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APPLICATION OF HOMOTOPY PERTURBATION METHOD TO REGULARIZATION OF SCALAR IMAGES

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Abstract. The homotopy perturbation method is implemented to solve nonlinear equations. Based on this method, a multi-step scheme is constructed for a kind of Hamilton–Jacobi formulations by assuming the homotopy parameter is a linear function of time. Using this multi-step scheme, a minimal surface regularization equation is solved, which designates a regularization process that doesn't smooth the image with the same weight in all the spatial directions. Some image denoisying examples illustrate its effectiveness and convenience.

1. Introduction

For several years, regularization algorithms have attracted a growing interest in the computer vision community. Isotropic linear regularization is a natural way to smooth and simplify data and has consequently been reached by several mathematical formulations: from the restoration scheme to the classical linear filtering of images, all these methods lead to the same regularization behavior, e.g. the signal is blurred little by little in an isotropic way during the PDE evolution. To overcome the limitations of linear methods leading to isotropic smoothing, several nonlinear extensions of the heat equations are proposed, which can

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be classified as the Hamilton–Jacobi formulation. Comparing with other image processing methods, the PDE based method becomes one of the major tools in computer vision and image processing [4]. In recent years, many excellent algorithms were developed such as TVD schemes, ENO and WENO schemes [14], but almost all of these methods were based on the finite difference method. The usual finite difference method can be executed with a narrow band-width, but have to face the problems of numerical stability and precision [25].

The homotopy perturbation method (HPM) proposed by He [9], [11] is constantly being developed and applied to solve various nonlinear problems by Ji-Huan He [7], [17] and by others [1], [2], [5], [6], [8], [10], [12], [15], [16], [18]–[22]. Unlike other analytical perturbation methods, HPM does not depend on small parameter which is difficult to be found. The variational iteration method was another simple and effective method for nonlinear equations proposed by He [8], [12], which can provide analytical approximations to a rather wide class of nonlinear equations [24], [25], [3] without linearization, perturbation, or discretization which can result in massive numerical computation.

In fact, the Hamilton–Jacobi formulation could be transformed to matrix differential equations derived by the semi-analytical method from the PDEs. In this paper, we try use the coupling technique of He's VIM and HPM to solve the matrix differential equation. The corresponding numerical result could be obtained by the precise integration method (PIM) proposed by Zhong [25]. In contrast to the traditional finite difference approximation, the numerical results of PIM for a set of simultaneous linear time-invariant ODEs have computer precision and also are free from the stiff problem.

2. Fundamental theory of coupling technique of VIM and HPM

Consider the nonlinear matrix differential equation as follows

$$L(\dot{\mathbf{V}}, \mathbf{V}, t) + N(\dot{\mathbf{V}}, \mathbf{V}, t) = \mathbf{G}(t) \tag{2.1}$$

where L is a linear operator, N a nonlinear operator and $\mathbf{G}$ an inhomogeneous term, $\mathbf{V}$ is an n-dimensional unknown vector, the dot stands for the differential with respect to the time t. For convenience, (2.1) can be rewritten as

$$\dot{\mathbf{V}} - \mathbf{H}\mathbf{V} - \mathbf{F}(\dot{\mathbf{V}}, \mathbf{V}, t) = 0 \tag{2.2}$$

Here $\mathbf{H}$ is a given $n \times n$ constant matrix, and $\mathbf{F}(\dot{\mathbf{V}}, \mathbf{V}, t)$ is a n-dimensional nonlinear external force vector. According to the variational iteration method [24], we can write down a correction functional as follows

$$\mathbf{V}_{n+1}(t) = \mathbf{V}_n(t) + \int_0^t \lambda[\dot{\mathbf{V}}_n(\tau) - \mathbf{H}\mathbf{V}_n(\tau) - \mathbf{F}(\dot{\widetilde{\mathbf{V}}}_n, \widetilde{\mathbf{V}}_n, \tau)]\, d\tau$$

where λ is a general Lagrange vector multiplier [24] which can be identified optimally via the variational theory. The subscript n denotes the nth approximation and $\widetilde{\mathbf{V}}_n$ is considered as a restricted variation, i.e. $\delta\widetilde{\mathbf{V}}_n = 0$.

Taking the exact analytic solution of $\dot{\mathbf{V}} - \mathbf{HV} = 0$ as the initial approximation, we have:

$$\mathbf{V}_{n+1}(t) = \mathbf{V}_n(t) + \int_0^t e^{\mathbf{H}\,(t-\tau)}\mathbf{F}(\dot{\widetilde{\mathbf{V}}}_n, \widetilde{\mathbf{V}}_n, \tau)\, d\tau$$

where the exponential matrix $e^{\mathrm{H}t}$ can be calculated accurately by PIM. A linear homotopy function for (2.2) can be constructed as

$$\dot{\mathbf{V}} - \mathbf{HV} - \mathbf{F}(\dot{\mathbf{V}}, \mathbf{V}, t) + \varepsilon[\mathbf{f}_0 - \mathbf{F}(\dot{\mathbf{V}}, \mathbf{V}, t)] = 0 \tag{2.3}$$

where $\mathbf{f}_0$ is a known initial value vector, and $\varepsilon \in [0,1]$ is the homotopy parameter. According to the perturbation theory, the solution of (2.3) can be expressed as the power series expansion of ε

$$\mathbf{V} = \mathbf{V}_0 + \varepsilon\mathbf{V}_1 + \varepsilon^2\mathbf{V}_2 + \dots \tag{2.4}$$

Substituting equation (2.4) into (2.3), we have:

$$\varepsilon^0 : \dot{\mathbf{V}}_0 = \mathbf{HV}_0 + \mathbf{f}_0, \tag{2.5}$$

$$\varepsilon^1 : \dot{\mathbf{V}}_1 = \mathbf{HV}_1 - \mathbf{f}_0 + \mathbf{f}_1, \tag{2.6}$$

$$\varepsilon^2 : \dot{\mathbf{V}}_2 = \mathbf{HV}_2 + \mathbf{f}_2 \tag{2.7}$$

where $\mathbf{f}_1$ and $\mathbf{f}_2$ are the terms containing ε^1 and ε^2, respectively in the expansion of $\varepsilon\mathbf{F}(\dot{\mathbf{V}}, \mathbf{V}, t)$. According to VIM, taking the iteration once and setting the initial values of $\mathbf{V}_1$ and $\mathbf{V}_2$ as 0, the general solutions of (2.5)–(2.7) could be obtained exactly. Substituting solutions of (2.5)–(2.7) into (2.4), and assuming $\varepsilon = 1$, the approximate analytic solution of (2.1) can be obtained subsequently. As the exponential matrix $e^{\mathbf{H}t}$ can be calculated accurately by PIM [25], the numerical solution of (2.1) can be obtained lastly.

3. The application of HPM in regularization of scalar images with PDEs

A scalar image can be defined as

$$I : \begin{cases} \Omega \subset \mathbb{R}^p \to \mathbb{R}, \\ \mathbf{x} \to I(\mathbf{x}), \end{cases}$$

where $\mathbf{x}= x$ when $p = 1$, $\mathbf{x}= (x, y)$ when $p = 2$, $\Omega \subset \mathbb{R}^p$ is a closed spatial domain of dimension p, Ω is the defined domain of images. The derivative of the image I with respect to the variable a is written as $I_a = \partial I/\partial a$, and the second derivative of a scalar image I with respect to a then to b is denoted by $I_{ab} = \partial^2 I/(\partial a \partial b)$.

Consider the diffusion PDE as follows

$$\begin{cases} I_{t=0} = I_{\text{noisy}}, \\ \dfrac{\partial I}{\partial t} = \operatorname{div}\left(\dfrac{\phi'(\|\nabla I\|)}{\|\nabla I\|}\nabla I\right), \end{cases} \tag{3.1}$$

where $\phi = 2\sqrt{1+s^2} - 2$, which called Hyper-surface schemes. Of course, different choices of function ϕ lead to different regularization methods.

According definitions of the image gradient ∇I and the image gradient norms $\|\nabla I\|$, (3.1) can be rewritten as

$$\begin{cases} I_{t=0} = I_{\text{noisy}}, \\ \dfrac{\partial I}{\partial t} = a(I_x, I_y)I_{xx} + b(I_x, I_y)I_{xy} + c(I_x, I_y)I_{yy}, \end{cases} \tag{3.2}$$

where

$$\begin{cases} a(I_x, I_y) = 2\dfrac{1 + {I_y}^2}{({I_x}^2 + {I_y}^2)\sqrt{{I_x}^2 + {I_y}^2}}, \\ b(I_x, I_y) = -\dfrac{4 I_x I_y I_{xy}}{({I_x}^2 + {I_y}^2)\sqrt{{I_x}^2 + {I_y}^2}}, \\ c(I_x, I_y) = 2\dfrac{1 + {I_x}^2}{({I_x}^2 + {I_y}^2)\sqrt{{I_x}^2 + {I_y}^2}}. \end{cases} \tag{3.3}$$

It is easy to see that $a(I_x, I_y)$, $b(I_x, I_y)$ and $c(I_x, I_y)$ are decreasing functions vanishing on edges in order to stop the diffusion. So, we can linearize these functions with respect to the time t approximately, and the influence of this approximation on the regularization of scalar images should be small.

Using the central difference scheme, we have

$$\begin{cases} I_x = \dfrac{I(x_{i+1}, y_j) - I(x_{i-1}, y_j)}{2h}, \\ I_y = \dfrac{I(x_i, y_{j+1}) - I(x_i, y_{j-1})}{2k}, \\ I_{xx} = \dfrac{I(x_{i+1}, y_j) - 2I(x_i, y_j) + I(x_{i-1}, y_j)}{h^2}, \\ I_{yy} = \dfrac{I(x_i, y_{j+1}) - 2I(x_i, y_j) + I(x_i, y_{j-1})}{k^2}, \\ I_{xy} = \dfrac{I(x_{i+1}, y_{j+1}) - I(x_{i-1}, y_{j+1}) - I(x_{i+1}, y_{j-1}) + I(x_{i-1}, y_{j-1})}{4hk}, \end{cases} \tag{3.4}$$

where $x_i = x_0 + ih, y_j = y_0 + jk$. Substituting (3.4) into (3.2), a system of nonlinear ODEs can be obtained as follows:

$$
\begin{aligned}
\text{(3.5)}\quad & \frac{dI(x_i, y_j, t)}{dt} = a_{i,j}\frac{I_{i+1,j} - 2I_{i,j} + I_{i-1,j}}{h^2} \\
& + b_{i,j}\frac{I_{i+1,j+1} - I_{i-1,j+1} - I_{i+1,j-1} + I_{i-1,j-1}}{4hk} + c_{i,j}\frac{I_{i,j+1} - 2I_{i,j} + I_{i,j-1}}{k^2}
\end{aligned}
$$

where $t \in [t_0, t_1]$, $I_{i,j}$ denotes $I(x_i, y_j, t_0)$; $a_{i,j}, b_{i,j}, c_{i,j}$ denote $a(I_x(x_i, y_j, t), I_y(x_i, y_j, t))$, $b(I_x(x_i, y_j, t), I_y(x_i, y_j, t))$ and $c(I_x(x_i, y_j, t), I_y(x_i, y_j, t))$, respectively. Let

$$
\begin{aligned}
\mathbf{I} &= ((I(x_0, y_0), \dots, I(x_n, y_0)), \dots, \\
&\quad (I(x_0, y_j), \dots, I(x_i, y_j), \dots, I(x_n, y_j)), \dots, I(x_n, y_m))^T, \\
\mathbf{A} &= \mathrm{diag}((a_{0,0}, a_{1,0}, \dots, a_{n,0}), (a_{0,1}, a_{1,1}, \dots, a_{n,1}), \dots, (a_{0,m}, a_{1,m}, \dots, a_{n,m})), \\
\mathbf{B} &= \mathrm{diag}((b_{0,0}, b_{1,0}, \dots, b_{n,0}), (b_{0,1}, b_{1,1}, \dots, b_{n,1}), \dots, (b_{0,m}, b_{1,m}, \dots, b_{n,m})), \\
\mathbf{C} &= \mathrm{diag}((c_{0,0}, c_{1,0}, \dots, c_{n,0}), (c_{0,1}, c_{1,1}, \dots, c_{n,1}), \dots, (c_{0,m}, c_{1,m}, \dots, c_{n,m})),
\end{aligned}
$$

$$
\mathbf{M}_1 = \frac{1}{h^2}\begin{pmatrix}
-2 & 1 & 0 & 0 & \dots & 0 \\
1 & -2 & 1 & 0 & \dots & 0 \\
0 & 1 & -2 & 1 & \dots & 0 \\
0 & \dots & 0 & 1 & -2 & 1 \\
\vdots & \ddots & \ddots & \vdots & \ddots & \vdots \\
0 & 0 & \dots & 0 & 1 & -2
\end{pmatrix}_{(n\times m)\times(n\times m)}
$$

$$
\mathbf{M}_2 = \frac{1}{4hk}\begin{pmatrix}
\overbrace{\begin{matrix}
0 & -1 & 0 & \dots & 0 \\
1 & 0 & -1 & 0 & \dots \\
0 & 1 & 0 & -1 & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \dots & 0 & 1 & 0
\end{matrix}}^{n-1} &
\overbrace{\begin{matrix}
0 & \dots & 0 \\
0 & \dots & 0 \\
0 & \dots & 0 \\
\vdots & \ddots & \vdots \\
1 & 0 & \dots
\end{matrix}}^{n} &
\begin{matrix}
-1 & 0 & 1 & 0 & \dots & 0 \\
0 & -1 & 0 & 1 & 0 & \dots \\
0 & 0 & -1 & 0 & 1 & 0 \\
\vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \dots & -1 & 0
\end{matrix}
\end{pmatrix}_{(n\times m)\times(n\times m)}
$$

$$
\mathbf{M}_3 = \frac{1}{k^2}\begin{pmatrix}
\overbrace{\begin{matrix}
1 & 0 & \dots & 0 \\
0 & 1 & 0 & \dots \\
\vdots & \ddots & \vdots & \ddots \\
0 & \dots & 0 & 1
\end{matrix}}^{n} &
\overbrace{\begin{matrix}
-2 & 0 & \dots & 0 \\
0 & -2 & 0 & \dots \\
\vdots & \ddots & \vdots & \ddots \\
0 & \dots & 0 & -2
\end{matrix}}^{n} &
\begin{matrix}
1 & 0 & \dots & 0 \\
0 & 1 & 0 & \dots \\
\vdots & \ddots & \vdots & \vdots \\
0 & \dots & 0 & 1
\end{matrix}
\end{pmatrix}_{(n\times m)\times(n\times m)}.
$$

And then (3.5) can be rewritten as the matrix differential equation:

$$
\frac{d}{dt}\mathbf{I} = (\mathbf{A}\mathbf{M}_1 + \mathbf{B}\mathbf{M}_2 + \mathbf{C}\mathbf{M}_3)\mathbf{I}.
$$

According to HPM, we can construct the linear homotopy function as follows

$$\left[\frac{d\mathbf{I}}{dt} - (\mathbf{A}_0\mathbf{M}_1 + \mathbf{B}_0\mathbf{M}_2 + \mathbf{C}_0\mathbf{M}_3)\mathbf{I}\right] + \varepsilon\left[\frac{d\mathbf{I}}{dt} - ((\mathbf{A}_0 - \mathbf{A})\mathbf{M}_1 + (\mathbf{B}_0 - \mathbf{B})\mathbf{M}_2 + (\mathbf{C}_0 - \mathbf{C})\mathbf{M}_3)\mathbf{I}\right] = 0 \tag{3.6}$$

where $\mathbf{A}_0$, $\mathbf{B}_0$, $\mathbf{C}_0$ are the known initial values, $\varepsilon \in [0,1]$ is the homotopy parameter. According to the homotopy perturbation theory, the solution of (3.6) can be expressed as the power series expansion of ε:

$$\mathbf{I} = \mathbf{I}_0 + \varepsilon\mathbf{I}_1 + \varepsilon^2\mathbf{I}_2 + \dots \tag{3.7}$$

Substituting (3.7) into (3.6), we have:

$$\varepsilon^0 : \frac{d\mathbf{I}_0}{dt} = (\mathbf{A}_0\mathbf{M}_1 + \mathbf{B}_0\mathbf{M}_2 + \mathbf{C}_0\mathbf{M}_3)\mathbf{I}_0, \tag{3.8}$$

$$\varepsilon^1 : \frac{d\mathbf{I}_1}{dt} = [(\mathbf{A} - \mathbf{A}_0)\mathbf{M}_1 + (\mathbf{B} - \mathbf{B}_0)\mathbf{M}_2 + (\mathbf{C} - \mathbf{C}_0)\mathbf{M}_3]\,\mathbf{I}_1. \tag{3.9}$$

Assuming $\varepsilon = t/\tau$, $t \in [0,\tau]$, $\mathbf{A}$ in (3.9) can be identified as $[A(\mathbf{I}_0)]_{(n\times m)\times(n\times m)}$ by the definition of Taylor series. So, the solution can be obtained by multi-step scheme, that is, we can calculate $\mathbf{I}_0$ from (3.8) firstly, and then obtaine the matrix $\mathbf{A}$ in (3.9). Secondly, we solve (3.9) and obtain $\mathbf{I}_1$ by PIM. Letting $\varepsilon = 1$, and substituting $\mathbf{I}_0$ and $\mathbf{I}_1$ into (3.7), the regularization image I can be obtained.

The comparison of HPM with the common difference method is shown in Figure 1. As this system has no exact analytic solution, the solution obtained by the Adams–Bashforth–Moulton method (ABM) which is built into Matlab 7.0 is taken as the exact solution. In the options of ABM, the scalar relative error tolerance 'RelTol' (1e-3 by default) is taken as 1e-6, and the vector of absolute error tolerances 'AbsTol' (all components 1e-6 by default) is taken as 1e-12. Solutions obtained by ABM at specific times 3, 6, 9 are shown in Figure 1 (c1)–(c3), respectively. The corresponding solutions obtained by HPM and the difference method (DM) are shown in Figure 1 (d1)–(d3) and Figure 1 (e1)–(e3), respectively. It should be noted that the time step τ in HPM is the same as in DM. Obviously, regularization results of a noisy image obtained by HPM and ABM are similar. But in the results with the difference method, a large amount of noise doesn't vanish and even is enhanced. This indicates that the difference method is invalid if a larger time step is taken.

4. Conclusion

The multi-step scheme based on HPM developed in this paper can solve nonlinear diffusion differential equations successfully. Comparison of scalar images regularization with the hyper-surfaces schemes reveals that HPM can obtain higher accuracy than the common difference method. Otherwise, HPM is not

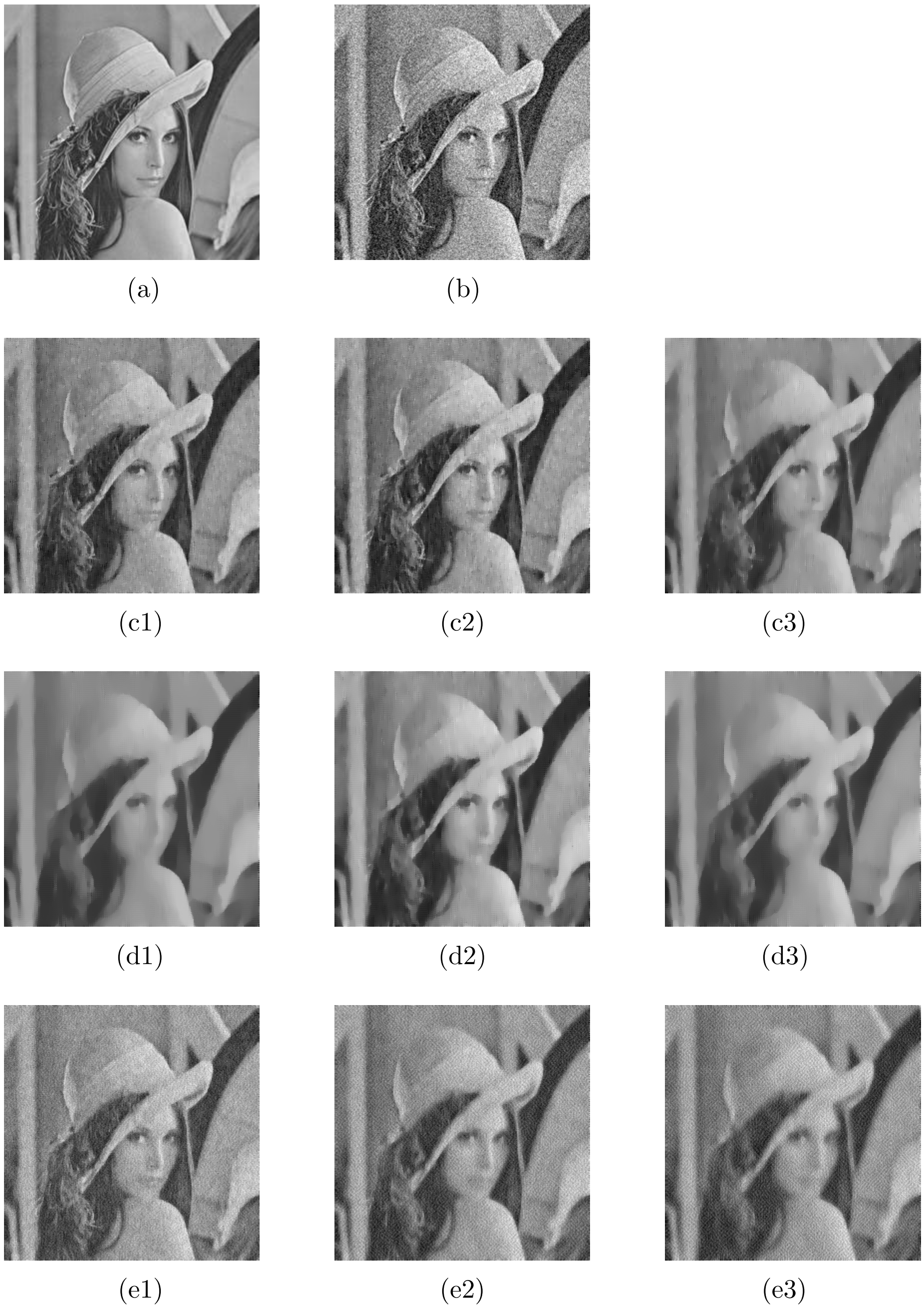

Figure 1. HPM and the difference method applied in regularization of a noisy scalar image. (a) Scalar image, (b) noisy image, (c1) $t = 30$, (c2) $t = 60$, (c3) $t = 90$, (d1) HPM, $\tau = 30$, 1 iteration, (d2) $\tau = 30$, 2 iterations, (d3) $\tau = 30$, 3 iterations, (e1) DM, $\tau = 30$, 1 iteration, (e2) $\tau = 30$, 2 iterations, (e3) $\tau = 30$, 3 iterations.

sensitive to the time step and possesses better stability. Actually, HPM for matrix differential equations has the uniform analytic solution format, and so it can be easily used to solve various nonlinear problems.

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