Tôhoku Math. Journ.
21(1969), 354-362

# HYPERSURFACES IN ALMOST CONTACT MANIFOLDS

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(Received April 16, 1968 ; revised July 23, 1968)

There have been several recent papers examining hypersurfaces in almost complex manifolds (e. g. [3, 5, 7]) ; in particular Y.Tashiro [5] has shown that an arbitrary hypersurface in an almost complex manifold is, in a natural way, an almost contact manifold. It would be natural to ask if a hypersurface in an almost contact manifold is an almost complex manifold. The study of this question has been initiated by Eum [2] and the present authors independently.

The problem here is somewhat different than that of a hypersurface in an almost complex manifold. In general it is not true that a hypersurface in an almost contact manifold is almost complex, for example, the 4-sphere $S^4$ is not an almost complex manifold yet it can be realized as a hypersurface in the almost contact (in fact normal contact metric) manifold $S^5$.

After several preliminaries we prove in §1 that a hypersurface in an almost contact manifold possesses a natural $f$-structure (Yano [6]) which is an almost complex structure if the normal to the hypersurface is in the same direction as the distinguished direction of the almost contact structure. In §2 we 'rotate' an almost contact structure to obtain the main result. A hypersurface in an almost contact manifold $M^{2n+1}$ is almost complex if its normal is the restriction of a non-vanishing vector field on $M^{2n+1}$.

Finally in §3 we show that if the almost contact manifold has additional structure, for example, a "compatible" metric, a normal almost contact structure, a quasi-Sasakian structure, then under some conditions a hypersurface inherits the corresponding analogus structure (e.g. almost Hermitian, complex, Kaehler, resp.).

**1.** A $(2n+1)$-dimensional $C^\infty$ manifold $M^{2n+1}$ is said to have an *almost contact structure* if there exists on $M^{2n+1}$ a tensor field $\varphi$ of type (1,1), a vector field $\xi$ and a 1-form $\eta$ satisfying

$$\begin{aligned}\eta(\xi) &= 1,\\ \varphi\xi &= 0,\\ \eta\varphi &= 0,\\ \varphi^2 &= -\text{identity} + \xi\otimes\eta.\end{aligned} \tag{1. 1}$$

If $M^{2n+1}$ has an almost contact structure $(\varphi, \xi, \eta)$ then we can find a Riemannian metric $g$ on $M^{2n+1}$ such that

$$\eta(X) = g(\xi, X), \tag{1. 2}$$
$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y)$$

where $X$ and $Y$ are vector fields on $M^{2n+1}$ (Sasaki [4]). In this case we say $M^{2n+1}$ has an *almost contact metric structure*.

An almost contact structure $(\varphi, \xi, \eta)$ is said to be *normal* if

$$[\varphi, \varphi](X, Y) + d\eta(X, Y)\xi = 0 \tag{1. 3}$$

where $[\varphi, \varphi]$ is the Nijenhuis torsion of $\varphi$. An almost contact metric structure $(\varphi, \xi, \eta, g)$ is called *quasi-Sasakian* if it is normal and the fundamental 2-form $\Phi$, defined by $\Phi(X, Y) = g(X, \varphi Y)$ is closed [1].

The analogous structures on an even-dimensional manifold $M^{2n}$ are well-known. $M^{2n}$ is *almost complex* if there exists on $M^{2n}$ a tensor field $J$ of type (1, 1) such that $J^2 = -$identity, *almost Hermitian* if there exists a metric $G$ satisfying $G(JX, JY) = G(X, Y)$, *complex* if $[J, J] = 0$ and Kaehlerian if $[J,J] = 0$ and $d\Omega = 0$ where $\Omega(X, Y) = G(X, JY)$, $X, Y$ vector fields on $M^{2n}$.

Suppose $M^{2n+1}$ has an almost contact structure $(\varphi, \xi, \eta)$ and that $M^{2n}$ is an orientable $C^\infty$ hypersurface imbedded in $M^{2n+1}$. Let $TM^{2n}$ denote the tangent bundle of $M^{2n}$ and $T_R M^{2n+1}$ the restriction of the tangent bundle of $M^{2n+1}$ to $M^{2n}$. We denote by $B$ the differential of the imbedding so that $B$ is a mapping of $TM^{2n}$ into $T_R M^{2n+1}$. Let $C$ be a vector field defined along $M^{2n}$ that is not tangent to $M^{2n}$ anywhere, i.e., at each point of $M^{2n}$, $C \in T_R M^{2n+1}$ and $C \notin TM^{2n}$. Then we can find a mapping $B^{-1}$ of $T_R M^{2n+1}$ into $TM^{2n}$ and a 1-form $C^*$ defined on $M^{2n}$ such that

$$\begin{aligned} &B^{-1}B = I, \\ &BB^{-1} = I - C \otimes C^*, \\ &C^*B = B^{-1}C = 0, \\ &C^*(C) = 1, \end{aligned}$$

where $I$ is the identity on $TM^{2n}$ or $T_R M^{2n+1}$. If in addition, $M^{2n+1}$ has a Riemannian metric $g$ satisfying (1. 2), then the induced metric $G$ on $M^{2n}$ is given by

$$G(X, Y) = g(BX, BY),$$

where $X$, $Y$ are vector fields on $M^{2n}$.

Throughout this paper $X$, $Y$, $Z$ will denote vector fields on $M^{2n}$ or $M^{2n+1}$, it being clear from the context which manifold is referred to. Furthermore, we will only consider hypersurfaces $M^{2n}$ for which $\xi$ restricted to $M^{2n}$ is everywhere tangent to $M^{2n}$ *or* nowhere tangent to $M^{2n}$. In the latter case, we may pick $C$ to be $\xi$.

THEOREM 1.1 *If $M^{2n}$ is a hypersurface in an almost contact manifold $M^{2n+1}$, then there exists a tensor field $f$ of type* $(1,1)$ *on $M^{2n}$ such that*

$$f^3+f=0$$

*($M^{2n}$ is then said to have an f-structure, Yano* [6]*) and the rank of $f$ is either $2n$ or $2n-2$.*

PROOF. Let

$$f=B^{-1}\varphi B.$$

Then $f$ is a tensor field of type (1, 1) on $M^{2n}$ and we only have to show that this $f$ has the required properties. If $X$ is a vector field on $M^{2n}$, then

$$\begin{aligned}(f^3+f)(X) &= f^3(X)+f(X)\\ &= B^{-1}\varphi B\, B^{-1}\varphi B\, B^{-1}\varphi B(X)+B^{-1}\varphi B(X)\\ &= B^{-1}\varphi(I-C\otimes C^*)\varphi(I-C\otimes C^*)(\varphi BX)+B^{-1}\varphi B(X)\\ &= B^{-1}\varphi(I-C\otimes C^*)(\varphi^2(BX)-C^*(\varphi BX)\varphi C)+B^{-1}\varphi B(X)\\ &= B^{-1}\varphi(I-C\otimes C^*)(-BX+\eta(BX)\xi-C^*(\varphi BX)\varphi C)+B^{-1}\varphi B(X)\\ &= -B^{-1}\varphi(BX)-C^*(\varphi BX)B^{-1}\varphi^2C-C^*(-BX+\eta(BX)\xi\\ &\quad -C^*(\varphi BX)\varphi C)B^{-1}\varphi C+B^{-1}\varphi B(X).\end{aligned}$$

Now there is no loss in generality in assuming $C^*(\varphi C)=0$ since, with any associated metric $g$, the tensor $\Phi$ defined by $\Phi(X,Y)=g(X,\varphi Y)$ is a 2-form. Hence, we have

$$(f^3+f)(X)=-C^*(\varphi BX)\eta(C)B^{-1}\xi-\eta(BX)C^*(\xi)B^{-1}\varphi C.$$

Now if $\xi=C$, then $B^{-1}\xi=0$, $\varphi C=0$ and so $f^3+f=0$. In this case, we can see that $f(X)=0$ implies $X=0$, since $f(X)=B^{-1}\varphi(BX)$ and $B$ maps no vector into the $C$ direction. Hence, $f$ is of rank $2n$.

Consider the case where $\xi$ is everywhere tangent to $M^{2n}$. Let $g$ be an associated metric on $M^{2n+1}$ and assume $C$ is the normal to $M^{2n}$. Then $\eta(C)=C^*(\xi)=0$ and thus $f^3+f=0$ (note that the metric is not necessary to get this $f$-structure). In this case, since $B$ is of maximum rank, $B$ will map some direction $X_1$ into the $\xi$ direction. Then $\varphi(BX_1)=0$ so $f(X_1)=0$. Also, since $\varphi C$ is tangent to $M^{2n}$, there is a direction $X_2$ such that $BX_2=\varphi C$ so that $fX_2=0$. Now let $X_3$ be any direction tangent to $M^{2n}$ and orthogonal to $X_1$ and $X_2$ with respect to the induced metric on $M^{2n}$. Then there is a direction $X_4$ such that $\varphi BX_4=BX_3$ and thus $fX_4=X_3$. Hence $f$ is of rank $2n-2$. This finishes the proof of the theorem.

Yano [6] has shown that an $f$-structure of maximal rank is an almost complex structure if this rank is even or an almost contact structure if this rank is odd. We then have the following corollary.

COROLLARY 1.2. *Let $M^{2n+1}$ be a manifold with almost contact structure $(\varphi, \xi, \eta)$ and let $M^{2n}$ be a hypersurface of $M^{2n+1}$. Then the natural tensor field $\varphi|M^{2n}=B^{-1}\varphi B$ defines an almost complex structure on $M^{2n}$ if and only if $\xi$ is nowhere tangent to $M^{2n}$.*

PROOF. The 'if' part is given by the first of the proof of Theorem 1.1. If $\xi$ is everywhere tangent to $M^{2n}$(recall that we are only considering hypersurfaces $M^{2n}$ where $\xi$ is everywhere *or* nowhere tangent to $M^{2n}$) then $B^{-1}\varphi B$ is an $f$-structure of rank $2n-2$ and hence is not an almost complex structure on $M^{2n}$.

**2.** The 'rotation' of an almost contact structure requires the existence of a certain tensor field which essentially produces the 'rotation'. Again let $M^{2n+1}$ be an almost contact manifold with structure tensors $\varphi$, $\xi$, $\eta$.

LEMMA 2.1. *For every non-vanishing vector field $\xi'$ on $M^{2n+1}$ that is nowhere in the $\xi$ direction there exists a non-singular tensor field $\mu$ of type* $(1,1)$ *such that*

$$\mu\xi'=\xi.$$

PROOF. Let $g$ be an arbitrary positive definite Riemannian metric on $M^{2n+1}$. Let $\{U_\alpha\}$ be an open covering of $M^{2n+1}$ by coordinate neighborhoods and in each $U_\alpha$ choose $2n-2$ vector fields $X_{(A)}$ such that $\{X_{(A)}\}$ are orthogonal to the span of $\xi'$, $\varphi\xi'$, $\xi$ on $U_\alpha$. Letting $\{\eta_{(A)}, \eta', \eta'', \eta'''\}$ denote the dual basis, we define $\mu$ on $U_\alpha$ by

$$\mu^i_j=\sum_{A=1}^{2n-2} X^i{}_{(A)}\eta_{(A)j}+\xi^i\eta'_j+\xi'^i\eta'''_j+(\varphi\xi')^i(\eta'')_j \tag{2. 1}$$

Then $\mu^i_j\ \xi'^j=\xi^i$, $i,j=1,2,\cdots,2n+1$.

Now similarly construct a basis $\{\bar{X}_{(A)}, \xi', \varphi\xi', \xi\}$ on $U_\beta$, then on $U_\alpha \cap U_\beta \neq \phi$ we have

$$\bar{X}^i_{(A)} = \sum_{B=1}^{2n-2} C_{AB} X^i_{(B)}$$

where $(C_{AB})$ is non-singular. We now have

$$\sum_{A=1}^{2n-2} \bar{X}^i_{(A)} \bar{\eta}_{(A)j} = \sum_{A=1}^{2n-2} \left( \sum_{B,C=1}^{2n-2} C_{AB} C^{-1}_{AC} \right) X^i_{(B)} \eta_{(C)j}$$

$$= \sum_{A=1}^{2n-2} X^i_{(A)} \eta_{(A)j}$$

from which we see that $\mu | U_\alpha$ and $\mu | U_\beta$ agree on $U_\alpha \cap U_\beta$, giving us the desired tensor field $\mu$ (cf. Sasaki [4]).

THEOREM 2.2. *Define a tensor field* $\varphi'$ *and a* 1-*form* $\eta'$ *on the almost contact manifold* $M^{2n+1}$ *by*

$$\varphi' X = \mu^{-1} \varphi \mu X,$$

$$\eta'(X) = \eta(\mu X)$$

*where* $\mu$ *is the tensor field of Lemma* 2.1. *Then* $(\varphi', \xi', \eta')$ *is an almost contact structure on* $M^{2n+1}$.

PROOF. We show that $\varphi'$, $\xi'$, $\eta'$ satisfy equations (1.1).

$$\eta'(\xi') = \eta(\mu\xi') = \eta(\xi) = 1,$$

$$\varphi'\xi' = \mu^{-1}\varphi\mu\xi' = \mu^{-1}\varphi\xi = 0,$$

$$\eta'(\varphi' X) = \eta(\mu\mu^{-1}\varphi\mu X) = \eta(\varphi\mu X) = 0,$$

$$\begin{aligned} \varphi'^2 X &= \mu^{-1}\varphi\mu\mu^{-1}\varphi\mu X \\ &= \mu^{-1}\varphi^2\mu X \\ &= \mu^{-1}(-\mu X + \eta(\mu X)\xi) \\ &= -X + \eta'(X)\xi'. \end{aligned}$$

Combining Corollary 1.2 and Theorem 2.2 we obtain the main theorem.

THEOREM 2.3. *If* $M^{2n}$ *is a hypersurface in an almost contact manifold* $M^{2n+1}$ *such that there is a non-vanishing vector field* $\xi'$ *on* $M^{2n+1}$ *such that* $\xi'$ *restricted to* $M^{2n}$ *is nowhere tangent to* $M^{2n}$, *then* $M^{2n}$ *has an almost* *complex structure given by $J$ satisfying $JX = B^{-1}\varphi' BX$.*

REMARK. The example of $S^4$ considered as a hypersurface in $S^5$ shows that in general some condition on the hypersurface is needed. The condition in Theorem 2.3 is sufficient but not necessary, for example $S^2$ and $S^6$ are almost complex hypersurfaces in $S^3$ and $S^7$, respectively but their unit normals are not the restrictions of a tangent vector field on the higher dimensional sphere.

PROPOSITION 2.4. *If $(\varphi, \xi, \eta)$ and $(\varphi', \xi', \eta')$ are the almost contact structures of Theorem 2.2, then the tensor field $f$ defined by $f=(\varphi-\varphi')/\sqrt{2}$ is an $f$-structure on $M^{2n+1}$.*

PROOF. The proof involves examining the action of $\mu$ on $TM^{2n+1}$. As in the proof of Lemma 2.1 we will assume $\mu$ is chosen such that $\mu\xi=\xi'$ and $\mu(\varphi\xi')=\varphi\xi'$. Then $\eta(\varphi'\xi)=\eta(\varphi\xi')=0$. Also, we may assume $\eta(\xi')=0$ since $\xi$ and $\xi'$ are linearly independent. First we show that $(f^3+f)(\xi)=(f^3+f)(\xi')=0$. Since $f\xi=-2^{-1/2}\varphi'\xi$ and $f\xi'=2^{-1/2}\varphi\xi'$, we see that $(f^3+f)(\xi) = -2^{-1/2}(f^2(\varphi'\xi)+\varphi'\xi) = -2^{-1/2}(2^{-1/2}f(\varphi\varphi'\xi+\xi)+\varphi'\xi) = -2^{-1/2}((-\varphi'\xi-\varphi'\varphi\varphi'\xi-\varphi'\xi)/2+\varphi'\xi) = 2^{-1/2}\varphi'\varphi\varphi'\xi/2$. Now $\varphi'\varphi\varphi'\xi=\varphi'\varphi\mu^{-1}\varphi\mu\xi=\varphi'\varphi\mu^{-1}\varphi\xi'=\varphi'\varphi^2\xi'=-\varphi'\xi'=0$. Similarly, we can show that $(f^3+f)(\xi')=0$.

Now, $f\varphi\xi'=2^{-1/2}(-\xi'-\varphi'\varphi\xi')=2^{-1/2}(-\xi'-\mu^{-1}\varphi\mu\varphi\xi')=2^{-1/2}(-\xi'+\xi)$, where we have made use of the fact that $\mu\varphi\xi'=\varphi\xi'$ and $\mu\xi=\xi'$ as can be seen from (2.1).

Also, $f^2\xi=-(\varphi-\varphi')\varphi'\xi/2=-(\varphi\varphi'\xi+\xi)/2=-(-\xi'+\xi)/2$ and $f^2\xi'=(\varphi-\varphi')\varphi\xi'/2=-(\xi'+\varphi'\varphi\xi')/2=-(\xi'-\xi)/2$. Therefore, $(f^3+f)(\varphi\xi')=0$. If $X$ is a vector with no component in the $\xi$, $\xi'$ or $\varphi\xi'$ directions then $\varphi' X=\mu^{-1}\varphi\mu X=\mu^{-1}\varphi X=\varphi X$ so that $fX=0$. Putting everything together we see that $f$ is an $f$-structure. Furthermore, since $\varphi'\xi=\mu^{-1}\varphi\mu\xi=\mu^{-1}\varphi\xi'$, we see that $f$ maps $TM^{2n+1}$ onto the subspace spanned by $\varphi\xi'$ and $\xi-\xi'$. Hence, $f$ is an $f$-structure of rank 2.

**3.** If $M^{2n+1}$ has almost contact metric structure $(\varphi, \xi, \eta, g)$ we define a new meric $g'$ on $M^{2n+1}$ using the map $\mu$ by

$$g'(X,Y)=g(\mu X, \mu Y).$$

THEOREM 3.1. *The tensors $\varphi'$, $\xi'$, $\eta'$, $g'$ given above are an almost contact metric structure on $M^{2n+1}$.*

PROOF : We have seen that $\varphi'$, $\xi'$, $\eta'$ give $M^{2n+1}$ an almost contact structure, hence it remains only to show that $g'$ satisfies equations (1.2). We have

$$
\begin{aligned}
g'(\xi', X) &= g(\mu\xi', \mu X) \\
&= \eta(\mu X) = \eta'(X)
\end{aligned}
$$

and

$$
\begin{aligned}
g'(\varphi' X, \varphi' Y) &= g(\mu\varphi' X, \mu\varphi' Y) \\
&= g(\varphi\mu X, \varphi\mu Y) \\
&= g(\mu X, \mu Y) - \eta(\mu X)\eta(\mu Y) \\
&= g'(X, Y) - \eta'(X)\eta'(Y).
\end{aligned}
$$

We now prove that the induced metric on the hypersurface $M^{2n}$ is (almost) Hermitian.

THEOREM 3.2. *Let $M^{2n+1}$ have an almost contact metric structure $(\varphi, \xi, \eta, g)$ and let $M^{2n}$ be a hypersurface of $M^{2n+1}$. If $\xi$ is orthogonal to $M^{2n}$ then $M^{2n}$ is an almost Hermitian manifold.*

PROOF. By Corollary 1.2, $M^{2n}$ has an almost complex structure given by $J = B^{-1}\varphi B$. Using the induced metric $G$ on $M^{2n}$ we see that

$$
\begin{aligned}
G(JX, JY) &= g(BJX, BJY) \\
&= g(\varphi BX, \varphi BY) \\
&= g(BX, BY) - \eta(BX)\eta(BY) \\
&= G(X, Y).
\end{aligned}
$$

THEOREM 3.3. *Let $M^{2n+1}$ have an almost contact metric structure $(\varphi, \xi, \eta, g)$ and let $M^{2n}$ be a hypersurface of $M^{2n+1}$. If there is a non-vanishing vector field $\xi'$ on $M^{2n+1}$ orthogonal to $\xi$ and to $M^{2n}$, then $M^{2n}$ is an almost Hermitian manifold.*

PROOF. By Theorem 2.3, $M^{2n}$ has an almost complex structure given by $J = B^{-1}\varphi' B$, where $\varphi' = \mu^{-1}\varphi\mu$. Using the metric $g'$ of Theorem 3.1 define a metric $G$ on $M^{2n}$ by $G(X, Y) = g'(BX, BY)$. Then we see that

$$
\begin{aligned}
G(JX, JY) &= g'(BJX, BJY) \\
&= g'(\varphi' BX, \varphi' BY) \\
&= g'(BX, BY) - \eta'(BX)\eta'(BY) \\
&= G(X, Y) - \eta(\mu BX)\eta(\mu BY).
\end{aligned}
$$

We can write $BX = Z+\alpha\xi$ where $g(Z,\xi)=0$. Then $\eta(\mu BX)=g(\mu BX,\xi)=g(Z+\alpha\xi',\xi)=0$ since, as can be seen from the proof of Lemma 2.1, $\mu$ can be chosen so that $\mu(Z)=Z$. Thus, we have shown that $G$ is an (almost) Hermitian structure.

If $\Phi$, $\Phi'$ denote the fundameatal 2-forms of the almost contact metric structures $(\varphi, \xi, \eta, g)$, $(\varphi', \xi', \eta', g')$ respectively, then an easy computation shows that $\Phi'(X, Y)=\Phi(\mu X, \mu Y)$. However, in general, $\mu[X,Y]\neq[\mu X,\mu Y]$ hence we do not have $d\Phi'(X,Y,Z)=d\Phi(\mu X,\mu Y,\mu Z)$. Similarly the tensor of the normality condition (1. 3) is not invariant under $\mu$. However for the hypersurface of Corollary 1. 2 we have the following results.

THEOREM 3. 4. *If $M^{2n}$ is an almost complex hypersurface in $M^{2n+1}$ as in Corollary 1. 2 and if the almost contact structure on $M^{2n+1}$ is normal, then the almost complex structure on $M^{2n}$ is integrable. Furthermore if $M^{2n+1}$ is quasi-Sasakian then $M^{2n}$ is Kaehlerian.*

PROOF. We need only show that equation (1. 3) implies $[J,J]=0$ and that $d\Phi=0$ implies $d\Omega=0$.

$$\begin{aligned}
[J,J](X,Y) &= -[X,Y]+[JX,JY]-J[JX,Y]-J[X,JY]\\
&= -B^{-1}[BX,BY]+B^{-1}[\varphi BX,\varphi BY]-B^{-1}\varphi[\varphi BX,BY]\\
&\quad -B^{-1}\varphi[BX,\varphi BY]\\
&= B^{-1}([\varphi,\varphi](BX,BY)-\eta[BX,BY])\xi)\\
&= B^{-1}([\varphi,\varphi](BX,BY)+d\eta(BX,BY)\xi)\\
&= 0.
\end{aligned}$$

$$\begin{aligned}
d\Omega(X,Y,Z) &= X\Omega(Y,Z)+Y\Omega(Z,X)+Z\Omega(X,Y)\\
&\quad -\Omega([X,Y],Z)-\Omega([Y,Z],X)-\Omega([Z,X],Y)\\
&= BX\Phi(BY,BZ)+BY\Phi(BZ,BX)+BZ\Phi(BX,BY)\\
&\quad -\Phi(B[X,Y],BZ)-\Phi(B[Y,Z],BX)-\Phi(B[Z,X],BY)\\
&= d\Phi(BX,BY,BZ)\\
&= 0
\end{aligned}$$

where we have used the fact $B[X,Y]=[BX,BY]$.

We close this section with an example of a quasi-Sasakian manifold $M^{2n+1}$ with a Kaehlerian hypersurface $M^{2n}$ whose normal is not the distinguished direction $\xi$. Let $M^{2n+1}$ be a quasi-Sasakian manifold with structure tensors $\varphi$, $\xi$, $\eta$, $g$ such that $\eta\wedge d\eta\neq 0$ but that $(d\eta)^2=0$. Now there exist maps $\Psi$ and $\theta$ [1] such that $g(X,\Psi Y)=d\eta(X,Y)$ and $\theta=\varphi-\Psi$ and we assume $[\theta,\theta]=0$. Then as was shown in [1], $M^{2n+1}$ is locally the product of a Sasakian (normal contact metric) manifold $N^3$ and a Kaehler manifold $N^{2n-2}$. Now let $M^{2n}$ be a hypersurface whose normal $\xi'$ is the restriction of a non-zero vector field on $M^{2n+1}$ which is locally a vector field on $N^3$.

The projection maps of the locally product structure on $M^{2n+1}$ are $-\Psi^2+\xi\otimes\eta$ and $-\theta^2$ [1]. Define maps $P$ and $Q$ on $M^{2n}$ by $PX=B^{-1}(-\Psi^2+\xi\otimes\eta)BX$, $QX=B^{-1}(-\theta^2)BX$, then $P^2=P, Q^2=Q, P+Q=$ identity, $PQ=QP=0$. We now show that $M^{2n}$ is locally the product of a surface $N^2$ and $N^{2n-2}$.

$$\begin{aligned}[Q,Q](X,Y)&=Q^2[X,Y]+[QX,QY]-Q[QX,Y]-Q[X,QY]\\&=B^{-1}(-\theta^2)^2[BX,BY]+B^{-1}[-\theta^2BX,-\theta^2BY]\\&\quad-B^{-1}(-\theta^2)[-\theta^2BX,BY]-B^{-1}(-\theta^2)[BX,-\theta^2BY]\\&=B^{-1}[-\theta^2,-\theta^2](BX,BY)\\&=0.\end{aligned}$$

Hence, giving the surface $N^2$ a Kaehler structure, $M^{2n}$ has the 'non-natural' Kaehler structure of the local Kaehlerian product of $N^2$ and $N^{2n-2}$.

The authors would like to express their appreciation to the referee for his useful suggestions.

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