \end{aligned} \tag{3.38}$$

Similarly, we know that there exists $T_5 > \bar{t} + \tau$, for $t > T_5$ that

$$S(t) > \frac{\rho e^{-(\beta e^{-\mu\tau} I^* + \mu)T}}{1 - e^{-(\beta e^{-\mu\tau} I^* + \mu)T}} - \varepsilon = S_\Delta. \tag{3.39}$$

Then the inequality $S(t) > S_\Delta$ holds true for $\bar{t} + \tau < t < \bar{t} + \psi$. On the other hand, we have from the second equation of (1.3) that

$$\begin{aligned} \dot{I}\left(\bar{t} + \tau + T_4\right) &> \left(\beta e^{-\mu\tau} \frac{S(t)}{1 + \alpha S(t)} - (d + r + \omega)\right) I^* e^{-(d+r+\omega)\tau} \\ &= (d + r + \omega)\left(\frac{\beta e^{-\mu\tau}}{d + r + \omega} \frac{S(t)}{1 + \alpha S(t)} - 1\right) I^* e^{-(d+r+\omega)\tau} \\ &> (d + r + \omega)\left(\frac{S_\Delta/(1 + \alpha S_\Delta)}{S^*/(1 + \alpha S^*)} - 1\right) I^* e^{-(d+r+\omega)\tau} > 0. \end{aligned} \tag{3.40}$$

This is a contradiction to $\dot{I}(\bar{t} + \tau + T_4) \leq 0$. Therefore, $I(t) \geq m_2$ for $t \in [\bar{t}, \bar{t} + \psi]$.

Since this kind of interval $[\bar{t}, \bar{t} + \psi]$ is arbitrarily chosen, we get that $I(t) \geq m_2$ for t being large enough. In view of our arguments previously, the choice of m_2 is independent of the positive solution of (1.3) which satisfies that $I(t) \geq m_2$ for sufficiently large t. This completes the proof. □

Theorem 3.4. *If* $\Re_2 > 1$, *the system* (1.3) *is permanent; that is, there exist two positive constants* m_1, m_2 *such that* $S(t) \geq m_1, I(t) \geq m_2$ *for t being large enough.*

Proof. Suppose that $X(t) = (S(t), I(t))$ is any positive solution of system (1.3). From the first and third equations of (1.3), we have that

$$\begin{aligned} \dot{S}(t) &> -\left(\beta e^{-\mu\tau} M' + \mu\right) S(t), \quad t \neq nT, \\ S(t^+) &= S(t) + \rho, \quad t = nT. \end{aligned} \tag{3.41}$$

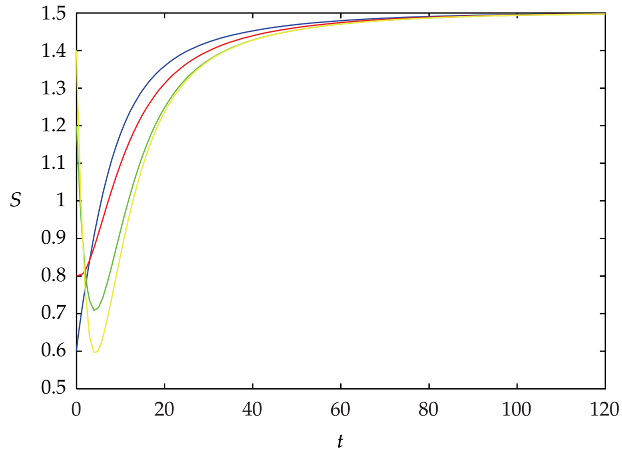

Figure 1: Time series of $S(t)$ with different initial values and parameters $\rho = 0.3$, $\mu = 0.2$, $\beta = 1.5$, $\alpha = 1$, $r = 0.5$, $d = 0.2$, $\omega = 0.1$, and $\tau = 1$.

Similarly, we can get such t large enough and $\varepsilon > 0$ small enough that

$$S(t) \geq \frac{\rho e^{-(\beta e^{-\mu\tau} M' + \mu)T}}{1 - e^{-(\beta e^{-\mu\tau} M' + \mu)T}} - \varepsilon = m_1. \tag{3.42}$$

Set

$$D = \left\{ (S, I) \in R^2 \mid m_1 \leq S(t) \leq M', m_2 \leq I(t) \leq M' \right\}. \tag{3.43}$$

Then D is a bounded compact region which has positive distance from coordinate axes. By Lemma 3.3, one obtains that every solution of system (1.3) eventually enters and remains in the region D. The proof of Theorem 3.4 is completed. □

4. Numerical Simulation and Conclusion

To verify the theoretical results obtained in this paper, we will give some numerical simulations.

Under the continuous control strategy, we consider the hypothetical set of parameter values as $\rho = 0.3$, $\mu = 0.2$, $\beta = 1.5$, $\alpha = 1$, $r = 0.5$, $d = 0.2$, and $\omega = 0.1$, with $S(0) = 1$, and $I(0) = 1$. Through calculation, we know $\tau_1 = 0.5889$ and $E_1(1.5, 0)$.

(i) If $\tau = 1 > \tau_1$, then according to Theorem 2.1, we know the disease-free equilibrium E_1 of system (1.2) is local stable for this case (see Figures 1, 2, and 3).

(ii) If $\tau = 0.5 < \tau_1$, through calculation, we know $E_2(1.4356, 0.0644)$. Then according to Theorem 2.2, the positive equilibrium E_2 of system (1.2) is local stable for this case (see Figures 4, 5, and 6).

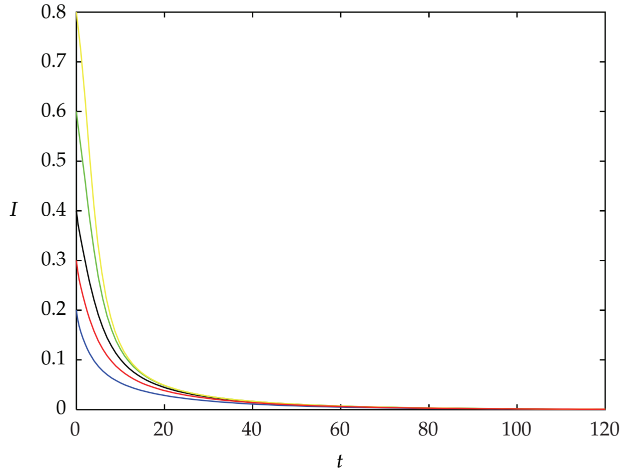

Figure 2: Time series of $I(t)$ with different initial values and parameters $\rho = 0.3$, $\mu = 0.2$, $\beta = 1.5$, $\alpha = 1$, $r = 0.5$, $d = 0.2$, $\omega = 0.1$, and $\tau = 1$.

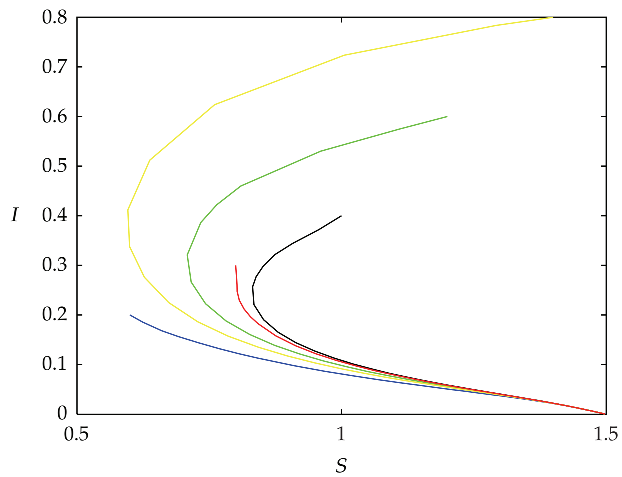

Figure 3: Phase diagram of $S(t)$ with different initial values and parameters $\rho = 0.3$, $\mu = 0.2$, $\beta = 1.5$, $\alpha = 1$, $r = 0.5$, $d = 0.2$, $\omega = 0.1$, and $\tau = 1$.

Its epidemiological implication is that if time delay τ is greater than some key value τ_1, then diseased plants will disappear in local scope. In contrast, if time delay τ is less than some key value τ_1, then susceptible plants and diseased plants will coexist in local scope.

Under the impulsive control strategy, We consider the hypothetical set of parameter values as $\mu = 0.2$, $\beta = 1.2$, $\alpha = 1$, $r = 0.6$, $d = 0.2$, $\omega = 0.1$, $T = 1$, and $\tau = 0$ with $S(0) = 1$, and $I(0) = 1$.

(i) We consider the susceptible plants rate $\rho = 0.8$. Through calculation, we have $\Re_2 = 1.2044$. Then according to Theorem 3.4, we know that system (1.3) is permanence, for this case (see Figures 10, 11, and 12).

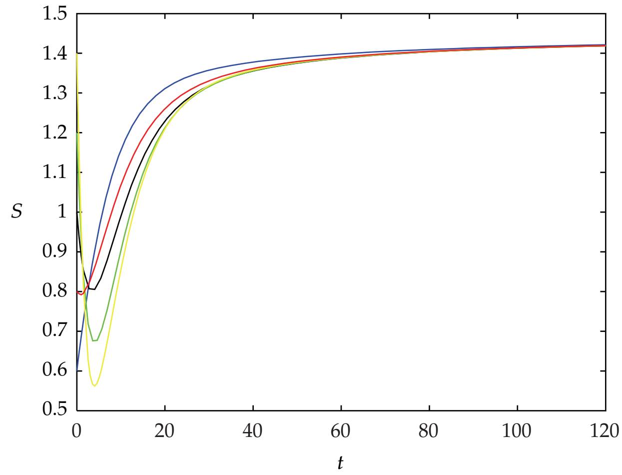

Figure 4: Time series of $S(t)$ with different initial values and parameters $\rho = 0.3$, $\mu = 0.2$, $\beta = 1.5$, $\alpha = 1$, $r = 0.5$, $d = 0.2$, $\omega = 0.1$, and $\tau = 0.5$.

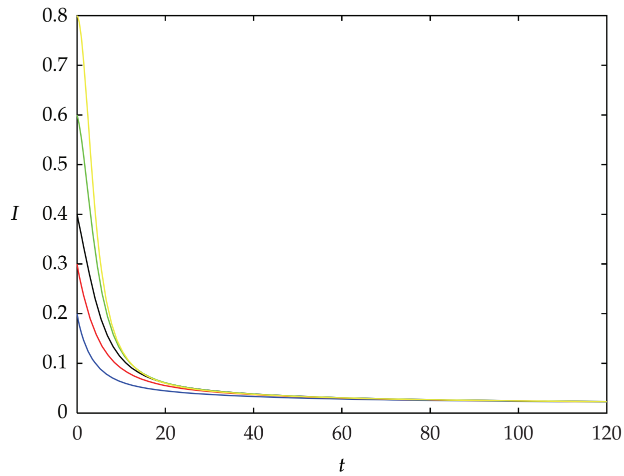

Figure 5: Time series of $I(t)$ with different initial values and parameters $\rho = 0.3$, $\mu = 0.2$, $\beta = 1.5$, $\alpha = 1$, $r = 0.5$, $d = 0.2$, $\omega = 0.1$, and $\tau = 0.5$.

(ii) If we decrease the susceptible plants rate ρ to 0.3, through calculation, we know $\Re_1 = 0.9505$. Then according to Theorem 3.1, the disease-free periodic solution of system is globally attractive, for this case (see Figures 7, 8, and 9).

We have other hypothetical parameter values under the impulsive control strategy as $\mu = 0.1$, $\beta = 1$, $\alpha = 1$, $r = 0.3$, $d = 0.2$, $\omega = 0.2$, $T = 0.8$, and $\tau = 0.2$. with $S(0) = 0.5$, and $I(0) = 0.1$

(i) We consider the susceptible plants rate $\rho = 5$. Through calculation, we have $\Re_2 = 1.2044$. Then according to Theorem 3.4, we know that system (1.3) is permanence, for this case (see Figures 16, 17, and 18).

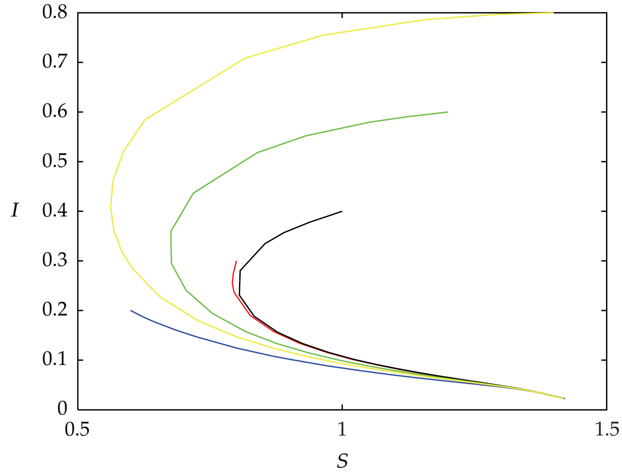

Figure 6: Phase diagram of $S(t)$ with different initial values and parameters $\rho = 0.3$, $\mu = 0.2$, $\beta = 1.5$, $\alpha = 1$, $r = 0.5$, $d = 0.2$, $\omega = 0.1$, and $\tau = 0.5$.

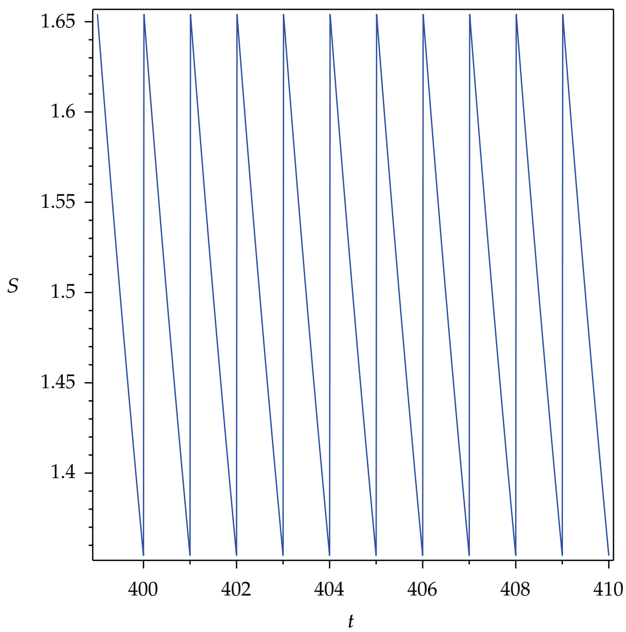

Figure 7: Time series of $S(t)$ with parameters $\rho = 0.8$, $\mu = 0.2$, $\beta = 1.2$, $\alpha = 1$, $r = 0.6$, $d = 0.2$, $\omega = 0.1$, $T = 1$, and $\tau = 0$.

(ii) If we decrease the susceptible plants rate ρ to 0.4, through calculation, we know $\Re_1 = 0.9505$. Then according to Theorem 3.1, the disease-free periodic solution of system is globally attractive, for this case (see Figures 13, 14, and 15).

Its epidemiological implication is that we took such a strategy by improving planting susceptible plants in practice; as a result, if the susceptible plants rate is greater than some key

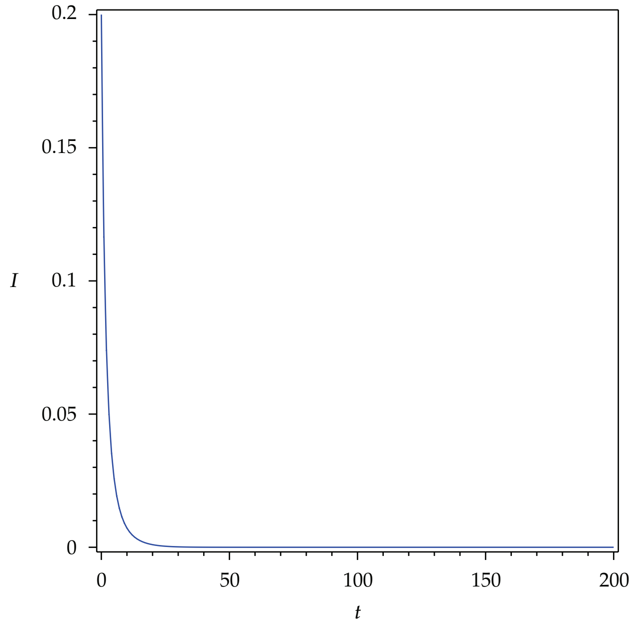

Figure 8: Time series of $I(t)$ with parameters $\rho = 0.8$, $\mu = 0.2$, $\beta = 1.2$, $\alpha = 1$, $r = 0.6$, $d = 0.2$, $\omega = 0.1$, $T = 1$, and $\tau = 0$.

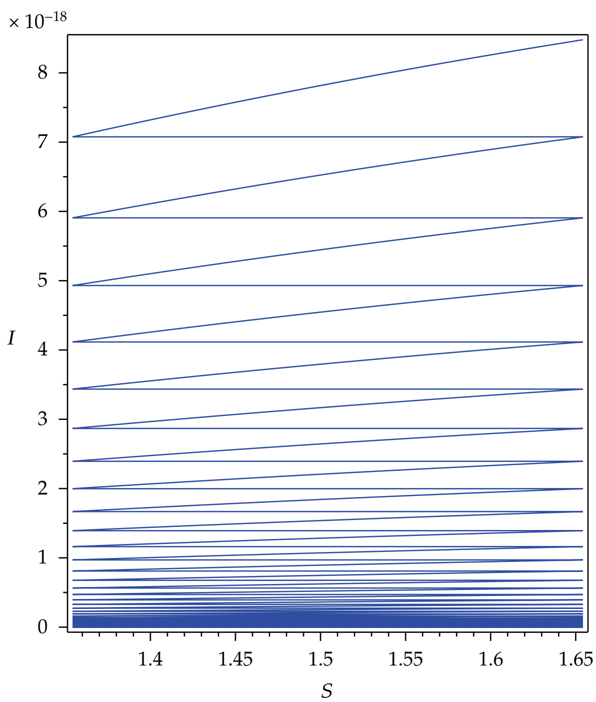

Figure 9: Phase diagram of $S(t)$ and $I(t)$ with parameters $\rho = 0.8$, $\mu = 0.2$, $\beta = 1.2$, $\alpha = 1$, $r = 0.6$, $d = 0.2$, $\omega = 0.1$, $T = 1$, and $\tau = 0$.

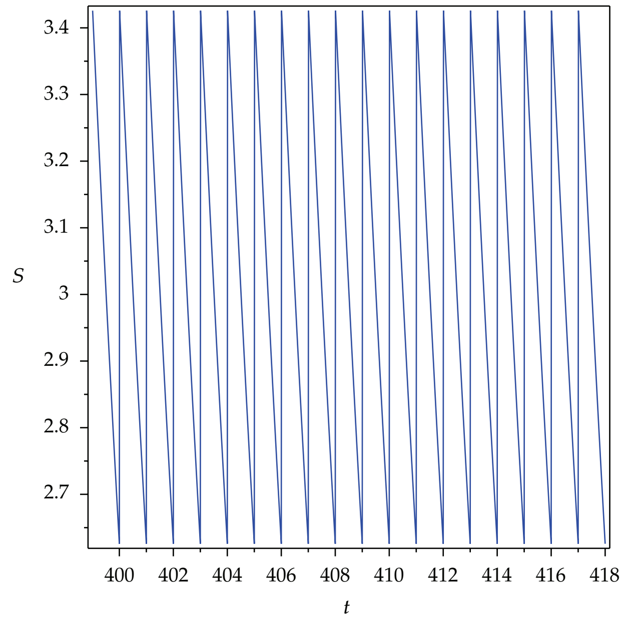

Figure 10: Time series of $S(t)$ with parameters $\rho = 0.3$, $\mu = 0.2$, $\beta = 1.2$, $\alpha = 1$, $r = 0.6$, $d = 0.2$, $\omega = 0.1$, $T = 1$, and $\tau = 0$.

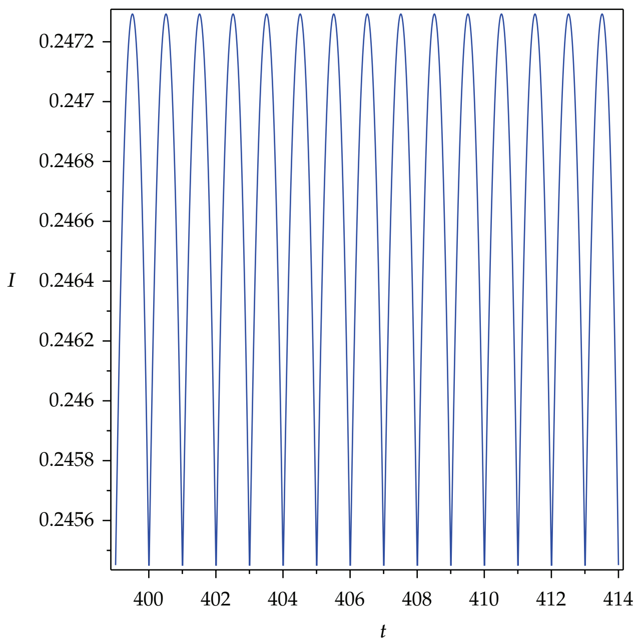

Figure 11: Time series of $I(t)$ with parameters $\rho = 0.3$, $\mu = 0.2$, $\beta = 1.2$, $\alpha = 1$, $r = 0.6$, $d = 0.2$, $\omega = 0.1$, $T = 1$, and $\tau = 0$.

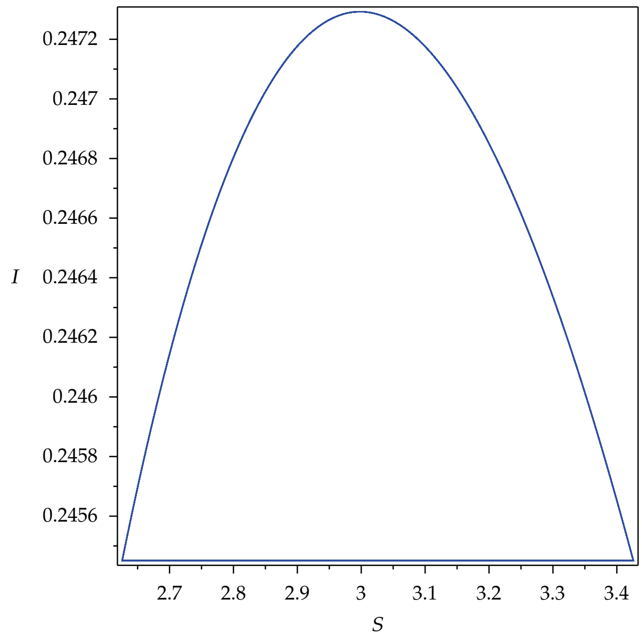

Figure 12: Phase diagram of $S(t)$ and $I(t)$ with parameters $\rho = 0.3$, $\mu = 0.2$, $\beta = 1.2$, $\alpha = 1$, $r = 0.6$, $d = 0.2$, $\omega = 0.1$, $T = 1$, and $\tau = 0$.

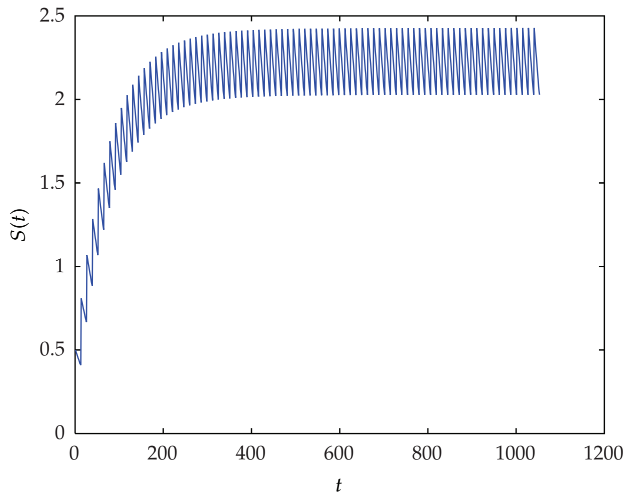

Figure 13: Time series of $S(t)$ with parameters $\rho = 0.4$, $\mu = 0.1$, $\beta = 1$, $\alpha = 1$, $r = 0.3$, $d = 0.2$, $\omega = 0.2$, $T = 0.8$, and $\tau = 0.2$.

value ρ^*, both susceptible plants and diseased plants will coexist. In contrast, if we decrease the susceptible plants rate ρ and make it less than some key value ρ_*, then diseased plants will die out at length. In a word, we find that the impulse plants rate has played a very important role in the actual plant epidemic prevention.

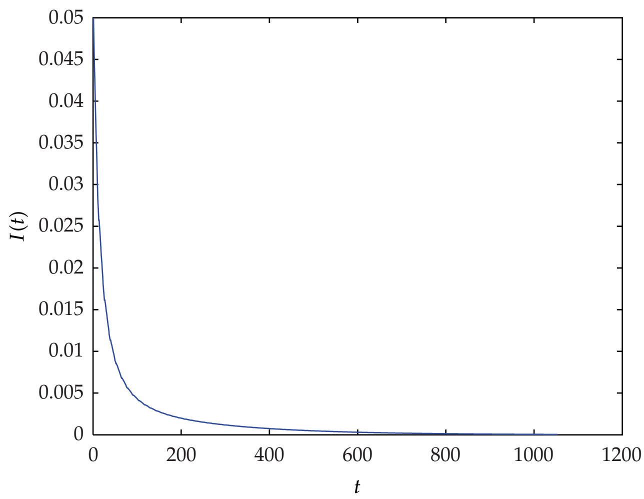

Figure 14: Time series of $I(t)$ with parameters $\rho = 0.4$, $\mu = 0.1$, $\beta = 1$, $\alpha = 1$, $r = 0.3$, $d = 0.2$, $\omega = 0.2$, $T = 0.8$, and $\tau = 0.2$.

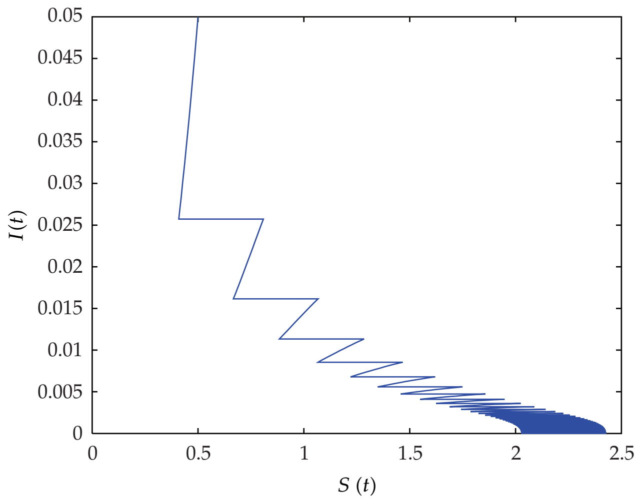

Figure 15: Phase diagram of $S(t)$ and $I(t)$ with parameters $\rho = 0.4$, $\mu = 0.1$, $\beta = 1$, $\alpha = 1$, $r = 0.3$, $d = 0.2$, $\omega = 0.2$, $T = 0.8$, and $\tau = 0.2$.

In this paper, delay SIS plant epidemic model is constructed and investigated. We proposed two different control strategies in the model. Our primary results are to compare the difference between the two control methods. Firstly, we consider continuous cultural control strategy by continuous replanting of healthy plants. We come to the conclusion that if $\rho < \rho_\Delta$, then diseased plants will disappear in local scope where $\rho_\Delta = \mu(d+r+\omega)/(\beta e^{-\mu\tau} - \alpha(d+r+\omega))$.

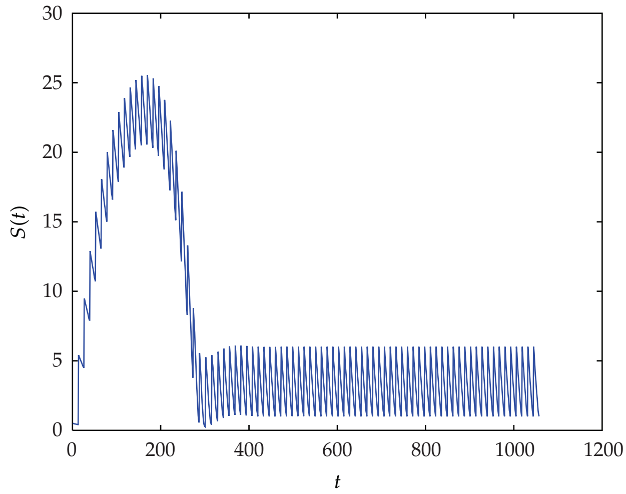

Figure 16: Time series of $S(t)$ with parameters $\rho = 5$, $\mu = 0.1$, $\beta = 1$, $\alpha = 1$, $r = 0.3$, $d = 0.2$, $\omega = 0.2$, $T = 0.8$, and $\tau = 0.2$.

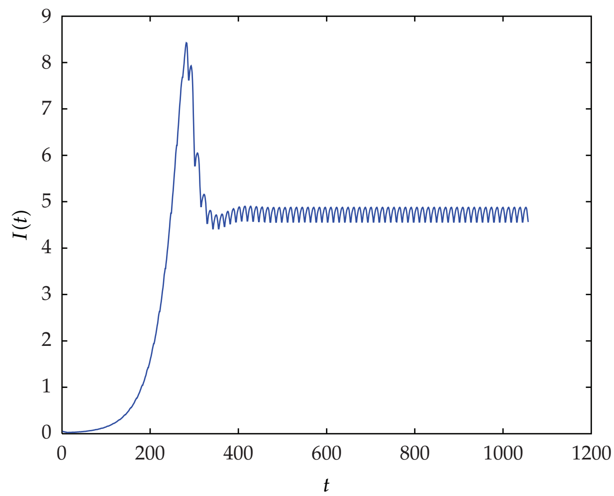

Figure 17: Time series of $I(t)$ with parameters $\rho = 5$, $\mu = 0.1$, $\beta = 1$, $\alpha = 1$, $r = 0.3$, $d = 0.2$, $\omega = 0.2$, $T = 0.8$, and $\tau = 0.2$.

And if $\rho > \rho_{\Delta}$, then diseased plants will exist for a long time in local scope. Secondly, impulsive control strategy of plant disease model is considered; in this case, we get that if $\rho < \rho_*$, then diseased plants will disappear finally where $\rho_* = ((1 - e^{-LT})/(\mu + \omega))(\mu(d + r + \omega)/(\beta e^{-\mu\tau} - \alpha(d + r + \omega)))$. And if $\rho > \rho^*$, then diseased plants will exist for a long time where

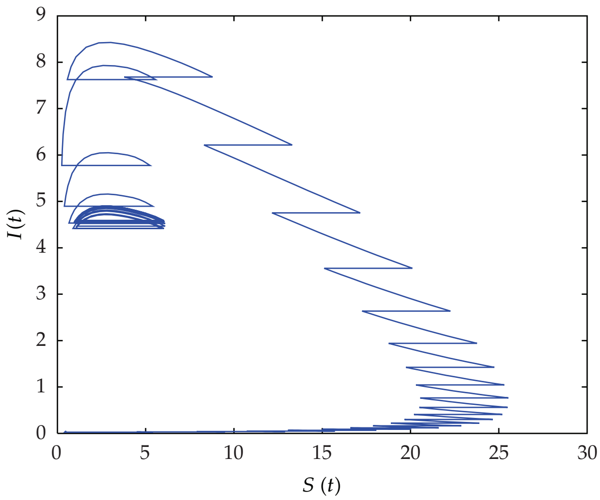

Figure 18: Phase diagram of $S(t)$ and $I(t)$ with parameters $\rho = 5$, $\mu = 0.1$, $\beta = 1$, $\alpha = 1$, $r = 0.3$, $d = 0.2$, $\omega = 0.2$, $T = 0.8$, and $\tau = 0.2$.

$\rho^* = (e^{\mu T} - 1)(\mu(d + r + \omega)/(\beta e^{-\mu\tau} - \alpha(d + r + \omega)))$. We think that our results will offer help to the actual plant infectious disease management.

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