TSUKUBA J. MATH.
Vol. 32 No. 2 (2008), 407–408

CORRECTION TO THE AUTOMORPHISM GROUP OF A CYCLIC p-GONAL CURVE

By

Naonori Ishii and Katsuaki Yoshida

In our paper [1], we have made an error about the conditions on which our arguments were built. More precisely, we presented a wrong assertion as Lemma 2.1 (ii) in [1]. Necessarily the assertion Lemma 2.1 (iii) that V is contained in the center of G is not correct either. In order to carry the whole argument through the paper, we have to assume that V is in the center of G, and we should rewrite Lemma 2.1 as follows.

The authors would like to thank Professor Aaron Wootton who pointed out this error.

Lemma 2.1. *Here the notations are same as in* [1].

(i) *The group H acts on $\mathscr{S}$.*

The following two conditions are equivalent.

(ii) *Let a_i and a_j be in $\mathscr{S}$. If there exists an element $T \in G$ satisfying $\tilde{T}a_i = a_j$, then we have $r_i = r_j$. Here we define r_{s+1} by $r_{s+1} \equiv -\sum_{i=1}^{s} r_i \pmod{p}$ and $0 < r_{s+1} < p$ when $\sum_{i=1}^{s} r_i \not\equiv 0 \pmod{p}$.*

(iii) *The automorphism V is contained in the center of G.*

Proof. The statement (i) and the implication (ii) $\Rightarrow$ (iii) have actually been proved in [1].

Proof of (ii) $\Leftarrow$ (iii).

Assume $\tilde{T}^* x = \zeta_n x = \zeta x$. Moreover assume $\mathscr{S} \cap \{0, \infty\} = \emptyset$. Let

$$\mathscr{S} = \langle \tilde{T} \rangle b_1 \cup \cdots \cup \langle \tilde{T} \rangle b_t = \bigcup_{k=1}^{t} \{b_k, \zeta_n^1 b_k, \zeta_n^2 b_k, \ldots, \zeta_n^{n-1} b_k\}$$

be the decomposition of $\mathscr{S}$ by the action of $\langle \tilde{T} \rangle$. Then M is defined by

$$y^p = \prod_{k=1}^{t} (x - b_k)^{u_{k,0}} (x - \zeta_n^1 b_k)^{u_{k,1}} \cdots (x - \zeta_n^{n-1} b_k)^{u_{k,n-1}}, \quad 1 \le u_{k,j} \le p-1. \quad (1)$$

Received February 2, 2008.

By acting T^* on (1), we have

$$(T^*y)^p = \prod_{k=1}^{t}(T^*x - b_k)^{u_{k,0}}(T^*x - \zeta_n^1 b_k)^{u_{k,1}}(T^*x - \zeta_n^2 b_k)^{u_{k,2}} \cdots (T^*x - \zeta_n^{n-1} b_k)^{u_{k,n-1}}$$

$$= \zeta_n^C \prod_{k=1}^{t}(x - \zeta_n^{n-1} b_k)^{u_{k,0}}(x - b_k)^{u_{k,1}}(x - \zeta_n^1 b_k)^{u_{k,2}} \cdots (x - \zeta_n^{n-2} b_k)^{u_{k,n-1}},$$

where $C = \sum_{k=1}^{t}\sum_{j=0}^{n-1} u_{k,j}$.

By the assumption that V is in the center, $\frac{T^*y}{y}$ is invariant under the action of V^*. Then

$$\frac{T^*y^p}{y^p} = \zeta_n^C \times \frac{\prod_{k=1}^{t}(x - \zeta_n^{n-1} b_k)^{u_{k,0}}(x - b_k)^{u_{k,1}}(x - \zeta_n^1 b_k)^{u_{k,2}} \cdots (x - \zeta_n^{n-2} b_k)^{u_{k,n-1}}}{\prod_{k=1}^{t}(x - b_k)^{u_{k,0}}(x - \zeta_n^1 b_k)^{u_{k,1}} \cdots (x - \zeta_n^{n-1} b_k)^{u_{k,n-1}}}$$

$$= \zeta_n^C \times \prod_{k=1}^{t}(x - b_k)^{u_{k,1} - u_{k,0}} \cdots (x - \zeta_n^{n-2} b_k)^{u_{k,n-1} - u_{k,n-2}}(x - \zeta_n^{n-1} b_k)^{u_{k,0} - u_{k,n-1}}$$

is p-th power of the rational function $\frac{T^*y}{y} \in \mathbf{C}(x)$. Therefore we have

$$u_{k,0} \equiv \cdots \equiv u_{k,n-1} \mod p.$$

As $u_{k,j} \leq p-1$, we have $u_{k,0} = \cdots = u_{k,n-1}$.

In case $\mathscr{S} \cap \{0, \infty\} \neq \emptyset$, we can carry the same argument as above. $\square$

According to this revised lemma, we have to correct the results in [1] as follows:

(1) we add the assumption that V is in the center of G to Theorem 2.1 [1];

(2) the curves listed in Theorems 3.1 and 5.1 are those with the condition that V is in the center of G.

References

[1] Ishii, N. and Yoshida, K., The automorphism group of a cyclic p-gonal curve, Tsukuba J. Math. Vol. **31**, No. 1 (2007), 1–37.

Mathematical Division of General Education
College of Science and Technology
Nihon University, Narashinodai 7-24-1
Funabashi-shi, Chiba 274-8501, Japan
ishii@penta.ge.cst.nihon-u.ac.jp
yoshida@penta.ge.cst.nihon-u.ac.jp