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Research Article

Global Behavior of the Max-Type Difference Equation $x_{n+1} = \max\{1/x_n, A_n/x_{n-1}\}$

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We study global behavior of the following max-type difference equation $x_{n+1} = \max\{1/x_n, A_n/x_{n-1}\}$, $n = 0, 1, \ldots$, where $\{A_n\}_{n=0}^{\infty}$ is a sequence of positive real numbers with $0 \le \inf A_n \le \sup A_n < 1$. The special case when $\{A_n\}_{n=0}^{\infty}$ is a periodic sequence with period k and $A_n \in (0,1)$ for every $n \ge 0$ has been completely investigated by Y. Chen. Here we extend his results to the general case.

1. Introduction

In the recent years, there has been a lot of interest in studying the global behavior of, the so-called, max-type difference equations; see, for example, [1–17] (see also references therein). In [1, 3–5, 7, 8], the second order max-type difference equation

$$x_{n+1} = \max\left\{\frac{1}{x_n}, \frac{A_n}{x_{n-1}}\right\}, \quad n = 0, 1, \ldots \tag{1.1}$$

has been studied for positive coefficients A_n, which are periodic with period k. The case $k = 1$ was studied in [1], the case $k = 2$ was studied in [3], the case $k = 3$ was studied in [4, 8], and the more difficult case $k = 4$ was studied in [7]. Chen [5] found that every positive solution of (1.1) is eventually periodic with period 2 when $\{A_n\}_{n=0}^{\infty}$ is a periodic sequence of positive real numbers with period $k \ge 2$ and $A_n \in (0,1)$ for all $n \ge 0$. These results were also included in the recent monograph [9] along with other related references. In this paper, we study global behavior of (1.1) when $\{A_n\}_{n=0}^{\infty}$ is a sequence of positive real numbers with $0 \le \inf A_n \le \sup A_n < 1$.

2. Main Results

The main results of this paper are established through the following lemmas.

Lemma 2.1. *Let* $\{x_n\}_{n=-1}^{\infty}$ *be a positive solution of* (1.1), *then*

(1) $x_{n+1}x_n \geq 1$ *for all* $n \geq 0$;

(2) *if* $x_{k+1}x_k > 1$ *for some* $k \geq 1$, *then* $x_{k+2}x_{k+1} = 1$.

Proof. (1) is obvious since $x_{n+1} \geq 1/x_n$ for all $n \geq 0$.

(2) If $x_{k+1}x_k > 1$ for some $k \geq 1$, then $x_{k+1}x_{k-1} = A_k$. Suppose for the sake of contradiction that $x_{k+2}x_{k+1} > 1$, then similarly we get $x_{k+2}x_k = A_{k+1}$ and

$$A_{k+1}A_k = x_{k+1}x_{k-1}x_{k+2}x_k \geq 1. \tag{2.1}$$

This is a contradiction since $A_{k+1} < 1$ and $A_k < 1$. The proof is complete. □

Lemma 2.2. *Let* $\{x_n\}_{n=-1}^{\infty}$ *be a positive solution of* (1.1) *and* $P_n = \max\{x_n, x_{n-1}\}$ *for all* $n \geq 1$. *Then*

(1) $x_{n+1} \leq P_n$ *and* P_n *is nonincreasing;*

(2) x_n *is bounded, and moreover* $1/P_1 \leq x_n \leq P_1$ *for any* $n \geq 1$.

Proof. By Lemma 2.1(1) and the assumption $A_n < 1$, we obtain that for any $n \geq 1$,

$$x_{n+1} = \max\left\{\frac{x_{n-1}}{x_n x_{n-1}}, \frac{A_n x_n}{x_n x_{n-1}}\right\} \leq \max\{x_{n-1}, x_n\} = P_n. \tag{2.2}$$

Hence

$$P_{n+1} = \max\{x_{n+1}, x_n\} \leq P_n, \tag{2.3}$$

which implies that for all $n \geq 1$,

$$x_n \leq P_1. \tag{2.4}$$

Furthermore, it follows that for all $n \geq 1$,

$$x_{n+1} = \max\left\{\frac{1}{x_n}, \frac{A_n}{x_{n-1}}\right\} \geq \frac{1}{x_n} \geq \frac{1}{P_1}. \tag{2.5}$$

The proof is complete. □

Remark 2.3. Note that from the proof of Lemma 2.2 we have that $P_1 \geq 1$.

Remark 2.4. Various sequences which satisfy inequality in Lemma 2.2(1), that is, $x_{n+1} \leq P_n$ have been studied, for example, in [18–24].

Lemma 2.5. *Let* $\{x_n\}_{n=-1}^{\infty}$ *be a positive solution of* (1.1) *and* $\lim_{n\to\infty} P_n = S$. *Then* $S = \limsup_{n\to\infty} x_n$.

Proof. Since P_n is a subsequence of x_n, it follows that

$$S \le \limsup_{n\to\infty} x_n. \tag{2.6}$$

On the other hand, by $x_{n+1} \le P_n$ for all $n \ge 1$, we obtain

$$\limsup_{n\to\infty} x_n \le \limsup_{n\to\infty} P_n = S. \tag{2.7}$$

The proof is complete. □

Remark 2.6. Let $\{x_n\}_{n=-1}^{\infty}$ be a positive solution of (1.1). By Lemma 2.2, we see that if $S = \limsup_{n\to\infty} x_n$ and $x_N < S$ for some $N > 0$, then $x_{N-1}, x_{N+1} \in [S, +\infty)$. For example, if it were $x_{N-1} < S$, then it would be $P_N < S$, which would imply $\limsup_{n\to\infty} x_n < S$.

Lemma 2.7. *Suppose that* $\{x_n\}_{n=-1}^{\infty}$ *is a positive solution of* (1.1) *and* $S = \limsup_{n\to\infty} x_n$. *Write*

$$\omega(x_n) = \left\{ x : \textit{there exist } -1 \le k_1 < k_2 < \cdots < k_n < \cdots \textit{ such that } \lim_{n\to\infty} x_{k_n} = x \right\}. \tag{2.8}$$

Then $\omega(x_n) = \{S, 1/S\}$.

Proof. If $\omega(x_n)$ contains only one point, we may assume by taking a subsequence that $A_{n_k} \to \mu (< 1)$. By taking the limit in the following relationship:

$$x_{n_k+1} = \max\left\{ \frac{1}{x_{n_k}}, \frac{A_{n_k}}{x_{n_k-1}} \right\}, \tag{2.9}$$

as $k \to \infty$, we obtain

$$S = \max\left\{ \frac{1}{S}, \frac{\mu}{S} \right\} = \frac{1}{S}, \tag{2.10}$$

which implies that $S = 1$.

If $\omega(x_n)$ contains at least two points, let $L \in \omega(x_n) - \{S\}$, then there exists a subsequence x_{n_k} of x_n such that

$$x_{n_k} \longrightarrow L < S. \tag{2.11}$$

By Remark 2.6, we see that there exists $N > 0$ such that for every $n_k > N$,

$$x_{n_k} < S, \qquad x_{n_k+1}, x_{n_k-1} \in [S, +\infty), \tag{2.12}$$

from which it follows that

$$x_{n_k+1} \longrightarrow S, \qquad x_{n_k-1} \longrightarrow S. \tag{2.13}$$

By taking a subsequence we may assume that $A_{n_k} \to \mu(< 1)$. By taking the limit in the following relationship:

$$x_{n_k+1} = \max\left\{\frac{1}{x_{n_k}}, \frac{A_{n_k}}{x_{n_k-1}}\right\}, \tag{2.14}$$

as $k \to \infty$, we obtain

$$S = \max\left\{\frac{1}{L}, \frac{\mu}{S}\right\} = \frac{1}{L}, \tag{2.15}$$

which implies

$$L = \frac{1}{S}. \tag{2.16}$$

The proof is complete. □

Theorem 2.8. *Let $\{x_n\}_{n=-1}^{\infty}$ be a positive solution of* (1.1) *and $S = \limsup_{n\to\infty} x_n$. Then one of the following two statements is true.*

(1) *If there exist infinitely many n such that $x_n \geq S$ and $x_{n+1} \geq S$, then $\{x_n\}_{n=-1}^{\infty}$ is eventually equal to 1.*

(2) *If there exists N such that $x_{N+2k} < S$ and $x_{N+2k-1} \geq S$ for all $k \geq 0$, then $x_{N+2k} \to 1/S$ and $x_{N+2k-1} \to S$.*

Proof. (1) We assume that there exists an infinite sequence $n_1 < n_2 < n_3 < \cdots < n_k < \cdots$ such that

$$x_{n_k} \geq S, \qquad x_{n_k+1} \geq S. \tag{2.17}$$

By taking a subsequence we may assume from Lemma 2.7 that

$$A_{n_k} \longrightarrow \mu < 1, \qquad x_{n_k-1} \longrightarrow l \in \left\{S, \frac{1}{S}\right\}. \tag{2.18}$$

By taking the limit in the following relationship:

$$x_{n_k+1}x_{n_k} = \max\left\{1, \frac{A_{n_k}x_{n_k}}{x_{n_k-1}}\right\}, \tag{2.19}$$

as $k \to \infty$, we get

$$S^2 = \max\left\{1, \frac{S\mu}{l}\right\}. \tag{2.20}$$

Since $S\mu/l \in \{\mu, \mu S^2\}$ and $\mu < 1$, it follows that $S^2 = 1$ and $\omega(x_n) = \{1\}$.

In the following, we show that $\{x_n\}_{n=-1}^{\infty}$ is eventually equal to 1. It only needs to prove that there exists $N \geq 0$ such that for all $n \geq N$,

$$\frac{1}{x_n} > \frac{A_n}{x_{n-1}}. \tag{2.21}$$

Indeed, if there exist infinitely many n_k such that

$$x_{n_k+1} = \frac{A_{n_k}}{x_{n_k-1}}, \tag{2.22}$$

by taking a subsequence we may assume that $A_{n_k} \to \mu < 1$, then it follows that

$$1 = \frac{\mu}{1}, \qquad \mu = 1, \tag{2.23}$$

which is a contradiction. Therefore there exists N such that for all $n \geq N$,

$$x_{n+1} = \frac{1}{x_n}. \tag{2.24}$$

Thus

$$\begin{aligned} x_n &= x_N, \quad \text{for } n = N + 2k, \\ x_n &= x_{N+1}, \quad \text{for } n = N + 2k + 1. \end{aligned} \tag{2.25}$$

Since $x_n \to 1$, we have $x_{N+1} = x_N = 1$.

(2) If $S = 1$, then the result follows from Lemma 2.7. In the following, we assume $S \neq 1$. Suppose for the sake of contradiction that there exists a subsequence x_{N+2k_i} of x_{N+2k} such that

$$x_{N+2k_i} \longrightarrow S. \tag{2.26}$$

By taking a subsequence we may assume that

$$A_{N+2k_i} \longrightarrow \mu. \tag{2.27}$$

By taking the limit in the following relationship:

$$x_{N+2k_i+1} = \max\left\{ \frac{1}{x_{N+2k_i}}, \frac{A_{N+2k_i}}{x_{N+2k_i-1}} \right\}, \tag{2.28}$$

as $k_i \to \infty$, we get

$$S = \max\left\{ \frac{1}{S}, \frac{\mu}{S} \right\}, \tag{2.29}$$

which implies

$$S = 1. \tag{2.30}$$

This is a contradiction. The proof is complete. □

Corollary 2.9. *Let $\{A_n\}_{n=0}^{\infty}$ be a periodic sequence of positive real numbers, then every positive solution of* (1.1) *is eventually periodic with period 2.*

Proof. Let $\{x_n\}_{n=-1}^{\infty}$ be a positive solution of (1.1) and $S = \limsup_{n\to\infty} x_n$. By Remark 2.6 and Theorem 2.8, we may assume without loss of generality that $x_{2k} < S$, $x_{2k-1} \geq S \geq 1$ for all $k \geq 0$. Suppose for the sake of contradiction that there exists a sequence $m_1 < m_2 < \cdots < m_k < \cdots$ such that

(1) $x_{m_k+1}x_{m_k-1} = A_{m_k}$, and $x_{m_k+1}x_{m_k} > 1$;

(2) $x_{n+1}x_n = 1$, for $n \neq m_k$.

Then m_k is odd for every $k \geq 1$. Let $m_k = 2n_k + 1$, then it follows from Lemma 2.1 that

$$x_{2n_k+2}x_{2n_k} = A_{2n_k+1} < 1 = x_{2n_k+1}x_{2n_k} < x_{2n_k+1}x_{2n_k+2}. \tag{2.31}$$

From this and by (2) it follows that

$$\frac{A_{2n_k+1}}{x_{2n_k+2}} = x_{2n_k} < x_{2n_k+2} = x_{2n_k+4} = \cdots = x_{2n_{k+1}} < x_{2n_{k+1}+2} = \frac{A_{2n_{k+1}+1}}{x_{2n_{k+1}}}. \tag{2.32}$$

Therefore for every $k \geq 1$,

$$A_{2n_k+1} < x_{2n_k+2}^2 = x_{2n_{k+1}}^2 < A_{2n_{k+1}+1}, \tag{2.33}$$

which is a contradiction since $\{A_n\}_{n=0}^{\infty}$ is a periodic sequence. The proof is complete. □

Remark 2.10. Corollary 2.9 is the main result of [5].

3. Example

In this section, we give an example for $\{A_n\}_{n=0}^{\infty}$ to be no periodic sequence.

Example 3.1. Consider

$$x_{n+1} = \max\left\{\frac{1}{x_n}, \frac{A_n}{x_{n-1}}\right\}, \quad n = 0, 1, \ldots, \tag{3.1}$$

where $A_{2n} = A_{2n+1} = (2 - 1/2^n)(2 - 1/2^{n+1})/16$ for any $n \geq 0$. Then solution $\{x_n\}_{n=-1}^{\infty}$ of (3.1) with the initial values $x_{-1} = 1/4$ and $x_0 = 4$ satisfies the following.

(1) $x_{2p-1}x_{2p} = 1$, for any $p \geq 0$.

(2) $x_{2p-1} < x_{2p+1} = \dfrac{A_{2p}}{x_{2p-1}} < \dfrac{1}{2} < 2 < x_{2p+2} < x_{2p}$, for any $p \geq 0$.

Proof. By simple computation, we have

$$A_{2p} = \frac{(2 - 1/2^p)(2 - 1/2^{p+1})}{16} > \begin{cases} x_{-1}^2, & \text{if } p = 0, \\ \left(\dfrac{A_0}{x_{-1}}\right)^2, & \text{if } p = 1, \\ \left(\dfrac{A_{2p-2}A_{2p-6}\cdots A_2}{A_{2p-4}A_{2p-8}\cdots A_0}x_{-1}\right)^2, & \text{if } p \geq 2 \text{ is even}, \\ \left(\dfrac{A_{2p-2}A_{2p-6}\cdots A_4A_0}{A_{2p-4}A_{2p-8}\cdots A_2x_{-1}}\right)^2, & \text{if } p \geq 2 \text{ is odd}. \end{cases} \tag{3.2}$$

It follows from (3.1) and (3.2) that

$$\begin{aligned}
x_1x_{-1} &= \max\left\{\frac{x_{-1}}{x_0}, A_0\right\} = \max\left\{x_{-1}^2, A_0\right\} = A_0, \\
x_2x_1 &= \max\left\{1, \frac{x_1A_1}{x_0}\right\} = \max\left\{1, \frac{A_0A_1}{x_{-1}x_0}\right\} = 1, \\
x_3x_1 &= \max\left\{\frac{x_1}{x_2}, A_2\right\} = \max\left\{\frac{x_1^2}{x_2x_1}, A_2\right\} = \max\left\{\left(\frac{A_0}{x_{-1}}\right)^2, A_2\right\} = A_2, \\
x_4x_3 &= \max\left\{1, \frac{x_3A_3}{x_2}\right\} = \max\left\{1, \frac{A_2A_3}{x_2x_1}\right\} = 1, \\
x_5x_3 &= \max\left\{\frac{x_3}{x_4}, A_4\right\} = \max\left\{\frac{x_3^2}{x_4x_3}, A_4\right\} = \max\left\{\left(\frac{x_3x_1}{x_1x_{-1}}x_{-1}\right)^2, A_4\right\} \\
&= \max\left\{\left(\frac{A_2}{A_0}x_{-1}\right)^2, A_4\right\} = A_4,
\end{aligned}$$

$$
\begin{aligned}
x_6x_5 &= \max\left\{1, \frac{x_5A_5}{x_4}\right\} = \max\left\{1, \frac{A_4A_5}{x_4x_3}\right\} = 1, \\
x_7x_5 &= \max\left\{\frac{x_5}{x_6}, A_6\right\} = \max\left\{\frac{x_5^2}{x_6x_5}, A_6\right\} = \max\left\{\left(\frac{x_5x_3x_1x_{-1}}{x_3x_1x_{-1}}\right)^2, A_6\right\} \\
&= \max\left\{\left(\frac{A_4A_0}{A_2x_{-1}}\right)^2, A_6\right\} = A_6, \\
x_8x_7 &= \max\left\{1, \frac{x_7A_7}{x_6}\right\} = \max\left\{1, \frac{A_6A_7}{x_6x_5}\right\} = 1.
\end{aligned}
\tag{3.3}
$$

By induction, we have from (3.1) and (3.2) that for any $p \geq 1$,

$$
\begin{aligned}
x_{4p+1}x_{4p-1} &= \max\left\{\frac{x_{4p-1}}{x_{4p}}, A_{4p}\right\} = \max\left\{\frac{x_{4p-1}^2}{x_{4p}x_{4p-1}}, A_{4p}\right\} = \max\left\{x_{4p-1}^2, A_{4p}\right\} \\
&= \max\left\{\left(\frac{x_{4p-1}x_{4p-3}x_{4p-5}\cdots x_1}{x_{4p-3}x_{4p-5}x_{4p-7}\cdots x_{-1}}x_{-1}\right)^2, A_{4p}\right\} \\
&= \max\left\{\left(\frac{A_{4p-2}A_{4p-6}\cdots A_2}{A_{4p-4}A_{4p-8}\cdots A_0}x_{-1}\right)^2, A_{4p}\right\} = A_{4p}, \\
x_{4p+2}x_{4p+1} &= \max\left\{1, \frac{x_{4p+1}A_{4p+1}}{x_{4p}}\right\} = \max\left\{1, \frac{A_{4p}A_{4p+1}}{x_{4p}x_{4p-1}}\right\} = 1, \\
x_{4p+3}x_{4p+1} &= \max\left\{\frac{x_{4p+1}}{x_{4p+2}}, A_{4p+2}\right\} = \max\left\{\frac{x_{4p+1}^2}{x_{4p+2}x_{4p+1}}, A_{4p+2}\right\} = \max\left\{x_{4p+1}^2, A_{4p+2}\right\} \\
&= \max\left\{\left(\frac{x_{4p+1}x_{4p-1}x_{4p-3}x_{4p-5}\cdots x_1x_{-1}}{x_{4p-1}x_{4p-3}x_{4p-5}x_{4p-7}\cdots x_1x_{-1}}\right)^2, A_{4p+2}\right\} \\
&= \max\left\{\left(\frac{A_{4p}A_{4p-4}\cdots A_4A_0}{A_{4p-2}A_{4p-6}\cdots A_2x_{-1}}\right)^2, A_{4p+2}\right\} = A_{4p+2}, \\
x_{4p+4}x_{4p+3} &= \max\left\{1, \frac{x_{4p+3}A_{4p+3}}{x_{4p+2}}\right\} = \max\left\{1, \frac{A_{4p+2}A_{4p+3}}{x_{4p+2}x_{4p+1}}\right\} = 1.
\end{aligned}
\tag{3.4}
$$

from which the result follows. The proof is complete. □

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