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ON THE ASYMPTOTIC BEHAVIORS OF THE POSITIVE SOLUTION OF $\Delta_p u + |u|^{q-2}u = 0$

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Abstract. In this paper, the unique positive solution of the nonlinear elliptic equation $\Delta_p u + |u|^{q-2}u = 0$, where $p \neq q$, is described and its behaviors relative to certain limiting conditions on p and q are discussed.

1. Introduction

For $p, q \in (1, \infty)$, with $p \neq q$, we consider the following one dimensional equation

$$(E)^{p,q}_{0,1} \qquad \begin{cases} \Delta_p u + |u|^{q-2}u = 0 \quad \text{on} \quad (0,1) & (1) \\ u(0) = u(1) = 0 & (2) \end{cases}$$

where $\Delta_p u = (|u_x|^{p-2}u_x)_x$.

A function u is said to be a *solution* of $(E)^{p,q}_{0,1}$ if $u \in W^{1,p}_0(0,1)$ and u satisfies (1) in the distribution sense.

Ôtani [8] showed the existence of the unique positive solution of $(E)^{p,q}_{0,1}$ and gave some detailed properties of the solution of $(E)^{p,q}_{0,1}$. Idogawa [5] studied the behavior of the maximum values of the solution of $(E)^{p,q}_{0,1}$ as $p, q \to 1^+$. In this paper, we give an explicit formula for the unique solution $u_{p,q}$ of $(E)^{p,q}_{0,1}$ and study the behaviors of the solution of $(E)^{p,q}_{0,1}$ as $p, q \to \infty$ and $p, q \to 1^+$ relative to some conditions on p and q.

In Ôtani's paper, he proved the following two theorems.

Theorem 1. [8] *Suppose that* $p, q > 1$ *and* $p \neq q$. *If* u *is a solution of* $(E)^{p,q}_{0,1}$, *then* u *satisfies the following*:

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i) $u \in C^{\alpha}([0,1]) \cap C^{<q>}([0,1] \backslash Z(u))$, *where*

$$Z(u) = \{x \in [0,1] \mid u_x(x) = 0\}; \quad \alpha = \min\left\{ < \frac{2-p}{p-1} > +1, < q > \right\}$$

and

$$< r > = \begin{cases} \infty & \text{if } r \text{ is an even integer} \\ \min\{n \mid n \geq r, \ n \text{ nonnegative integer}\} & \text{otherwise.} \end{cases}$$

ii) $\frac{p-1}{p}|u_x(x)|^p + \frac{1}{q}|u_x(x)|^q =$ *constant for all* $x \in [0,1]$

iii) $\lim_{t \to 0^+} u_x(t) = \lim_{t \to 1^-} [-u_x(t)]$

iv) $\|u_x\|^p_{L^p(0,1)} = \|u\|^q_{L^q(0,1)} = \frac{q(p-1)}{pq-q+p}|\lim_{t \to 0^+} u_x(t)|^p$.

Theorem 2. [8] *Suppose that* $p, q > 1$ *and* $p \neq q$. *Then* $(E)^{p,q}_{0,1}$ *has a unique positive solution* $u_{p,q}$. *Furthermore, for the functional* R *defined by*

$$R(v) = \frac{\|v\|_{L^q(0,1)}}{\|v_x\|_{L^p(0,1)}},$$

we have $R(u_{p,q}) = \sup\{R(v) \mid \ v \in W^{1,p}_0(0,1)$ *and* $v \neq 0\}$.

Remark: The solution $u_{p,q}$, in addition, satisfy the following:

i) $u_{p,q}(x) = u_{p,q}(1-x)$ for any $x \in [0,1]$.

ii) $(u_{p,q})_x$ is positive and decreasing on $[0, \frac{1}{2})$ with $(u_{p,q})_x(\frac{1}{2}) = 0$.

iii) $u_{p,q}(\frac{1}{2}) = \max_{x \in [0,1]} u_{p,q}(x)$.

2. Main Results

We first give an explicit formula for $u_{p,q}$. For this, recall that the Beta function $B(k,l)$ for $k, l > 0$ is defined by

$$\begin{aligned} B(k,l) &= \int_0^1 s^{k-1}(1-s)^{l-1} ds \\ &= \frac{1}{k}\int_0^1 (1-t^{\frac{1}{k}})^{l-1} dt \qquad \text{using the substitution } t = s^k. \end{aligned}$$

Let

$$f_{p,q}(s) = \int_0^s (1-t^q)^{\frac{-1}{p}} dt$$

for any $s \in [0, 1]$. It follows from

$$\int_0^s (1 - t^q)^{\frac{-1}{p}} dt \le \int_0^1 (1 - t^q)^{\frac{-1}{p}} dt = q^{-1} B(q^{-1}, 1 - p^{-1}) =: b_{p,q}$$

that $f_{p,q}$ is well-defined on [0,1]. Now since

$$\frac{d}{ds} f_{p,q}(s) = (1 - s^q)^{\frac{-1}{p}} > 0 \quad \text{on } (0, 1);$$

thus $f_{p,q}$ is increasing on $[0, 1]$, and hence $f_{p,q}$ must have an inverse $f_{p,q}^{-1}$ defined on $[0, b_{p,q}]$.

Let

$$w_{p,q}(x) = \left[\frac{q(p-1)}{p}\right]^{\frac{1}{q-p}} f_{p,q}^{-1}(x)$$

for $x \in [0, b_{p,q}]$. Then on $(0, b_{p,q})$, we have

$$
\begin{aligned}
\frac{d}{dx} w_{p,q}(x) &= \left[\frac{q(p-1)}{p}\right]^{\frac{1}{q-p}} \frac{d}{dx} f_{p,q}^{-1}(x) \\
&= \left[\frac{q(p-1)}{p}\right]^{\frac{1}{q-p}} \left[\frac{d}{ds} f_{p,q}(s)\Big|_{s = f_{p,q}^{-1}(x)}\right]^{-1} \qquad (2.1)\\
&= \left[\frac{q(p-1)}{p}\right]^{\frac{1}{q-p}} \left[1 - \left[\left[\frac{q(p-1)}{p}\right]^{\frac{-1}{q-p}} w_{p,q}\right]^q\right]^{\frac{1}{p}};
\end{aligned}
$$

and $(w_{p,q})_x > 0$ on $(0, b_{p,q})$ because $w_{p,q} < [\frac{q(p-1)}{p}]^{\frac{1}{q-p}}$ on $(0, b_{p,q})$.

Observe that

$$\lim_{x \to 0^+} (w_{p,q})_x(x) = \left[\frac{q(p-1)}{p}\right]^{\frac{1}{q-p}} \quad \text{and} \quad \lim_{x \to b_{p,q}^-} (w_{p,q})_x(x) = 0. \qquad (2.2)$$

Since $w_{p,q}$ is continuous on $[0, b_{p,q}]$, we have $w_{p,q} \in L^p(0, b_{p,q})$. It follows from (2.1) that $(w_{p,q})_x \in L^p(0, b_{p,q})$.

From (2.1), we see, that on $(0, b_{p,q})$,

$$(w_{p,q})_x^p + \left[\frac{q(p-1)}{p}\right]^{-1} w_{p,q}^q = \left[\frac{q(p-1)}{p}\right]^{\frac{p}{q-p}}. \qquad (2.3)$$

Differentiating both sides of (2.3), we get

$$p(w_{p,q})_x^{p-1}(w_{p,q})_{xx} + \frac{p}{p-1} w_{p,q}^{q-1}(w_{p,q})_x = 0 \qquad \text{on } (0, b_{p,q}). \tag{2.4}$$

Multiplying both sides of (2.4) by $(p-1)(p(w_{p,q})_x)^{-1}$, we obtain

$$(((w_{p,q})_x)^{p-1})_x + w_{p,q}^{q-1} = 0 \qquad \text{on } (0, b_{p,q}).$$

Set

$$v_{p,q}(x) = \begin{cases} w_{p,q}(x) & \text{if } x \in [0, b_{p,q}] \\ w_{p,q}(2b_{p,q} - x) & \text{if } x \in [b_{p,q}, 2b_{p,q}]. \end{cases}$$

Then $v_{p,q} \in W^{1,p}(0, 2b_{p,q})$. Since $v_{p,q}(0) = v_{p,q}(2b_{p,q}) = 0$, we have $v_{p,q} \in W_0^{1,p}(0, 2b_{p,q})$. Thus, $v_{p,q}$ is the unique positive solution of $(E)_{0,2b_{p,q}}^{p,q}$ and consequently,

$$u_{p,q}(x) = [2b_{p,q}]^{\frac{p}{q-p}} v_{p,q}(2b_{p,q}x) \qquad x \in [0, 1] \tag{2.5}$$

is in $W_0^{1,p}(0, 1)$ and is the unique solution of $(E)_{0,1}^{p,q}$.

We therefore obtain the following theorem.

Theorem 3. *If* $u_{p,q}$ *is the unique positive solution of* $(E)_{0,1}^{p,q}$*, then for any* $x \in [0, \frac{1}{2}]$*, we have*

$$u_{p,q}(x) = (2q^{-1}B(q^{-1}, 1-p^{-1}))^{\frac{p}{q-p}} \left(\frac{q(p-1)}{p}\right)^{\frac{1}{q-p}} f_{p,q}^{-1}(2xq^{-1}B(q^{-1}, 1-p^{-1})),$$

where B *is the Beta function and* $f_{p,q}(s) = \int_0^s (1-t^q)^{-\frac{1}{p}}dt$ for $s \in [0, 1]$.

Corollary 4. *The best possible constant for the Sobolev-Poincare type inequality*

$$\|v\|_{L^q(0,1)} \leq C\|v_x\|_{L^p(0,1)} \quad \text{for all } v \in W_0^{1,p}(0, 1)$$

is given by

$$C_{p,q} = \frac{p^{\frac{1}{q}} q^{1-\frac{1}{p}} (pq - q + p)^{\frac{1}{p}-\frac{1}{q}}}{2(p-1)^{\frac{1}{p}} B(q^{-1}, 1-p^{-1})}.$$

Proof. For each $x \in [0, \frac{1}{2}]$, we have from (2.5)

$$u_{p,q}(x) = [2b_{p,q}]^{\frac{p}{q-p}} w_{p,q}(2b_{p,q}x) \tag{2.6}$$

and on $(0, \frac{1}{2})$, we have

$$(u_{p,q})_x(x) = [2b_{p,q}]^{\frac{q}{q-p}}(w_{p,q})_x(2b_{p,q}x).$$

From (2.2), we obtain

$$\lim_{x \to 0^+}(u_{p,q})_x(x) = [2b_{p,q}]^{\frac{q}{q-p}}\left[\frac{q(p-1)}{p}\right]^{\frac{1}{q-p}}. \tag{2.7}$$

Thus, using Theorem 1(iv), Theorem 2 and (2.7), we get

$$\begin{aligned} R(u_{p,q}) &= \frac{\|u_{p,q}\|_{L^q(0,1)}}{\|(u_{p,q})_x\|_{L^p(0,1)}} = \|u_{p,q}\|_{L^q(0,1)}^{1-\frac{q}{p}} \\ &= \left[\frac{q(p-1)}{pq-q+p}\left[(2b_{p,q})^{\frac{q}{q-p}}\left(\frac{q(p-1)}{p}\right)^{\frac{1}{q-p}}\right]^p\right]^{\frac{p-q}{pq}} \end{aligned}$$

which completes the proof of Corollary 4. ∎

The behaviors of the unique positive solution $u_{p,q}$ of $(E)_{0,1}^{p,q}$ as $p \to \infty$ and $q \to \infty$ are given in the following theorem.

Theorem 5. (i) *Suppose* $\alpha(p)$ *is a function defined on* $(1,\infty)$ *with* $1 < p\alpha(p) \neq p$ *and* $\lim_{p\to\infty}\alpha(p) = a$. *If* $q = p\alpha(p)$, *then*

$$\lim_{p\to\infty} u_{p,q}(x) = 2^{\frac{a}{a-1}}\left(\frac{1}{2} - \left|x - \frac{1}{2}\right|\right) \qquad \text{for all} \quad x \in [0,1].$$

(ii) *Let* $\alpha(q)$ *be a function defined on* $(1,\infty)$ *such that* $1 < q\alpha(q) \neq q$ *for all* $q \in (1,\infty)$ *and* $\lim_{q\to\infty}\alpha(q) = b$. *If* $p = q\alpha(q)$, *then*

$$\lim_{q\to\infty} u_{p,q}(x) = 2^{\frac{a}{1-b}}\left(\frac{1}{2} - \left|x - \frac{1}{2}\right|\right) \qquad \text{for all} \quad x \in [0,1].$$

Proof. Due to the symmetry of $u_{p,q}$, it suffices to prove the Theorem only on the interval $[0, \frac{1}{2}]$.

To prove (i), let $q = p\alpha(p)$ where the function α is defined on $(1,\infty)$ with $1 < p\alpha(p) \neq p$ and $\lim_{p\to\infty}\alpha(p) = a$. First we recall that

$$\lim_{p\to\infty}(1-t^q)^{\frac{-1}{p}} = \begin{cases} \infty & \text{if} \quad t = 1 \\ 1 & \text{if} \quad t \in [0,1). \end{cases}$$

By the Lebegue's Dominated Convergence Theorem, we have

$$\lim_{p\to\infty} f_{p,q}(s) = \lim_{p\to\infty} \int_0^s (1-t^q)^{\frac{-1}{p}} dt = \int_0^s 1 dt = s \quad \text{for all} \quad s \in [0,1].$$

Thus,

$$\lim_{p\to\infty} f_{p,q}^{-1}(x) = x \qquad \text{for all} \quad x \in [0,1].$$

Since $1 \le (1-t^q)^{\frac{-1}{p}}$ for all $t \in [0,1]$, we have $f_{p,q}(s) \to s^+$ as $p \to \infty$ for all $s \in [0,1]$. In particular $b_{p,q} = f_{p,q}(1) \to 1^+$ as $p \to \infty$.

Now, since $b_{p,q} > 1$ for all $p > 1$ and $f_{p,q}^{-1}$ is increasing on $[0,1]$, we have for all $x \in (0, \frac{1}{2})$

$$f_{p,q}^{-1}(2x) < f_{p,q}^{-1}(2xb_{p,q}). \tag{2.8}$$

Let $\varepsilon > 0$ be such that $2x(1+\varepsilon) < 1$, and let p_ε be such that for all $p > p_\varepsilon$ we have $b_{p,q} < (1+\varepsilon)$. Then

$$f_{p,q}^{-1}(2xb_{p,q}) < f_{p,q}^{-1}(2x(1+\varepsilon)). \tag{2.9}$$

It follows from (2.8) and (2.9) that, for all $p > p_\varepsilon$, we have

$$f_{p,q}^{-1}(2x) < f_{p,q}^{-1}(2xb_{p,q}) < f_{p,q}^{-1}(2x(1+\varepsilon)).$$

Taking the limit as $p \to \infty$, we obtain

$$2x \le \lim_{p\to\infty} f_{p,q}^{-1}(2xb_{p,q}) \le 2x(1+\varepsilon).$$

Hence,

$$\lim_{p\to\infty} f_{p,q}^{-1}(2xb_{p,q}) = 2x.$$

From (2.6), we have

$$u_{p,q}(x) = [2b_{p,q}]^{\frac{1}{\alpha(p)-1}} [(p-1)\alpha(p)]^{\frac{1}{p(\alpha(p)-1)}} f_{p,q}^{-1}(2xb_{p,q})$$

so that if $\lim_{p\to\infty} \alpha(p) = a$, then

$$\lim_{p\to\infty} u_{p,q}(x) = 2^{\frac{1}{a-1}+1} x \qquad \text{for all} \quad x \in \left[0, \frac{1}{2}\right].$$

We thus proved (i).

(ii) can be proved in an analogous manner. ■

The behaviors of the maximum values of the unique positive solution of $(E)_{0,1}^{p,q}$ as $p \to 1^+$ and $q \to 1^+$ are given in the following theorem.

Theorem 6. (Idogawa) [5] *Let* $D_{p,q} = \max_{x\in[0,1]} u_{p,q}(x)$. *Then*

(i) *for fixed* $q > 1$, *we have*

$$\lim_{p \to 1^+} D_{p,q} = 2^{\frac{1}{q-1}}.$$

(ii) *for fixed* $p > 1$, *we have*

$$\lim_{q \to 1^+} D_{p,q} = 2^{\frac{-p}{p-1}} \left(\frac{p-1}{p} \right).$$

In addition, we describe the behaviors of the unique positive solution of $(E)_{0,1}^{p,q}$ as $p \to 1^+$ and $q \to 1^+$ in the following two theorems.

Theorem 7. *For some* $\delta > 1$, *let* $\alpha(p)$ *be a function defined on* $(1, 1+\delta]$ *such that* $p \neq \alpha(p) > 1$ *for all* $p \in (1, 1+\delta]$ *and* $\lim_{p \to 1^+} \alpha(p) = a > 1$. *If* $q = \alpha(p)$, *then*, *for any* $x \in (0, \frac{1}{2}]$

$$\lim_{p \to 1^+} u_{p,q}(x) = 2^{\frac{1}{a-1}}.$$

Proof. From Theorem 3, we have, for any $x \in [0, \frac{1}{2}]$

$$u_{p,q}(x) = 2^{\frac{p}{q-p}} \left(\frac{1}{q} B\left(\frac{1}{q}, 1 - \frac{1}{p} \right) \right)^{\frac{p}{q-p}} \left(\frac{q(p-1)}{p} \right)^{\frac{1}{q-p}} f_{p,q}^{-1} \left(\frac{2x}{q} B\left(\frac{1}{q}, 1 - \frac{1}{p} \right) \right).$$

Let $q = \alpha(p)$ and $\lim_{p \to 1^+} \alpha(p) = a > 1$. For any fixed $s \in [0, 1)$, we have

$$\begin{aligned} \lim_{p \to 1^+} f_{p,q}(s) &= \lim_{p \to 1^+} \int_0^s (1 - t^q)^{-\frac{1}{p}} dt \\ &= \int_0^s (1 - t^a)^{-1} dt < \infty, \end{aligned}$$

which implies that $\lim_{p \to 1^+} f_{p,q}(s) = \infty$ if and only if $s = 1$.

Since, by definition, $B(\frac{1}{q}, 1 - \frac{1}{p}) = \frac{\Gamma(\frac{1}{q})\Gamma(1-\frac{1}{p})}{\Gamma(\frac{1}{q}+1-\frac{1}{p})}$, where $\Gamma(\cdot)$ is the Gamma function, we have

$$\begin{aligned} \lim_{p \to 1^+} B\left(\tfrac{1}{q}, 1 - \tfrac{1}{p} \right) &= \lim_{p \to 1^+} \frac{\Gamma(\frac{1}{q})}{\Gamma(\frac{1}{q} + 1 - \frac{1}{p})} \cdot \lim_{p \to 1^+} \Gamma\left(1 - \frac{1}{p}\right) \\ &= \frac{\Gamma(\frac{1}{a})}{\Gamma(\frac{1}{a})} \cdot \infty = \infty. \end{aligned}$$

Therefore, for any $x \in (0, \frac{1}{2}]$, we have $\lim_{p \to 1^+} \frac{2x}{q} B(\frac{1}{q}, 1 - \frac{1}{p}) = \infty$, and hence,

$$\lim_{p \to 1^+} f_{p,q}^{-1}\left(\frac{2x}{q} B\left(\frac{1}{q}, 1 - \frac{1}{p}\right)\right) = 1.$$

Note that

$$\begin{aligned}&\left[\frac{1}{q} B\left(\frac{1}{q}, 1 - \frac{1}{p}\right)\right]^{\frac{p}{q-p}} \left[\frac{q(p-1)}{p}\right]^{\frac{1}{q-p}} \\ &= \left(\frac{1}{q}\right)^{\frac{p}{q-p}} \left(\frac{q}{p}\right)^{\frac{1}{q-p}} \left(\frac{\Gamma(\frac{1}{q})}{\Gamma(\frac{1}{q} + 1 - \frac{1}{p})}\right)^{\frac{p}{q-p}} \left(\Gamma\left(1 - \frac{1}{p}\right)\right)^{\frac{p}{q-p}} (p-1)^{\frac{1}{q-p}}.\end{aligned}$$

Since $\frac{1}{p}$ is not an integer, we see that

$$\left(\Gamma\left(1 - \frac{1}{p}\right)\right)^{\frac{p}{q-p}} (p-1)^{\frac{1}{q-p}} = \left(\left(\frac{\pi}{(\sin\frac{\pi}{p})\,\Gamma(\frac{1}{p})}\right)^{p} (p-1)\right)^{\frac{1}{q-p}}.$$

From

$$\lim_{p \to 1^+} \frac{p-1}{(\sin\frac{\pi}{p})^{p}} = \frac{1}{\pi},$$

it follows easily that

$$\lim_{p \to 1^+} \left[\frac{1}{q} B\left(\frac{1}{q}, 1 - \frac{1}{p}\right)\right]^{\frac{p}{q-p}} \left[\frac{q(p-1)}{p}\right]^{\frac{1}{q-p}} = 1.$$

Hence, for each $x \in (0, \frac{1}{2}]$, we have $\lim_{p \to 1^+} u_{p,q}(x) = 2^{\frac{1}{a-1}}$. ■

Theorem 8. *For some* $\delta > 1$, *let* $\beta(q)$ *be a function defined on* $(1, 1+\delta]$ *such that* $q \neq \beta(q) > 1$ *for all* $q \in (1, 1+\delta]$ *and* $\lim_{q \to 1^+} \beta(q) = b > 1$. *If* $p = \beta(q)$, *then for any* $x \in (0, \frac{1}{2}]$,

$$\lim_{q \to 1^+} u_{p,q}(x) = 2^{\frac{-b}{b-1}} \left(\frac{b-1}{b}\right) (1 - (1-2x)^{\frac{b}{1-b}}).$$

Proof. Since $p = \beta(q)$ and $\lim_{q \to 1^+} \beta(q) = b > 1$, we have for each $s \in [0, 1]$

$$\begin{aligned}\lim_{q \to 1^+} f_{p,q}(s) &= \int_0^s (1-t)^{\frac{-1}{b}} dt \\ &= \left(\frac{b}{b-1}\right) (1 - (1-s)^{\frac{b-1}{b}}) =: g(s).\end{aligned}$$

Now if $y = g(s)$ where $y \in [0, \frac{b}{b-1}]$, then $g^{-1}(y) = 1 - (1 - (\frac{b-1}{b})s)^{\frac{b}{b-1}}$, and thus

$$\lim_{q \to 1^+} f_{p,q}^{-1}(y) = 1 - \left(1 - \left(\frac{b-1}{b}\right) y\right)^{\frac{b}{b-1}}.$$

Hence, for each $x \in (0, \frac{1}{2}]$, Theorem 3 yields,

$$\begin{aligned} &\lim_{q \to 1^+} u_{p,q}(x) \\ &= \left(2B\left(1, 1 - \frac{1}{b}\right)\right)^{\frac{b}{1-b}} \left(\frac{b-1}{b}\right)^{\frac{1}{1-b}} \left(1 - \left(1 - 2x\left(\frac{b-1}{b}\right) B\left(1, 1 - \frac{1}{b}\right)\right)^{\frac{b}{1-b}}\right) \\ &= 2^{\frac{-b}{b-1}} \left(\frac{b-1}{b}\right) (1 - (1 - 2x)^{\frac{b}{1-b}}). \end{aligned}$$

Thus the theorem is proved. ■

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