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Research Article

A Unique Common Triple Fixed Point Theorem for Hybrid Pair of Maps

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We obtain a unique common triple fixed point theorem for hybrid pair of mappings in metric spaces. Our result extends the recent results of B. Samet and C. Vetro (2011). We also introduced a suitable example supporting our result.

1. Introduction

The study of fixed points for multivalued contraction mappings using the Hausdorff metric was initiated by Nadler [1].

Let (X, d) be a metric space. We denote $CB(X)$ the family of all nonempty closed and bounded subsets of X and $CL(X)$ the set of all nonempty closed subsets of X. For $A, B \in CB(X)$ and $x \in X$, we denote $D(x, A) = \inf\{d(x, a) : a \in A\}$. Let H be the Hausdorff metric induced by the metric d on X, that is,

$$H(A, B) = \max\left\{\sup_{x\in A} d(x, B), \sup_{y\in B} d(y, A)\right\}, \tag{1.1}$$

for every $A, B \in CB(X)$.

It is clear that for $A, B \in CB(X)$ and $a \in A$, we have $d(a, B) \leq H(A, B)$.

Definition 1.1. An element $x \in X$ is said to be a fixed point of a set-valued mapping $T : X \to CB(X)$ if and only if $x \in Tx$.

In 1969, Nadler [1] extended the famous Banach contraction principle [2] from single-valued mapping to multivalued mapping and proved the following fixed point theorem for the multivalued contraction.

Theorem 1.2 (see, Nadler [1]). *Let (X, d) be a complete metric space and let T be a mapping from X into $CB(X)$. Assume that there exists $c \in [0, 1)$ such that*

$$H(Tx, Ty) \leq cd(x, y), \tag{1.2}$$

for all $x, y \in X$. Then, T has a fixed point.

Lemma 1.3 (see, Nadler [1]). *Let $A, B \in CB(X)$ and $\alpha > 1$. Then for every $a \in A$, there exists $b \in B$ such that $d(a, b) \leq \alpha H(A, B)$.*

Lemma 1.4 (see, Nadler [1]). *Let $\alpha > 0$. If $A, B \in CB(X)$ with $H(A, B) \leq \alpha$, then for each $a \in A$, there exists $b \in B$ such that $d(a, b) \leq \alpha$.*

Lemma 1.5 (see, Nadler [1]). *Let $\{A_n\}$ be a sequence in $CB(X)$ with $\lim_{n\to+\infty} H(A_n, A) = 0$, for $A \in CB(X)$. If $x_n \in A_n$ and $\lim_{n\to+\infty} d(x_n, x) = 0$, then $x \in A$.*

The existence of fixed points for various multivalued contractive mappings has been studied by many authors under different conditions. For details, we refer the reader to [1, 3–11] and the references therein.

The concept of coupled fixed point for multivalued mapping was introduced by Samet and Vetro [12], and later several authors, namely, Hussain and Alotaibi [13], Aydi et al. [14], and Abbas et al. [15], proved coupled coincidence point theorems in partially ordered metric spaces.

Definition 1.6 (see, Samet and Vetro [12]). Let $F : X \times X \to CL(X)$ be a given mapping. We say that $(x, y) \in X \times X$ is a coupled fixed point of F if and only if

$$x \in F(x, y), \quad y \in F(y, x). \tag{1.3}$$

Definition 1.7 (see, Hussain and Alotaibi [13]). Let the mappings $F : X \times X \to CB(X)$ and $g : X \to X$ be given. An element $(x, y) \in X \times X$ is called

(1) a coupled coincidence point of a pair $\{F, g\}$ if $gx \in F(x, y)$ and $gy \in F(y, x)$;

(2) a coupled common fixed point of a pair $\{F, g\}$ if $x = gx \in F(x, y)$ and $y = gy \in F(y, x)$.

Berinde and Borcut [16] introduced the concept of triple fixed points and obtained a tripled fixed point theorem for single valued map.

Now we give the following.

Definition 1.8. Let X be a nonempty set, $T : X \times X \times X \to 2^X$ (collection of all nonempty subsets of X). $f : X \to X$.

(i) The point $(x, y, z) \in X \times X \times X$ is called a tripled fixed point of T if

$$x \in T(x, y, z), \quad y \in T(y, x, y), \quad z \in T(z, y, x). \tag{1.4}$$

(ii) The point $(x, y, z) \in X \times X \times X$ is called a tripled coincident point of T and f if

$$fx \in T(x, y, z), \quad fy \in T(y, x, y), \quad fz \in T(z, y, x). \tag{1.5}$$

(iii) The point $(x, y, z) \in X \times X \times X$ is called a tripled common fixed point of T and f if

$$x = fx \in T(x, y, z), \quad y = fy \in T(y, x, y), \quad z = fz \in T(z, y, x). \tag{1.6}$$

Definition 1.9. Let $T : X \times X \times X \to 2^X$ be a multivalued map and f be a self map on X. The Hybrid pair $\{T, f\}$ is called w-compatible if $f(T(x, y, z)) \subseteq T(fx, fy, fz)$ whenever (x, y, z) is a tripled coincidence point of T and f.

2. Main Results

Theorem 2.1. *Let (X, d) be a metric space and let $T : X \times X \times X \to CB(X)$ and $f : X \to X$ mappings satisfying*

(2.1.1) $H(T(x, y, z), T(u, v, w)) \leq jd(fx, fy) + kd(fy, fv) + ld(fz, fw)$, *for all $x, y, z, u, v, w \in X$ and $j, k, l \in [0, 1)$ with $j + k + l \leq h < 1$, where h is a fixed number,*

(2.1.2) $T(X \times X \times X) \subseteq f(X)$ *and $f(X)$ is a complete subspace of X.*

Then the maps T and f have a tripled coincidence point.

Further, T and f have a tripled common fixed point if one of the following conditions holds.

(2.1.3) *(a) $\{T, f\}$ is w-compatible, there exist $u, v, w \in X$ such that $\lim_{n\to\infty} f^n x = u$, $\lim_{n\to\infty} f^n y = v$ and $\lim_{n\to\infty} f^n z = w$, whenever (x, y, z) is a tripled coincidence point of $\{T, f\}$ and f is continuous at u, v, w.*

(b) There exist $u, v, w \in X$ such that $\lim_{n\to\infty} f^n u = x$, $\lim_{n\to\infty} f^n v = y$ and $\lim_{n\to\infty} f^n w = z$ whenever (x, y, z) is a tripled coincidence point of $\{T, f\}$ and f is continuous at x, y, and z.

Proof. Let $x_0, y_0, z_0 \in X$. From (2.1.2), there exist sequences $\{x_n\}$, $\{y_n\}$, and $\{z_n\}$ in X such that $fx_{n+1} \in T(x_n, y_n, z_n)$, $fy_{n+1} \in T(y_n, x_n, y_n)$ and $fz_{n+1} \in T(z_n, y_n, x_n)$, $n = 0, 1, 2, 3, \ldots$.

For simplification, denote

$$d_n^x = d(fx_{n-1}, fx_n), \qquad d_n^y = d(fy_{n-1}, fy_n), \qquad d_n^z = d(fz_{n-1}, fz_n). \tag{2.1}$$

From (2.1.1), we obtain

$$
\begin{aligned}
d_2^x &= d(fx_1, fx_2) \\
&\le H(T(x_0, y_0, z_0), T(x_1, y_1, z_1)) + h \\
&\le jd(fx_0, fx_1) + kd(fy_0, fy_1) + ld(fz_0, fz_1) + h \\
&= jd_1^x + kd_1^y + ld_1^z + h,
\end{aligned} \tag{i}
$$

$$
\begin{aligned}
d_2^y &= d(fy_1, fy_2) \\
&\le H(T(y_0, x_0, y_0), T(y_1, x_1, y_1)) + h \\
&\le jd(fy_0, fy_1) + kd(fx_0, fx_1) + ld(fy_0, fy_1) + h \\
&= kd_1^x + (j + l)d_1^y + h,
\end{aligned} \tag{ii}
$$

$$
\begin{aligned}
d_2^z &= d(fz_1, fz_2) \\
&\le H(T(z_0, y_0, x_0), T(z_1, y_1, x_1)) + h \\
&\le jd(fz_0, fz_1) + kd(fy_0, fy_1) + ld(fx_0, fx_1) + h \\
&= ld_1^x + kd_1^y + jd_1^z + h,
\end{aligned} \tag{iii}
$$

$$
\begin{aligned}
d_3^x &= d(fx_2, fx_3) \\
&\le H(T(x_1, y_1, z_1), T(x_2, y_2, z_2)) + h^2 \\
&\le jd(fx_1, fx_2) + kd(fy_1, fy_2) + ld(fz_1, fz_2) + h^2 \\
&= jd_2^x + kd_2^y + ld_2^z + h^2 \\
&\le j\left(jd_1^x + kd_1^y + ld_1^z + h\right) + k\left(kd_1^x + (j + l)d_1^y + h\right) \\
&\quad + l\left(ld_1^x + kd_1^y + jd_1^z + h\right) + h^2 \\
&= \left(j^2 + k^2 + l^2\right)d_1^x + (2jk + 2lk)d_1^y + (2jl)d_1^z + h^2 + (j + k + l)h \\
&= \left(j^2 + k^2 + l^2\right)d_1^x + (2jk + 2lk)d_1^y + (2jl)d_1^z + 2h^2,
\end{aligned} \tag{iv}
$$

$$
\begin{aligned}
d_3^y &= d(fy_2, fy_3) \\
&\le H(T(y_1, x_1, y_1), T(y_2, x_2, y_2)) + h^2 \\
&\le jd(fy_1, fy_2) + kd(fx_1, fx_2) + ld(fy_1, fy_2) + h^2 \\
&= kd_2^x + (j + l)d_2^y + h^2 \\
&\le k\left(jd_1^x + kd_1^y + ld_1^z + h\right) + (j + l)\left(kd_1^x + (j + l)d_1^y + h\right) + h^2 \\
&= (2jk + lk)d_1^x + \left[(j + l)^2 + k^2\right]d_1^y + kld_1^z + (j + k + l)h + h^2 \\
&\le (2jk + lk)d_1^x + \left[(j + l)^2 + k^2\right]d_1^y + kld_1^z + 2h^2,
\end{aligned} \tag{v}
$$

$$\begin{aligned}
d_3^z &= d(fz_2, fz_3)\\
&\leq H(T(z_1,y_1,x_1),T(z_2,y_2,x_2)) + h^2\\
&\leq jd(fz_1,fz_2) + kd(fy_1,fy_2) + ld(fx_1,fx_2) + h^2\\
&= jd_2^z + kd_2^y + ld_2^x + h^2 = ld_2^x + kd_2^y + jd_2^z + h^2\\
&\leq l\left(jd_1^x + kd_1^y + ld_1^z + h\right) + k\left(kd_1^x + (j+l)d_1^y + h\right)\\
&\quad + j\left(ld_1^x + kd_1^y + jd_1^z + h\right) + h^2\\
&= \left(2jl + k^2\right)d_1^x + 2[jk + lk]d_1^y + \left(j^2 + l^2\right)d_1^z + (j+k+l)h + h^2\\
&\leq \left(2jl + k^2\right)d_1^x + 2[jk + lk]d_1^y + \left(j^2 + l^2\right)d_1^z + 2h^2.
\end{aligned} \tag{vi}$$

Let $A = \begin{bmatrix} j & k & l \\ k & j+l & 0 \\ l & k & j \end{bmatrix}$ denoted by $\begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & e_1 & f_1 \\ g_1 & b_1 & h_1 \end{bmatrix}$.

Clearly, $a_1 + b_1 + c_1 = d_1 + e_1 + f_1 = g_1 + b_1 + h_1 = (j+k+l) \leq h < 1$.

Then,

$$A^2 = \begin{bmatrix} j^2+k^2+l^2 & 2jk+2lk & 2jl \\ 2jk+lk & (j+l)^2+k^2 & kl \\ 2jl+k^2 & 2jk+2lk & j^2+l^2 \end{bmatrix} \text{ denote } A^2 \text{ by } \begin{bmatrix} a_2 & b_2 & c_2 \\ d_2 & e_2 & f_2 \\ g_2 & b_2 & h_2 \end{bmatrix}. \tag{2.2}$$

It is clear that $a_2 + b_2 + c_2 = d_2 + e_2 + f_2 = g_2 + b_2 + h_2 = (j+k+l)^2 \leq h^2 < 1$.

Now we prove by induction that

$$A^n = \begin{bmatrix} a_n & b_n & c_n \\ d_n & e_n & f_n \\ g_n & b_n & h_n \end{bmatrix}, \tag{2.3}$$

where

$$a_n + b_n + c_n = d_n + e_n + f_n = g_n + b_n + h_n = \left(j+k+l\right)^n \leq h^n < 1. \tag{2.4}$$

Equation (2.3) is true for $n = 1, 2$.

Assume that (2.3) is true for some n. Consider

$$\begin{aligned}
A^{n+1} = A^n \cdot A &= \begin{bmatrix} a_n & b_n & c_n \\ d_n & e_n & f_n \\ g_n & b_n & h_n \end{bmatrix} \begin{bmatrix} j & k & l \\ k & j+l & 0 \\ l & k & j \end{bmatrix}\\
&= \begin{bmatrix} ja_n + kb_n + lc_n & ka_n + (j+l)b_n + kc_n & la_n + jc_n \\ jd_n + ke_n + lf_n & kd_n + (j+l)e_n + kf_n & ld_n + jf_n \\ jg_n + kb_n + lh_n & kg_n + (j+l)b_n + kh_n & lg_n + jh_n \end{bmatrix}.
\end{aligned} \tag{2.5}$$

We have

$$a_{n+1}+b_{n+1}+c_{n+1}=(j+k+l)(a_n+b_n+c_n)=(j+k+l)^{n+1}\leq h^{n+1}<1. \tag{2.6}$$

Similarly, we have

$$d_{n+1}+e_{n+1}+f_{n+1}=g_{n+1}+b_{n+1}+h_{n+1}=(j+k+l)^{n+1}\leq h^{n+1}<1. \tag{2.7}$$

Thus (2.3) is true for all $+^{\text{ve}}$ integer values of n.

Now from (i)–(vi) and continuing this process, we get

$$\begin{bmatrix} d^x_{n+1} \\ d^y_{n+1} \\ d^z_{n+1} \end{bmatrix} \leq \begin{bmatrix} a_n & b_n & c_n \\ d_n & e_n & f_n \\ g_n & b_n & h_n \end{bmatrix} \begin{bmatrix} d^x_1 \\ d^y_1 \\ d^z_1 \end{bmatrix} + \begin{bmatrix} nh^n \\ nh^n \\ nh^n \end{bmatrix}, \tag{2.8}$$

for all $n=1,2,3,\ldots$. That is,

$$\begin{aligned} d^x_{n+1} &\leq a_n d^x_1 + b_n d^y_1 + c_n d^z_1 + nh^n, \\ d^y_{n+1} &\leq d_n d^x_1 + e_n d^y_1 + f_n d^z_1 + nh^n, \\ d^z_{n+1} &\leq g_n d^x_1 + b_n d^y_1 + h_n d^z_1 + nh^n, \\ &\qquad\qquad \forall n=1,2,3,\ldots. \end{aligned} \tag{2.9}$$

For $m>n$, we have

$$\begin{aligned} d(fx_m, fx_n) &\leq d(fx_m, fx_{m-1}) + d(fx_{m-1}, fx_{m-2}) \\ &\quad + \cdots + d(fx_{n+2}, fx_{n+1}) + d(fx_{n+1}, fx_n) \\ &= d^x_m + d^x_{m-1} + \cdots + d^x_{n+2} + d^x_{n+1} \\ &\leq a_{m-1}d^x_1 + b_{m-1}d^y_1 + c_{m-1}d^z_1 + (m-1)h^{m-1} \\ &\quad + a_{m-2}d^x_1 + b_{m-2}d^y_1 + c_{m-2}d^z_1 + (m-2)h^{m-2} \\ &\quad + \cdots + a_{n+1}d^x_1 + b_{n+1}d^y_1 + c_{n+1}d^z_1 + (n+1)h^{n+1} \\ &\quad + a_n d^x_1 + b_n d^y_1 + c_n d^z_1 + nh^n \end{aligned}$$

$$\begin{aligned}
&\leq (a_{m-1} + a_{m-2} + \cdots + a_{n+1} + a_n)d_1^x \\
&\quad + (b_{m-1} + b_{m-2} + \cdots + b_{n+1} + b_n)d_1^y \\
&\quad + (c_{m-1} + c_{m-2} + \cdots + c_{n+1} + c_n)d_1^z \\
&\quad + \left[(m-1)h^{m-1} + (m-2)h^{m-2} + \cdots + (n+1)h^{n+1} + nh^n\right] \\
&\leq \left(h^{m-1} + h^{m-2} + \cdots + h^{n+1} + h^n\right)\left(d_1^x + d_1^y + d_1^z\right) + \sum_{j=n}^{m-1} jh^j \\
&\leq \frac{h^n}{1-h}\left(d_1^x + d_1^y + d_1^z\right) + \sum_{j=n}^{m-1} jh^j \longrightarrow 0 \text{ as } n \longrightarrow \infty, \\
&\quad \text{because } 0 \leq h < 1.
\end{aligned} \tag{2.10}$$

Hence $\{fx_n\}$ is a Cauchy. Similarly, we can show that $\{fy_n\}$ and $\{fz_n\}$ are Cauchy.

Suppose $f(X)$ is complete, the sequences $\{fx_n\}$, $\{fy_n\}$, and $\{fz_n\}$ are convergent to some α, β, γ in $f(X)$, respectively. There exist $x, y, z \in X$ such that $\alpha = fx$, $\beta = fy$, and $\gamma = fz$.

Now, we have

$$\begin{aligned}
d(T(x,y,z),\alpha) &\leq d(T(x,y,z), fx_{n+1}) + d(fx_{n+1},\alpha) \\
&\leq H(T(x,y,z), T(x_n,y_n,z_n)) + d(fx_{n+1},\alpha) \\
&\leq jd(fx, fx_n) + kd(fy, fy_n) + ld(fz, fz_n) + d(fx_{n+1},\alpha) \\
&= jd(\alpha, fx_n) + kd(\beta, fy_n) + ld(\gamma, fz_n) + d(fx_{n+1},\alpha).
\end{aligned} \tag{2.11}$$

Letting $n \to \infty$, we get $d(T(x,y,z),\alpha) \leq 0$ so that $\alpha \in T(x,y,z)$. That is, $fx \in T(x,y,z)$. Similarly, we can show that $fy \in T(y,x,y)$ and $fz \in T(z,y,x)$. Thus (x,y,z) is a tripled coincidence point of T and f. Suppose (2.1.3) (a) holds.

Since (x,y,z) is a tripled coincidence point of T and f, there exist $u,v,w \in X$ such that $\lim_{n\to\infty} f^n x = u$, $\lim_{n\to\infty} f^n y = v$ and $\lim_{n\to\infty} f^n z = w$.

Since f is continuous at u, v and w, we have $fu = u$, $fv = v$ and $fw = w$.

Since $fx \in T(x,y,z)$, we have $f^2x \in f(T(x,y,z)) \subseteq T(fx,fy,fz)$.

Since $fy \in T(y,x,y)$, we have $f^2y \in f(T(y,x,y)) \subseteq T(fy,fx,fy)$.

Since $fz \in T(z,y,x)$, we have $f^2z \in f(T(z,y,x)) \subseteq T(fz,fy,fx)$.

Then (fx,fy,fz) is tripled coincidence point of T and f.

Similarly, we can show that (f^nx, f^ny, f^nz) is a tripled coincidence point of T and f.

Also it is clear that

$$\begin{aligned}
&f^n x \in T\left(f^{n-1}x, f^{n-1}y, f^{n-1}z\right), \\
&f^n y \in T\left(f^{n-1}y, f^{n-1}x, f^{n-1}y\right), \\
&f^n z \in T\left(f^{n-1}z, f^{n-1}y, f^{n-1}x\right).
\end{aligned} \tag{2.12}$$

From (2.1.1), we have

$$\begin{aligned} d(fu, T(u,v,w)) &\le d(fu, f^n x) + d(f^n x, T(u,v,w)) \\ &\le d(fu, f^n x) + H\Big(T\Big(f^{n-1}x, f^{n-1}y, f^{n-1}z\Big), T(u,v,w)\Big) \\ &\le d(fu, f^n x) + jd(f^n x, fu) + kd(f^n y, fv) + ld(f^n z, fw). \end{aligned} \tag{2.13}$$

Letting $n \to \infty$, we obtain

$$d(fu, T(u,v,w)) \le 0, \tag{2.14}$$

which implies that

$$fu \in T(u,v,w). \tag{2.15}$$

Thus $u = fu \in T(u,v,w)$. Similarly, we can show that $v = fv \in T(v,u,v)$ and $w = fw \in T(w,v,u)$. Thus (u,v,w) is a tripled common fixed point of T and f. Suppose (2.1.3) (b) holds.

Since (x,y,z) is a tripled coincidence point of $\{T,f\}$, there exist $u,v,w \in X$ such that $\lim_{n\to\infty} f^n u = x$, $\lim_{n\to\infty} f^n v = y$ and $\lim_{n\to\infty} f^n w = z$.

Since f is continuous at x, y and z, we have $fx = x$, $fy = y$ and $fz = z$. Thus $x = fx \in T(x,y,z)$, $y = fy \in T(y,x,y)$ and $z = fz \in T(z,y,x)$. Hence (x,y,z) is a tripled common fixed point of $\{T,f\}$.

The following example illustrates Theorem 2.1. □

Example 2.2. Let $X = [0,1]$, $T : X \times X \times X \to CB(X)$ and $f : X \to X$ defined as $T(x,y,z) = [0, (1/8)\sin x + (1/4)\sin y + (1/3)\sin z]$ and $fx = (7/8)x$. Then

$$\begin{aligned} H(T(x,y,z), T(u,v,w)) &= \left| \left(\frac{1}{8}\sin x + \frac{1}{4}\sin y + \frac{1}{3}\sin z \right) \right. \\ &\qquad \left. - \left(\frac{1}{8}\sin u + \frac{1}{4}\sin v + \frac{1}{3}\sin w \right) \right| \\ &\le \frac{1}{8}|\sin x - \sin u| + \frac{1}{4}|\sin y - \sin v| \\ &\quad + \frac{1}{3}|\sin z - \sin w| \\ &\le \frac{1}{8}|x-u| + \frac{1}{4}|y-v| + \frac{1}{3}|z-w| \\ &= \frac{1}{7}\left|\frac{7}{8}x - \frac{7}{8}u\right| + \frac{2}{7}\left|\frac{7}{8}y - \frac{7}{8}v\right| + \frac{8}{21}\left|\frac{7}{8}z - \frac{7}{8}w\right| \\ &= \frac{1}{7}d(fx, fu) + \frac{2}{7}d(fy, fv) + \frac{8}{21}d(fz, fw). \end{aligned} \tag{2.16}$$

It is clear that all conditions of Theorem 2.1 are satisfied and $(0, 0, 0)$ is the tripled common fixed point of T and f.

The following example shows that T and f have no tripled common fixed point if (2.1.3) (a) or (2.1.3) (b) is not satisfied.

Example 2.3. Let $X = [0, 4]$, $T(x, y, z) = [1.5, 2]$ and $fx = 2-(1/2)x$. Then $(0, 1/2, 1)$ is a tripled coincidence point of T and f. Clearly T and f have no tripled common fixed point.

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