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The Centralizers of Semisimple Elements of the Chevalley Groups E_7 and E_8

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The purpose of this paper is to give in detailed tables all the centralizers and their orders of semisimple elements of the finite Chevalley groups E_7 and E_8 . These tables are very useful since they give also the character degrees of the semisimple irreducible complex representations constructed by Deligne and Lusztig [7] for the finite Chevalley groups of adjoint type. For, these degrees can be obtained if we know what subgroups of the finite Chevalley groups of universal type are centralizers of semisimple elements (see [7]). Similar tables giving these centralizers for the classical groups and for the groups G_2 , F_4 and E_6 have been obtained in [5], [6], [12] and [11] respectively.

A considerable amount of detailed work was involved in the compilation of our tables which has not been included in the paper. However we outline below the general results on which we relied heavily for our calculations.

Let G be a simple linear algebraic Chevalley group of rank l defined over the algebraic closure K of the prime field F_p of p elements. Let \varPhi be a root system of G with respect to a maximal torus T_0 of G which splits over F_p . Consider the highest root r_0 in \varPhi and let $\widetilde{\varDelta} = \varDelta \bigcup \{-r_0\}$ where $\varDelta = \{r_i; i=1, \dots, l\}$ is a fixed fundamental basis of \varPhi . Also we put $I_0 = \{0, 1, 2, \dots, l\}$.

We have shown [8] that, except for the bad primes of G (see [1, p. 178]), a connected reductive subgroup G_1 of maximal rank in G is the connected centralizer of a semisimple element if and only if some proper subset of the roots in $\widetilde{\Delta}$ is equivalent under the Weyl group $W(=W(\Phi))$ to a fundamental basis of the root system of G_1 . Thus every connected centralizer of a semisimple element in G is in some $C_J, J \cong I_0$, where by C_J we denote the set of all connected centralizers of semisimple elements

Received April 16, 1981 Revised December 14, 1982 of G which are G-conjugate to the connected centralizers whose root system is Φ_J , the root system generated by $\Delta_J = \{r_j; j \in J\}$. All the centralizers belonging in a given C_J , $J \subsetneq I_0$, have the same Dynkin diagram the type of which we denote E_J . Notice (See [10].) that if two proper subsets Δ_J , $\Delta_{J'}$ of $\widetilde{\Delta}$ have the same Dynkin diagrams then Δ_J and $\Delta_{J'}$ are W-conjugate, apart from a few exceptions when $\widetilde{\Delta}$ is of type E_7 or E_8 .

We consider now the Frobenius endomorphism σ of G which raises every matrix coefficient to its q^{th} power where $q = p^m$. Then the group G_{σ} of the fixed points under σ is a Chevalley group over the field F_q of q elements. To work within G_{σ} we have to consider first the connected centralizers of σ -stable semisimple elements (which are, of course, σ stable subgroups) and the question is if each set $C_J(J \subsetneq I_0)$ contains such a centralizer. Thus for a given $J \subsetneq I_0$ we consider the set \mathscr{C}_J of all σ stable centralizers in C_J . Then the group G_{σ} acts on \mathscr{C}_J and let \mathscr{C}_J/G_{σ} be the set of G_{σ} -orbits in \mathscr{C}_J . Finally we denote by Ω_J the normalizer in W of the set Δ_J of the simple roots $r_j, j \in J$. Now the structure of the group of the fixed points of a centralizer in \mathscr{C}_J under σ has been determined by Carter [4] as follows: Let $G_J \in \mathscr{C}_J$. Then

(a) Each conjugacy class [w] of Ω_J gives rise to the orbit $\overline{G}_J^{\overline{g}}$ in \mathscr{C}_J/G_σ represented by the conjugate G_J^{σ} of G_J , where $\pi(g^{-1}\sigma(g)) = w, \pi$ being the natural homomorphism of the normalizer $N_G(T_0)$ of T_0 onto W. The map $[w] \rightarrow \overline{G}_J^{\overline{g}}$ is a bijection.

(b) If M is the semisimple part of G_J , then the group $(M^g)_{\sigma}$ is isomorphic to the subgroup of the finite Chevalley group of type Mobtained by combining the graph automorphism τ of the Dynkin diagram of Δ_J induced by w with σ and taking the fixed points of the product $\sigma\tau$.

(c) If S is the identity component of the centre of G_J , then the group $(S^{\sigma})_{\sigma}$ is isomorphic to the group $X/\bar{P}_J/(qw-1)(X/\bar{P}_J)$. Here X denotes the group (considered as an additive group) of the K-rational characters of T_0 and \bar{P}_J is the subgroup of X consisting of all rational linear combinations of roots in Φ_J .

Let G_J be as above. Then $G_J = MS$ and $M \bigcap S = A$ is finite. Also M and S are σ -stable, being characteristic subgroups of G_J and G_J is F_q -isogenous to the direct product $M \times S$ (since both the connected groups G_J and $M \times S$ are F_q -isogenous to $M/A \times S/A$ which is isomorphic to $(M \times S)/(A \times A)$. In general, it is known that if H_1 , H_2 are two connected algebraic groups defined over k, a finite subfield of K, and H_1 is k-isogenous to H_2 then the groups of the k-rational points of H_1 and H_2

respectively have the same order. Therefore $|(G_J)_{\sigma}| = |M_{\sigma}| |S_{\sigma}|$. In particular, if $w = \pi(g^{-1}\sigma(g))$, then $|(G_J^{\sigma})_{\sigma}| = |(M^{\sigma})_{\sigma}| |(S^{\sigma})_{\sigma}|$, where the order $|(M^{\sigma})_{\sigma}| = |M_{\sigma\tau}|$ is well known (See [2, ch. 12].) and the order $|(S^{\sigma})_{\sigma}|$ is f(q)/g(q), f(t) and g(t) being the characteristic polynomials of w on $X \otimes R$ and on $\bar{P}_J \otimes R$ respectively.

From the above discussion we see that each orbit in \mathscr{C}_J/G_{σ} is characterized by the type E_J of the Dynkin diagram of the centralizers in \mathscr{C}_J and by some conjugacy class [w] in Ω_J . If we are given such an orbit we can now ask whether this orbit contains a centralizer of some σ -stable semisimple element. Carter [4, Cor. 20] has shown that this depends on whether the group $X/P_J + (qw-1)X$ has a character which does not annihilate any root in $\bar{\Phi}_J - \Phi_J$ for sufficiently large values of q, where $\bar{\Phi}_J = \Phi \bigcap \bar{P}_J$. We note that instead of this we have given in [8] a practical method (using the Brauer complex of G, [9]) to determine the conditions which have to be imposed on q for the occurrence of such a centralizer in a given orbit in \mathscr{C}_J/G_{σ} .

To determine, for each proper subset J of I_0 , the structure and its conjugacy classes of the group Ω_J we made great use of the material of Carter's paper [3].

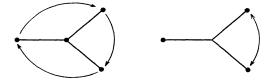
In the tables which follow one row corresponds to each orbit in \mathscr{C}_J/G_{σ} . The column headed with Δ_J gives the type E_J of the Dynkin diagram of the centralizers in \mathscr{C}_{J} . The column headed with Ω_{J} gives the abstruct type of group which is isomorphic to Ω_{J} . The columns headed with $|(M^g)_{\sigma}|$ and $|(S^g)_{\sigma}|$ give respectively the order of the semisimple and toral parts of the fixed points of the centralizers under σ belonging to a given orbit in \mathscr{C}_J/G_{σ} . In particular from the semisimple part we can deduce what kind of graph automorphism is induced by the elements of Ω_{J} . The last column gives the conditions which have to be imposed on q for the occurrence in the G_{σ} -orbits in \mathscr{C}_{J} of centralizers of semisimple elements of G_{σ} . In this last column whenever there is no indication of condition for occurrence this will mean that in the corresponding G_{σ} -orbit they do occur as centralizers of semisimple elements of G_{σ} for all q sufficiently large. For the group E_{τ} we shall distinguish the simply-connected case from the adjoint one by putting "sc" for the former and "ad" for the latter.

When the group Ω_J is not too small, in the tables there is a column headed with [w]. In this column we give a representative element wfor each conjugacy class in Ω_J which corresponds to the G_{σ} -orbit in \mathcal{C}_J parametrized by [w] and Δ_J so that one can distinguish the rows which have the same Δ_J , Ω_J , $|(M^g)_{\sigma}|$ and $|(S^g)_{\sigma}|$. For these cases, we indicate in D. I. DERIZIOTIS

the 2nd, 3rd and 4th row of the first column the chosen Δ_J , the type of the root subsystem Φ_J^{\perp} which is orthogonal to Φ_J and a fundamental basis Δ_J^{\perp} of Φ_J^{\perp} respectively. We give also in the 2nd and 3rd row of the second column the abstruct type of the group $\operatorname{Aut}_{W}(\Delta_J)$ and generators of this group respectively. These generators are given by their action on a suitable bases of Φ so that the reader can easily see the symmetries of Δ_J induced by them.

Our notation for the types of the Dynkin diagrams will be as that The root system of type E_8 is considered of Dynkin's paper [10]. to be embedded in the real vector space R^{s} with orthonormal basis The fundamental roots are chosen to be the vectors $1/2(\varepsilon_1-\varepsilon_2 \{\varepsilon_i\}_{1\leq i\leq 8}.$ $\varepsilon_3 - \varepsilon_4 - \varepsilon_5 - \varepsilon_6 - \varepsilon_7 + \varepsilon_8$, $\varepsilon_2 + \varepsilon_1$, $\varepsilon_2 - \varepsilon_1$, $\varepsilon_3 - \varepsilon_2$, $\varepsilon_4 - \varepsilon_8$, $\varepsilon_5 - \varepsilon_4$, $\varepsilon_6 - \varepsilon_5$ and $\varepsilon_7 - \varepsilon_6$ with respect to which the positive roots are the vectors $\pm \varepsilon_i + \varepsilon_j$, i < j and the vectors $1/2(\varepsilon_8 + \sum_{i=1}^7 (-1)^{v_i}\varepsilon_i)$ such that $\sum_{i=1}^7 v_i$ is even where $v_i = 0, 1$. The root system of type E_{τ} is the root subsystem of E_{s} consisting of the roots $\pm (\pm \varepsilon_i + \varepsilon_j)$, $1 \le i < j \le 6$, $\pm (\varepsilon_8 - \varepsilon_7)$ and $\pm 1/2(\varepsilon_8 - \varepsilon_7 + \sum_{i=1}^6 (-1)^{v_i} \varepsilon_i)$ such that $\sum_{i=1}^{6} v_i$ is odd. In the tables below, the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22 and 23 will denote respectively the roots $1/2(\varepsilon_1-\varepsilon_2-\varepsilon_3-\varepsilon_4-\varepsilon_5-\varepsilon_6-\varepsilon_7+\varepsilon_8)$, $\varepsilon_1+\varepsilon_2$, $\varepsilon_2-\varepsilon_1$, $\varepsilon_3-\varepsilon_2$, $\varepsilon_4-\varepsilon_3$, $\varepsilon_5-\varepsilon_4$, $\varepsilon_6-\varepsilon_5, \varepsilon_7-\varepsilon_6, \varepsilon_7-\varepsilon_8, -\varepsilon_7-\varepsilon_8, 1/2(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4+\varepsilon_5-\varepsilon_6-\varepsilon_7+\varepsilon_8), \varepsilon_3+\varepsilon_2, \varepsilon_4+\varepsilon_3, \varepsilon_6+\varepsilon_7+\varepsilon_8$ $\varepsilon_5, \varepsilon_4 + \varepsilon_1, 1/2(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \varepsilon_5 + \varepsilon_6 - \varepsilon_7 + \varepsilon_8), 1/2(-\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4 + \varepsilon_5 + \varepsilon_6 - \varepsilon_7 + \varepsilon_8),$ $\varepsilon_7+\varepsilon_6$, $\varepsilon_8-\varepsilon_5$, $\varepsilon_8+\varepsilon_5$, $1/2(-\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4-\varepsilon_5-\varepsilon_6-\varepsilon_7+\varepsilon_8)$, $1/2(\varepsilon_1+\varepsilon_2+\varepsilon_8-\varepsilon_4-\varepsilon_5-\varepsilon_6-\varepsilon_7+\varepsilon_8)$ $\varepsilon_6 - \varepsilon_7 + \varepsilon_8$) and $\varepsilon_4 - \varepsilon_1$. Also the letters $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta, \theta, \kappa, \lambda, \mu, \nu, \xi, \pi, \rho$, $\tau, \varphi, x, y, z, \omega, v$ and i will denote respectively the reflections in the hyperplanes orthogonal to the roots 1 up to 23.

We shall denote by H_1 and H_2 the following groups. Let us write the symmetric group S_4 as the semi-direct product of $Z_2 \times Z_2$ by S_3 . Then H_1 denotes the semi-direct product of $Z_2 \times Z_2 \times Z_2$ by S_4 , where S_4 acts on the normal part such that its Klein subgroup $Z_2 \times Z_2$ acts trivially and S_3 purmutes the components in all possible ways. H_2 denotes the semidirect product of the Weyl group $W(D_4)$ of type D_4 by S_4 , where here the Klein subgroup acts trivially on $W(D_4)$ and S_3 acts as in the figure:



Notice that if $J = \emptyset$ then the groups of the σ -fixed points of the centralizers in \mathscr{C}_{ϕ} are the maximal tori $(T_w)_{\sigma}, w \in \Omega_{\phi}$, in G_{σ} determined from

	The	structure ai	nd the orders	The structure and the orders of connected centralizers of semisimple elements in E_r .	ыы ы centralize	rs of semi	simple ele	ements in	$E_{7}.$	
d_J Ω_J	[m]	$ (M^g)_o $	$ (S^g)_\sigma $	Condition for occurrence sc. ad.	d_{J}	Ω_J	[m]	$ (M^g)_o $	$ (S^q)_\sigma $	Condition for occurrence sc. ad.
$A_1 \qquad W(D_6)$	H	$ A_1(q) $	$(q-1)^6$				κπατβε		$(q-1)^2(q^2+1)^2$)2
{7} 1	α		$(q^2 - 1)(q - 1)^4$				αδη βε		$(q+1)^2(q^2+1)^2$	[]2
D_6	-		$(q^3-1)(q-1)^3$				βεκαδ		$(a^2-1)(a^4+1)$	
{1, 2, 3, 4, 5, 9}	αβ		$(q-1)^2(q^2-1)^2$	12			κίπαε		$(a^2 + a + 1)(a^4)$	4-1)
	αδγ		$(q-1)^2(q^4-1)$				rador		$(q+1)(q^2-1)$	
	αγβ		$(q-1)(q^2-1)$				עווסמו		$\times (q^3+1)$	
	αδγε		$\times (q^{5}-1)$ $(q-1)(q^{5}-1)$				βγκαδε		$(q-1)(q^2+1) \times (q^3+1)$	
	KTE		$(q^2 - 1)^3$				κπατε		$(q+1)(q^5+1)$	
	$\kappa r \beta$		$(q^2 - 1)^3$				κπδατε		$(q^2+1)(q^4-1)$	
	κίαε		$(q^2 - 1)(q^4 - 1)$				κπδίατ		$(q^2+1)(q^4+1)$, (I
	κγαβ		$(q^2 - 1)(q^4 - 1)$				βεακγ		$(q^3+1)^2$	
	κδαε		$(q^3-1)^2$		$2A_1$	$\stackrel{W(B_4)}{\scriptstyle \lor \ \ $	1	$ A_1(q) ^2$	$(a-1)^{5}$	
	κΓεαδ		q^6-1		{12 OF	× 22	Q			
	κΥβαδ		q^6-1		11, <i>2</i> 7	, 5 5	с;		$(q^{2}-1)(q-1)^{\circ}$	0
	βε		$(q+1)^2(q-1)^4$		$D_4 + A_1$	\$ \$	Te		$(q-1)(q^2-1)^2$)2
	βεπκ		$(q+1)^2(q^2-1)^2$	2	{2, 3, 4, 5, 14]		β		$(q-1)(q^2-1)^2$)2
	βεπκξΥ		$(a+1)^6$			3 ⇔ 5	18		$(q-1)^2(q^3-1)$	(1
	βεγ		$(a^2 - 1)^3$			4-→4	Yeð		$(q-1)(q^4-1)$	•
	ΒεπκΓ		$(a-1)(a+1)^5$			6↔2	βrð		$(q-1)(q^4-1)$	
	βελξα		$(a+1)^2(a^4-1)$				βγεξ		$(q^2-1)(q+1)^3$)3
	γξα		$(q-1)^2(q^4-1)$				BTe 227 2		$(q+1)(q^2-1)^2$)2
	5		$(a+1)(a^2-1)$				BI EÙ		$(q^2 - 1)(q^3 + 1)$	(1
	pear		$\times (q^{3}-1)$				βγδυ		$(q-1)(q^2+1)^2$)2
	βεακπγ		$(q+1)^3(q^3+1)$				μ		$(q^2-1)(q-1)^3$)3
	κίπα		$(q-1)(q^2-1)$				βπ		$(q-1)(q^2-1)^2$)2
	Dert		$\times (q^{\circ}+1)$				γεπ		$(q+1)(q^2-1)^2$)2
	rear		(T - b)(T + b)				βγπ		$(q+1)(q^2-1)^2$)2
	I çae		$(1 - \frac{1}{2})(d_{\pi} - 1)_{\pi}$	2			$r \delta \pi$		$(q^2 - 1)(q^3 - 1)$	(1
	κπατ		$(q+1)^2(q^4-1)$				Υεδπ		$(q+1)(q^4-1)$	

TABLE 1

CHEVALLEY GROUPS

$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	(Continued)	(mani										
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	<i>As</i>	Ω_J	[m]	$ (M^{g})_{\sigma} $	$ (S^{q})_{\sigma} $	Condition for occurrence sc. ad.	<i>d</i> _J	Ω_J	[<i>m</i>]	$ (M^g)_\sigma $	$ (S^{q})_{\sigma} $	Condition for occurrence sc. ad.
$ \begin{array}{llllllllllllllllllllllllllllllllllll$			BTôπ		$(a+1)(a^4-1)$			6-←6			$(q^{2}+q+1)(q$	³ -1)
$ \begin{array}{llllllllllllllllllllllllllllllllllll$			RYet m		$(\alpha+1)^{5}$				BTKÖ		$(q+1)(q^4-1)$	~
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			pi es n BY en		$(a-1)(a+1)^4$				ĸĩað		(q^5-1)	
$ \begin{array}{llllllllllllllllllllllllllllllllllll$			pren RYedar		$(a+1)^2(a^3+1)$				κΥβαδ		$(q^2+q+1)(q$	3+1)
$ \begin{array}{lcccccccccccccccccccccccccccccccccccc$			βΓδνπ		$(q+1)(q^2+1)^2$				S	$ {}^{2}A_{2}(q) ^{2}$	$(q+1)^{5}$	
$ \begin{array}{llllllllllllllllllllllllllllllllllll$			- ∞	$ A_1(q^2) $	$(q-1)(q^2-1)^2$				βs		$(q^{2}-1)(q+1)$	° (
$\begin{array}{llllllllllllllllllllllllllllllllllll$			Υ 8		$(q-1)(q^4-1)$				ĸĩs		$(q+1)(q^{2}-1)$	4. (
$ \begin{array}{llllllllllllllllllllllllllllllllllll$			i8		$(q+1)(q^2-1)^2$				KAS		$(q+1)^{2}(q^{0}+$	[]
$ \begin{array}{llllllllllllllllllllllllllllllllllll$			õis		$(q^2-1)(q+1)^3$				κĩ βs		$(q-1)(q^2-1)$	4
$ \begin{array}{llllllllllllllllllllllllllllllllllll$			<i>ð</i> ۲8		$(q^2-1)(q^3+1)$				ĸ7 <i>0</i> 8		$(d^{2}-1)(d^{2}+1)$	()
$ \begin{array}{llllllllllllllllllllllllllllllllllll$			Bôs		$(q+1)^2(q^3-1)$				B7 88		$(q+1)(q^{2}-1)$	
$\begin{array}{llllllllllllllllllllllllllllllllllll$			BYs		$(q+1)(q^4-1)$				βκαδε		$(q^2 - q + 1)(q$	*+1)
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			Bues		$(q+1)(q^4+1)$				Breds		$(q-1)(q^{*}-1)$	•
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			BEes		$(q^2+1)(q+1)^3$				ĸľaðs		$(q^{*}+1)$	1
$\begin{array}{llllllllllllllllllllllllllllllllllll$			π8		$(q^2-1)(q-1)^3$	•					$(q^{z}-q+1)(q$	°-1)
$\begin{array}{llllllllllllllllllllllllllllllllllll$			$\gamma_{\pi 8}$		$(q^2+1)(q-1)^3$		$ 3A_1 '$	$W(B_s) \times Z_2$		$ A_1(q) ^3$	$(q-1)^*$	
$\begin{array}{llllllllllllllllllllllllllllllllllll$			iпs		$(q-1)(q^2-1)^2$		[3, 5, 7]	Ŝ	B		$(q^{z}-1)(q-1)$	2(
$ \begin{array}{llllllllllllllllllllllllllllllllllll$			dins		$(q+1)(q^2-1)^2$		$4A_1$		y.		$(q^{z}-1)(q-1)$	2(
$ \begin{array}{llllllllllllllllllllllllllllllllllll$			δίπ8		$(q-1)^2(q^3+1)$		2, 9, 13, 14		کر ،		(d ² 1) ²	
$\begin{array}{lclcrcr} \beta \pi s & (q-1)(q^{4}-1) & & & & & & & & & & & & & & & & & & &$			βδπs		$(q^2-1)(q^3-1)$			7↔7	βκ		(d1)- (d1)-	0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			βľπs		$(q-1)(q^4-1)$			6 ⊷ 6	βξκ		(1 + b)(1 - b)	, () ()
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			βνεπε		$(q-1)(q^4+1)$]4↔]4 2 11 10			$(b^{+}-1)(1-b^{+})$	-(
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			Βέεπε		$(q+1)(q^4-1)$		8	: 2→14→13		14 (2)1	-(T+b)	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	А.	$S_a \times Z_b$		$ A_2(q) $	$(q-1)^{5}$			→2	81	$\times A_1(q) \propto A_1(q^2) $	$(q^2 - 1)(q - 1)$	2(
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	{e. 7}	, "Z	Ø		$(q+1)(q-1)^4$			3→7→5→3	ξ8 ₁		$(q^2 - 1)^2$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	A.	8: 1→-			$(q-1)(q^2-1)^2$			6←6	πs_1		$(q^2 - 1)^2$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	{ 1.2.3.4 .		-2 Ka		$(q-1)^2(q^3-1)$				K 81		$(q^2-1)^2$	
$\begin{array}{ccccc} 1 & \kappa 7 \delta & (q^2 - 1)(q^3 - 1) \\ B T \delta & (q - 1)(q^4 - 1) \\ \end{array} \qquad \qquad$		Т С	-3 KTB		$(q-1)^{2}(q+1)^{3}$				ПК81		$(q^2 - 1)(q + 1)$)2
$\beta \gamma \delta$ $(q-1)(q^4-1)$ $\xi \kappa s_1$		+ ↑ 1	4 k78		$(q^2 - 1)(q^3 - 1)$				ξπ81		$(q^2 - 1)(q + 1)$)2
		2↔9	BTð		$(q-1)(q^4-1)$				ξ κ 81		$(q^{2}-1)(q+1)$	2(

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$ (M^q)_\sigma = (S^q)_\sigma = \begin{array}{c} C_{01} \\ O_{02} \\ SC_{03} \\ SC_{03}$	Condition for occurrence sc. ad.	<i>4</i> ,	Ω_J	[w]	$ (M^q)_\sigma $	$ (S^g)_{\sigma} $	Condition for occurrence, sc. ad.
		. *		ξφ82		$(q^2 - q + 1)^2$	
				$\pi \omega s_2$		$(d^{+}+q+1)^{+}$	
$(q+1)(q^{0}-1)$		$A_2 + A_1$	$S_4 imes Z_2$	-	$ A_1(q) \times A_n(q) $	$(q-1)^4$	
$(1 - 1)(a^3 + 1)$		{1, 2, 3}	$oldsymbol{Z}_{2}$	ŝ		$(q+1)(q-1)^3$	
		A_{3}	8: 1↔3	eر ۲		$(q-1)(q^3-1)$	
		[5, 6, 7]	2→2	εη		$(q^2 - 1)^2$	
$(a^2 - 1)^2$			4-→4	εηζ		$q^4 - 1$	
í í			$5 \rightarrow -5$	s	$ A_1(q) \times ^2 A_2(q^2) $	$(q+1)^4$	
$(q-1)(q^3-1)$			99	68		$(q+1)^2(q^2-1)$	
$q^4 - 1$			77	εζs		$(q+1)(q^3+1)$	
$(q+1)^4$				ens.		$(q^2 - 1)^2$	
$(q-1)(q+1)^3$				εηζε		q^4-1	
$(q+1)(q^3+1)$		Д.	$S_4 \times (Z_2)^2$	-	A ₀ (<i>a</i>)	$(\alpha - 1)^4$	·
$(q^2+1)^2$			Z_2	4	1/5/0		
$(q^2-1)(q-1)^2$		$\{3, 4, 5\}$ $A_3 + A_1$	s: 3⇔5 4→4	<i>1</i> и		$(q+1)(q-1)^{5}$ $(q-1)(q^{3}-1)$	
$(q^2+1)(q-1)^2$	<u> </u>	[7, 9, 11, 14]		yle		$(q^2 - 1)^2$	
$(q^2 - 1)^2$			9-→-9	nth		q^4-1	
$(q^2-1)(q+1)^2$			11→−11	μ		$(q+1)(q-1)^3$	
$(q-1)(q^3+1)$			14→14	ılι		$(q^2 - 1)^2$	
$(q+1)(q^3-1)$				udlı		$(q+1)(q^3-1)$	
$q^4 - 1$				TKT		$(q^2 - 1)(q + 1)^2$	
q^4+1				npun		$(q^{2}+1)(q+1)^{2}$	
$(q^2+1)(q+1)^2$				s	$ {}^{2}A_{3}(q^{2}) $	$(q+1)^2(q^2-1)$	
$(q-1)(q^3-1)$				1/8		$(q^2 - 1)^2$	
$(q-1)(q^3+1)$				87/4		$(q^3+1)(q-1)$	
$(q+1)(q^3-1)$				njk 8		$(q-1)^2(q^2-1)$	
$(q+1)(q^3+1)$				npixs		$(q-1)^2(q^2+1)$	
$1 \sim 4 = \alpha^2 \pm 1$				ļ		1	
$ A_1(q A_1(q) \times A_1(q) \times A_1(q) \times A_1(q) $	$ (S^{9})_{a} (S^{9})_{a} (S^{9})_{a} (q^{2}+1)(q^{+}1)^{2} (q^{-}1)(q^{3}-1) (q^{2}+1)(q^{3}-1) (q^{-}1)(q^{-}1)(q^{-}1)^{3} (q^{-}1)^{2} (q^{-}1)^{2} (q^{-}1)^{2} (q^{-}1)^{2} (q^{-}1)^{2} (q^{-}1)^{2} (q^{-}1)^{2} (q^{-}1)^{2} (q^{-}1)^{2} (q^{-}1)(q^{-}1)^{2} (q^{-}1)^{2} (q^{-}1$	$ (S^{0})_{a} $ cocurrence $ (S^{0})_{a} $ sc. ad. $ (q^{2}+1)(q^{3}-1) (q-1)(q^{3}-1) (q-1)(q^{3}+1) (q-1)^{4} (q+1)(q^{3}+1) (q-1)^{4} (q-1)(q^{3}-1) (q^{2}-1)^{2} (q^{2}-1)^{2} (q^{2}-1)^{2} (q^{2}-1)^{2} (q^{2}-1)^{2} (q^{2}-1)^{2} (q^{2}-1)(q^{3}+1) (q^{2}-1)(q^{3}-1) (q^{2}-1)(q^{2}-1) (q^{2}-1)(q^{$	$ (S^{0})_{a} $ cocurrence $ (S^{0})_{a} $ sc. ad. $ (q^{2}+1)(q^{3}-1) (q-1)(q^{3}-1) (q-1)(q^{3}+1) (q-1)^{4} (q+1)(q^{3}+1) (q-1)^{4} (q-1)(q^{3}-1)^{3} (q^{2}-1)^{2} (q^{2}-1)^{2} (q^{2}-1)^{2} (q^{2}-1)^{2} (q^{2}-1)^{2} (q^{2}-1)^{2} (q^{2}-1)(q^{2}-1) (q^{2}-1)(q^{2}-1) (q^{2}-1)(q^{2}-1) (q^{2}-1)(q^{2}-1) (q^{2}-1)(q^{2}-1) (q^{2}-1)(q^{2}-1) (q^{2}-1)(q^{2}-1) (q^{2}-1)(q^{2}-1) (q^{2}-1)(q^{3}-1) (q^{2}-1)(q^{2}-1) (q^{2}-1)(q^{2}-1) (q^{2}-1)$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$

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<i>J</i> _J	Ω_J	[m]	$ (M^g)_\sigma $	$ (S^g)_o $	Condition for occurrence sc. ad.	d_J	Ω_J	[m]	$ (M^g)_\sigma $	(S ⁰) ₀	Condition for occurrence sc. ad.
		πη8		$(q+1)^2(q^2-1)$			3→3 9→14	81	$\frac{ A_1(q) }{\times A_1(q^3) }$	$q^{3}-1$	
		πημ8 π υτα		(4+1)(4 ⁻ +1) ²			$\rightarrow 13 \rightarrow 9$	£ 81	-	q^3+1	
		ttrifues		(T - 1)			$8_2:2 \rightarrow 2$	8 2	$ A_1(q) ^2 \\ < A_1(a^2) $	$ A_1(q) ^2$ $(q-1)(q^2-1)$	
[4A1] [′]	H_1	Ţ	$ A_1(q) ^4$	$(q-1)^{3}$	$2 q-1 \ 2 q-1 $		54-7	r 0,	1/ hiter	$(a+1)(a^{2}-1)$	
{3, 5, 7, 9}	S,	B		$(q-1)(q^2-1)$	2 q-1 2 q-1		6 ← 6	68°		$(a-1)(a^2+1)$	
$3A_1$	8	βξ		$(q+1)(q^2-1)$	$2 q-1 \ 2 q-1 $		13~14	r€82		$(q+1)(q^2+1)$	
{2, 13, 14}	3↔9	βξπ		$(q+1)^{3}$	2 q-1 $2 q-1 $	4.4.94.	(Z.) ³	-	$ A_2(q) $	$(n-1)^{3}$	
	5-→5	81	$ A_1(q) ^2 \times A_1(q^2) $	$\begin{vmatrix} 2 \\ (a^2) \end{vmatrix} (q-1)(q^2-1)$	$2 q-1 \ 2 q-1 $	107 L 210 0	(27)	-1 :	$\times A_1(q)^2 $	(4 T)	
	7→7	βs_1		$(q+1)(q^2-1)$	2 q-1 $2 q-1 $	(2, 0, 0, 0]	(≰2) •. • 2++3	4	A ₀ (<i>a</i>)	(T - b)(T - b)	
	13↔14	ξ8 <u>1</u>		$(q-1)(q^2+1)$ $2 q-1$) $2 q-1 \ 2 q-1 $	A_1	4↔4	81	$\times A_1(q^2) $	$ A_1(q^2) (q^2) (q-1)(q^2-1)$	
	8₂: 2↔2	$\beta \pi s_1$		$(q+1)(q^2+1)$	$(q+1)(q^2+1) \ 2 q-1 \ 2 q-1 $	{6 }	5↔6	K 81		$(q+1)(q^2-1)$	
	3→3	8183	$\frac{ A_1(q) }{\times A_1(q^3) }$	$q^{3}-1$	2 q-1 $2 q-1 $		$7 \rightarrow -14$ $9 \rightarrow 9$	82	$ {}^{2}A_{2}(q^{2}) imes A_{1}(q) ^{2}$	$\begin{vmatrix} {}^{2}A_{2}(q^{2}) \\ \times A_{1}(q) ^{2} & (q+1)(q^{2}-1) \end{vmatrix}$	
	5↔7	$\beta s_1 s_3$			2 q-1 $2 q-1 $		8°:2→2			1.1.18	
	6←6	818382	$ A_1(q^4) $	$(q-1)(q^2-1)$) $2 q-1 \ 2 q-1 $		3→3	K82		(T+h)	
	13↔14 ° 10	$\beta 8_1 8_3 8_2$		$(q-1)(q^2+1)$	2 q-1 2 q		$4 \rightarrow -(\varepsilon_2 + \varepsilon_5)$ $5 \leftrightarrow 6$) 818°	$ {}^{2}A_{2}(q^{2}) $	$\begin{vmatrix} ^{2}A_{2}(q^{2}) \\ & A_{2}(q^{2}) \end{vmatrix}$, $(q-1)(q^{2}-1)$	
	8 ₃ : 2⇔13 9 F	5818382 Dec 2 2		$(q+1)(q^{2}-1)$	1-b = 1 - b = 1		7↔ε ₃ — ε ₆		$\times A_1(q^*) $		
	00	p5818382 e.e.	: A .(n ²) ²	$(q-1)^3$	2 a-1 2 a		6←6	K8 182		$(q+1)(q^2-1)$	
	6 ← 6	0102 B818,	1/ Avler	$(a-1)(a^2-1)$	2 a-1	$2A_{2}$	$S_3 imes (\boldsymbol{Z}_2)^2$	Ţ	$ A_{2}(q) ^{2}$	$(q-1)^{3}$	
	14→14	ξ8182		$(q-1)(q^2-1)$	2 q-1	$2 q-1 {2, 4, 6, 7}$	$(\mathbf{Z}_2)^2$	ø		$(q+1)(q-1)^2$	-
		ξπ8182		$(q+1)(q^2-1)$	2 q-1	A_2	$s_1: 1 \rightarrow -1$	ar		$q^{3}-1$	
		β ξ 818 2 25		$(q+1)(q^2-1)$	2 q-1	نې س	<u> </u>	_			
1/L A .]//	W(R.)	þξπ8₁8₂ 1	$ A,(a) ^{4}$	$(q+1)^{3}$ $(q-1)^{3}$	T-b Z T-b Z		6 1 2 4 9	81	$ {}^{2}A_{2}(q^{2}) ^{2}$	$ ^{2}A_{2}(q^{2}) ^{2}$ $(q+1)(q^{2}-1)$	
[[1772]		• •		$(n+1)(n-1)^2$	67		• • •				
(Z, ð, 0, 1) 3.4.	υ₃ a.: 2→ħ→7	4. 4		$(q+1)(q^{-1})$			8₂: I→−1 2↔7	αs_1		$(q^{3}+1)$	
9, 13, 14}	→2	κĘπ		$(q+1)^{3}$			4⇔6	ak81		$(q+1)(q^2-1)$	

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d_J	Ω_{J}	[m]	$ ^o(M^o) $	$ (S^g)_\sigma $	Condition for occurrence sc. ad.	\mathcal{A}_J	Ω_J	[m]	$ (M^{\varrho})_{\sigma} $	$ (S^{q})_{\sigma} $	Condition for occurrence sc. ad.
	5-→5 2	ő	$ A_{\circ}(q^{2}) $	$(a^2 - 1)(a - 1)$		A_2	8:1→−1	ak		q ³ -1	
	n - ← n		•	$(a-1)(a^2-a+1)$	- 1)	{1,9}	$2 \rightarrow -(arepsilon_1+arepsilon_6) \ A \leftrightarrow 7$	6) 8	$ {}^{2}A_{4}(q^{2}) $	$(q+1)^{3}$	
		avz aks2		$(q^2-1)(q-1)$	À		5↔6	as		$(q+1)(q^2-1)$	
		8182		$(q+1)(q-1)^2$			6←6	ak8		q^3+1	
		$\alpha s_1 s_2$		$(q+1)(q^{2}-1)$		D_4	$W(B_3)$	-	$]D_4(q) $	$(q-1)^{3}$	
		CK8182		$(q^2+q+1)(q+1)$		{2, 3, 4, 5}	S,	h		$(q-1)(q^2-1)$	· (1
$[A_3 + A_1]'$	$(\mathbf{Z}_2)^3$	1	$ A_{s}(q) \times A_{s}(d) $	$(q-1)^{3}$		3A1	81: 2→2	(LA)		$(q+1)(q^2-1)$	(
{3, 5, 6, 7}	\mathbf{Z}_{2}^{*}	¥	l/Avterly	$(q+1)(q-1)^2$		{1, 9, 14}	3⇔0 4⊸4	njurit 8.	$ {}^{2}D_{2}(\alpha^{2}) $	$(q+1)^{-}$ $(a+1)(a^{2}-1)$	
	8: 2→2	β		$(q+1)(q-1)^2$			6↔2	181		$(q+1)(q^2+1)$	
	3→3	ĸβ		$(q-1)(q+1)^2$			14→−14			$(q-1)(q^2+1)$	
	$4 \rightarrow -(\varepsilon_2 + \varepsilon_6) s$	8	$ {}^{2}A_{3}(q^{2}) \times A_{-}(q) $	$(q-1)(q^2-1)$			8₂: 2→3→5			$(q-1)(q^2-1)$	
	5↔7	K8	I/F/T+r ~	$(q+1)(q^2-1)$			4→4	S2	$ {}^{3}D_{4}(q^{3}) $	$q^{3}-1$	
	9←9	βs		$(q+1)(q^2-1)$			7-→9-→14			-	
	6←6	$\kappa \beta s$		$(q^{3}+1)$			L←	ηs_2		q~+1	
$[A_3 + A_1]''$	$S_4 \times Z_9$		$ A_{s}(q) $	$(q-1)^{3}$		$5A_1$		1	$ A_1(q) ^5$	$(q-1)^{2}$	2 q-1 2 q-1
12 2 2 0		ł	$\times A_1(q) $	1 - 1 1 / 2 - 1 / 8	· · · ·	{2, 3, 5, 7, 9}		ŝ		$q^2 - 1$	2 q-1 2 q-1
ح, ט, 0, <i>1</i>] ۸	2, 1)1	a *		(T - b)(T + b)		$2A_1$	$s_1:2{ ightarrow}2$	ξπ	•	$(q+1)^2$	2 q-1 2 q-1
1.3.9}	3• 1 → 1 2 → 2	nte nte		(q+1)(q-1)		{13, 14}	3-→3	81	$ A_1(q^2) \times A_2(\alpha) ^3$	q^2-1	2 q-1 $2 q-1$
	3→-3	αĨ		(3 - 1 - 1)			5↔7	ξ 8 1	1/E/Terly	q^2+1	2 q-1 $2 q-1$
	5↔7	80	$egin{array}{c c c } {}^2A_3(q^2) \ imes A_1(q) \end{array}$	$(q+1)^{3}$			$9 \rightarrow 9$ $13 \leftrightarrow 14$	8182	$\frac{ A_1(q^4) }{\times A_1(q) }$	q^2+1	never $2 q-1$ occurs
	99	αs		$(q+1)(q^2-1)$			8₂: 2→2				
	66	<i>Тк</i> в		$(q-1)(q^2-1)$			3↔5 7+05	ξ8182		q^2-1	occurs $2 q-1$
		αľ κ8 ž		$(q-1)(q^2+1)$			13→13		$ A,(a^2) ^2$	1	never of
		α[8 •		q"+1			1414	4 S ₂	$\times A_1(q) $	q ^z -1	occurs 2 q-1
A4 {4 5.6.7}	03× ∠ 2 Z °	- 2	(<i>b</i>)+ <i>V</i>	$(q-1)^{2}$ $(a+1)(a-1)^{2}$	· · · · ·	·		ξ 8 2		$(q+1)^2$	never 2 q-1

CHEVALLEY GROUPS

Д у	σ	[w]	$ (M^g)_\sigma $	(S ^g) ₀	Condition for occurrence sc. ad.	4,	Ω_{J}	$ (M^g)_\sigma $	(S ^g),	Condition for occurrence sc. ad.
		$\pi 8_2$		$(q-1)^{2}$	never $2 q-1$ occurs $2 q-1$			$ ^{2}A_{a}(a^{2}) A_{a}(a) ^{2}$	$q^{2}-1$ (a+1) ²	
		ξπ82		$q^2 - 1$	never 2 q-1				q^2-1	
		$(8_18_2)^2$		$(q + 1)^2$	$2 q-1 \ 2 q-1 $	A_3+A_2	$(Z_2)^2$	$ A_3(q) A_2(q) $	$(q-1)^2$	
		$\xi(s_1s_2)^2$		$q^{2}-1$	2 q-1 $2 q-1$			$ {}^{2}A_{3}(\sigma^{2}) {}^{2}A_{3}(\sigma) $	$(a+1)^2$	
	1	$\xi \pi (8_1 8_2)^2$	-	$(q-1)^2$	2 q-1 2 q-1				q^2-1	
$A_{2}+3A_{1}$	$S_3 \times Z_2$		$ A_2(q) A_1(q) ^3$	$ A_1(q) ^3 (q-1)^2$		$A_4 + A_1$	Z_{2}	$ A_4(q) A_1(q) $	$(q-1)^2$	
			$ X ^{2} A_1(q) \times A_1(q) $	q^{z-1}		;		$ {}^{2}A_{4}(q^{2}) A_{1}(q) $	$(q+1)^2$	
			$ {}^{2}A_{2}(q^{2}) $	•		$[A_5]'$	$(\mathbf{Z}_2)^2$	$ A_5(q) $	$(q-1)^2$	
			$\times A_1(q^{\gamma}) \times A_1(q) $	т <i>.</i> ћ				12 A . (m ²)	$(q + 1)^2$	
			$ {}^{2}A_{2}(q^{2}) $	$q^{2}-1$				1/ 5/927 1	q^2-1	
			$\times A_1(q^2) $ $ ^2A_n(q^2) $	412		[A ₅]′′	$S_3 imes Z_2$	$ A_5(q) $	$(q-1)^2$	
			$\times A_1(q) ^3$	(d +1)*					$(q^{2}-1)$	
			$egin{array}{c} A_2(q) \ imes A_1(q^3) \end{array}$	$q^{2}+q+1$				$ {}^{2}A_{*}(a^{2}) $	$q^{4}+q+1$ $(q+1)^{2}$	
$2A_{s}+A_{1}$	$(\mathbf{Z}_2)^2$	$ A_2 $	$ A_2(q) ^2 A_1(q) $	$(q-1)^2$					$q^{2}-1$	
		$ A_2 $	$ A_2(q^2) A_1(q) $	q^2-1					q^2-q+1	
		$ A_2 $	$ A_2(q^2) A_1(q) $	$q^{2}-1$		D_4+A_1	$W(B_2)$	$ D_4(q) A_1(q) $	$(q-1)^2$	
		$ ^{2}A_{2}($	$(q^2) ^2 A_1(q) $	$(q+1)^2$					$q^{2}-1$	
$[A_{3}+2A_{1}]'$	$(Z_2)^3$	A ₃ (6	$ A_3(q) A_1(q) ^2$	$(q-1)^{2}$	$2 q-1 \ 2 q-1 $				$(q+1)^{2}$	
				$q^{2}-1$	2 q-1 $2 q-1 $			$ {}^{2}D_{4}(q^{2}) A_{1}(q) $	$q^{2}-1$	
		$ A_{3} $	$ A_3(q) A_1(q^2) $	$(q+1)^2$	$2 q-1 \ 2 q-1 $				$q^{2}+1$	
				$q^{2}-1$	2 q-1 2 q-1	$D_{ m s}$	$(\mathbf{Z}_2)^2$	$ D_{5}(q) $	$(q-1)^2$	
		$ ^{2}A_{3}($	$ {}^{2}A_{3}(q^{2}) A_{1}(q^{2}) $	$(q-1)^2$	$2 q-1 \ 2 q-1$				q^2-1	
				$q^2 - 1$	2 q-1 $2 q-1$			$ {}^{2}D_{6}(q^{2}) $	$(q+1)^2$	
		$ ^{2}A_{3}($	$ {}^{2}A_{3}(q^{2}) A_{1}(q) ^{2}$	$(q+1)^2$	2 q-1 2 q-1				$q^{2}-1$	
				$q^{2}-1$	$2 q-1 \ 2 q-1$	$3A_2$	$S_3 imes Z_2$	$ A_2(q) ^3$	q-1	3 q-1 $3 q-1$
$[A_{3}+2A_{1}]''$	$(\mathbf{Z}_2)^2$	$ A_3 $	$ A_3(q) A_1(q) ^2$	$(q-1)^2$				$ {}^{2}A_{2}(q^{2}) ^{3}$	q+1	$3 q+1 \ 3 q+1$

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dr	Q,	°(øW)	$ (S^g)_o $	Condition for occurrence sc. ad.	<i>4</i> ,	Ω_{f}	°(aW)	(S ⁹) ₀	Condition for occurrence sc. ad.
		$ A_2(q^2) A_2(q) $	q+1	3 q-1 3 q-1			$ ^{2}A_{5}(q^{2}) A_{1}(q) $	q+1	
		$ A_{2}(q^{3}) $	q-1	3 q-1 3 q-1	A_6	\mathbf{Z}_{2}	$ A_6(q) $	q-1	
•		$ {}^{2}A_{2}(q^{2}) A_{2}(q^{2}) $	q-1	3 q+1 $3 q+1 $			$ {}^{2}A_{6}(q^{2}) $	q+1	
		$ {}^{2}A_{2}(q^{0}) $	q+1		D_4+2A_1	$(\boldsymbol{Z}_2)^2$	$ D_4(q) A_1(q) ^2$	q-1	2 q-1 2 q-1
A_3+3A_1	$(\mathbf{Z}_2)^2$	$ A_3(q) A_1(q) ^3$	q-1	2 q-1 $2 q-1 $				q+1	2 q-1 2 q-1
		$ {}^{2}A_{3}(q^{2}) A_{1}(q^{2}) imes X A_{1}(q) $	q-1	2 q-1 $2 q-1$			$ ^{2}D_{4}(q^{2}) A_{1}(q^{2}) $	q-1	never $2 q-1$ occurs
		$ {}^{2}A_{3}(q^{2}) A_{1}(q) ^{3}$	q+1	2 q-1 $2 q-1$				<i>a</i> +1	never 2 a-1
		$ A_{3}(q) A_{1}(q^{2}) \times A_{1}(q) $	q+1	2 q-1 $2 q-1$	$D_{1} \pm A_{1}$	Z	D_(a) 4_(a)		occurs -14 -
	t	(<i>n</i>) A ₀ (<i>n</i>)	,		14	Ň	$ ^{2}D_{-}(\alpha^{2}) A_{-}(\alpha) $	т н г + и	
$A_3 + A_2 + A_1 Z_2$	Z2	$\times A_1(q) $	q - 1		D_{s}	Z,	$ D_{\alpha}(\sigma) $	4 - 1 0 − 1	
		$ {}^{2}A_{3}(q^{2}) {}^{2}A_{2}(q^{2}) $	0+1		5	3		q+1	
	17 18	$\times [A_1(q)]$	• •		$E_{ m s}$	Z.	$ E_6(q) $	q-1	
ZA_3	$(\mathbf{Z}_2)^{\circ}$	$ A_{3}(q) ^{2}$	d-1	4 d-1 Z d-1			$[{}^{2}E_{a}(a^{2})]$	a+1	
			q+1	2 q-1	94.44	(Z)2	$ A_{a}(a) ^{2} A_{a}(a) $	i . 	Ala-1 Ala-1
		$ A_3(q^2) $	q-1	7	Iu ⊥ εus	(37)	1/5/10/1/5/80/	4	тћ њ тћ њ
			q+1	$4 q+1 \ 2 q-1$			$ A_3(q^2) A_1(q) $	1	never $4 q-1$ occurs
			q-1	-1 2 q				-	
			q+1	$4 q-1 \ 2 q-1 $				-1	occurs 4/9 - 1
		$ {}^{2}A_{3}(q^{2}) ^{2}$	q-1	$4 q+1 \ 2 q-1 $			$ {}^{2}A_{3}(q^{2}) {}^{2} A_{1}(q) $	H	4 q+1 4 q+1
			q+1	$4 q+1 \ 2 q-1 $	A_5+A_2	\mathbf{Z}_{2}	$ A_5(q) A_2(q) $	1	3 q-1 3 q-1
$A_4 + A_2$	Z_2	$ A_4(q) A_2(q) $	q-1				$ {}^{2}A_{5}(q^{2}) {}^{2}A_{2}(q^{2}) $	-	$3 q+1 \ 3 q+1$
		$ {}^{2}A_{4}(q^{2}) {}^{2}A_{2}(q^{2}) $	q+1		A_7	$oldsymbol{Z}_{2}$	$ A_7(q) $		4 q-1 4 q-1
$[A_5+A_1]'$	\mathbf{Z}_{2}	$ A_{5}(q) A_{1}(q) $	q - 1	$2 q-1 \ 2 q-1 $			$ {}^{2}A_{7}(q^{2}) $	H	4 q+1 4 q+1
		$ {}^{2}A_{5}(q^{2}) A_{1}(q) $		$2 q-1 \ 2 q-1 $	$D_6 + A_1$	1	$ D_{6}(q) A_{1}(q) $	H	$2 q-1 \ 2 q-1$
$[A_{5}+A_{1}]^{\prime\prime}$	Z_{z}	$ A_{5}(q) A_{1}(q) $	q - 1		E_{7}	Ħ	$ E_7(q) $		

The structure and the orders of the certification of $M^{0}_{\sigma} $ $ (M^{0})_{\sigma} $ $ (S^{0})_{\sigma} $ Condition $ (M^{0})_{\sigma} $ $ (M^{0})_{\sigma} $ $ (S^{0})_{\sigma} $ Condition d_{J} Q_{J} Q_{J} $[w]$ $ (M^{0})_{\sigma} $ $ (S^{0})_{\sigma} $ $ (S^{0})_{\sigma} $ contribute of d_{J}	$\begin{array}{llllllllllllllllllllllllllllllllllll$			$ \begin{array}{llllllllllllllllllllllllllllllllllll$			κ $\pi \alpha \epsilon \eta$ $(q^2 - 1)(q + 1)^2$ × $(q^3 + 1)$ κ $\pi \alpha \delta \eta$ $(q + 1)(q^2 - 1)$ × $(q^4 + 1)$	κπδατη $(q^{3}+1)(q^{4}-1)$ κ ⁷ βεαδ $(q^{2}-1)(q^{5}+1)$ κπδατε $(q-1)(q^{2}+1)$	$\begin{array}{llllllllllllllllllllllllllllllllllll$
dr dr									
Condition for occurrence									
$ (S_{\theta})^{\sigma} $	$(q-1)^r$ $(q+1)(q-1)^6$ $(q+1)^2(q-1)^5$	$(q^{3}-1)(q-1)^{4}$ $(q-1)(q^{2}-1)^{3}$ $(q-1)(q^{2}-1)^{3}$	$\begin{array}{c} (q-1)^3(q^2-1) \\ \times (q^3-1) \\ (q-1)^3(q^4-1) \\ (q+1)(a^2-1)^3 \end{array}$	$(q^{3}-1)(q^{2}-1)^{3}$ $(q^{3}-1)(q^{2}-1)^{3}$ $(q^{3}-1)(q^{2}-1)^{2}$	$(q - 1)(q^2 - 1)$ × $(q^4 - 1)$ × $(q^4 - 1)$ × $(q^4 - 1)$	$(q-1)^2(q^5-1)$ $(q+1)(q^3+1)$ $ imes (q-1)^3$	$(q-1)^3(q^2+1)^2$ $(q-1)^2(q+1)^5$ $(q+1)^2(q^2-1)$ $\times (a^3-1)$	$(q+1)(q^3-1)^2 \ (q+1)(q^2-1) \ imes (q^4-1) \ imes (q^4-1$	$(q^{\pm}_{1})(q^{2}_{-})$ × (q^{4}_{-}) $(q^{3}_{-})(q^{4}_{-})$ $(a^{2}_{-})(a^{5}_{-})$
	$ A_1(q) $								
[<i>m</i>]	1 αβ	αϊ κιζε κιζβ	αγβ αδγ «Υεν	ki By ki By aBie	κίαε κίαβ	αδγε κγπα	κπατ κΓβεη αβΓεη	κδαεη κί αεη	κγαβη κγζαε κγαδτ
Ω,	$W(E_7)$ 1								
<i>A</i> _J	$egin{array}{c} A_1 \ \{10\} \ E_7 \end{array}$	{1, 2, 3, 4, 5, 6, 7}							

TABLE 2 e orders of the centralizers of semisimple elem

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D. I. DERIZIOTIS

d,	Ω_J	[<i>m</i>]	$ (M^g)_\sigma $	$ (S^g)_\sigma $	Condition for occurrenec	d,	Ω_J	[m]	$ (M^g)_\sigma $	$ (S^g)_\sigma $	Condition for occurrence
		κπραζτ		$(q^3-1)(q^2-q+1)^2$	+1)2			γξαε		$(q^2-1)(q^4-1)$	(
		κπί βεηξ		$(q+1)^{7}$				αδγε		$(q-1)(q^{5}-1)$	
		κγζαεηβ		$(q+1)^3(q^2+1)^2$)2			кĨта		$(q-1)(q^2-1)$	<u>.</u>
		κγβζαδη		$(q^3+1)(q^2+q)$	+1) ²					$\times (q^{n+1})$	·
		r Tenañ		$(q+1)(q^2+1)$				κπατ		$(q-1)^{z}(q^{z}+1)^{z}$	z(
		500/1212		$\times (q^{*}+1)$				βεπκί		$(q-1)(q+1)^{0}$	_
		κΥπαεηβ		$(q^3+1)(q+1)^4$	+			βεακγ		$(q+1)^2(q^4-1)$	•
		κΥπβαδη		$(q+1)^2(q^5+1)$				βεγξα		$(q+1)^2(q^4-1)$	-
		κπρατζη		$(q+1)(q^3+1)^2$	61			Bernd		$(q+1)(q^2+1)$	
		κγ βεζαδ		$(q+1)(q^6-q^3+1)$	+1)					$(\mathbf{I} - \mathbf{J}) \times$	
		κπδαβζτ		q^{r+1}				ĸĩeað		q°1	
		κπνδαζτ		$(q^3+1)(q^4-q^2+1)$	² +1)			κζπαε		$(q-1)(q+1)^{2}$	_
		κπρατζβ		$(q^2-q+1)(q^5+1)$	+1)					$\times (d + 1)$	q
		κπιρατζ		$(q+1)(q^2-q+1)^3$	+1) ³			κπατε		$(q^2 - 1)(q^2 + 1)^2$	
$2A_1$	$W(B_6)$		$ A_1(q) ^2$	$(q-1)^6$				αδΥβε		$(q^2-1)(q^4+1)$.
{7, 10}	\mathbf{Z}_2	α		$(q+1)(q-1)^5$				κπδατ		$(q-1)(q^2+1)$	_
D_{6}	8: 1→1	αβ		$(q+1)^2(q-1)^4$	*			D		(T 5)~	
{1, 2, 3, 4,	2→2	βε		$(q+1)^2(q-1)^4$	4			perice		(4 T L) (~ 1 1)2/~2 1 1)2	1/2
5, 9}	3-→3	αĩ		$(q^3-1)(q-1)^3$	8			pernar		(4 Τ Ι) (4 Τ.	
	5→5	ĸĨe		$(q^2 - 1)^3$				pearn 1		(d+T)~(q~+L) /~8 + 1/2	1
	7↔10	βεγ		$(q^2-1)^3$				KTOTAT S		(1 + 1) (22 + 1)/24 + 1	_
	9↔14	aĩB		$(q-1)(q^2-1)$				RTLANC		(T + b)(T + b)	
	;			(T-,b)×				pr would	1 1 10211	$g(1-\alpha)(1+\alpha)$	2
		ασί		$(T - b)_{*}(T - b)$				ŝ	11 BITE	(T B)/T B)	4
		γξα		$(q-1)^{2}(q^{4}-1)$	-			α8		$(\mathbf{T} - \mathbf{h})_{-}(\mathbf{T} + \mathbf{h})$	_
		βεατ		$(q+1)^2(q^2-1)^2$)2			K8		$(q^{2}+1)(q-1)^{4}$)4
		βεπκ		$(q+1)^2(q^2-1)^2$.)2			õas		$(q^2 - 1)^3$	
		RewT		$(q-1)(q+1)^2$				εβ 3		$(q^2 - 1)^3$	
		mad		$\times (q^{3}-1)$				ð78		$(q-1)(q^2-1)$	•
		AUGE		(T - b)							

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	(Continued)	nued)										
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	<i>d</i> ₁	Ω_J	[<i>w</i>]	$ {}^{o}(M^{g})^{o} $	(S ⁰)_0	Condition for occurrence	<i>d</i> ,	Ω,	[<i>m</i>]	$ (M^g)_\sigma $	[(S ⁰),	Condition for occurrence
$ \begin{array}{llllllllllllllllllllllllllllllllllll$			K (1)		$(q-1)^3(q^3+1)$		(1, 2, 3, 4,	$2 \rightarrow -2$	18		$(q^3-1)(q-1)$	
$ \begin{array}{llllllllllllllllllllllllllllllllllll$			γεβ 8		$(q-1)^2(q+1)^4$		5, 6}	$3 \rightarrow -3$	γεβ		$(q^{2}-1)^{3}$	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			εβκε		$(q^2 - 1)(q^4 - 1)$			4→4	γεζ		$(q-1)(q^2-1)$	_
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			еľкs		$(q^2 - 1)(q^4 - 1)$ $(n+1)(n^2 - 1)$			5→−5	Yeõ		$(q-1)^{2}(q^{4}-1)$	()
$\begin{array}{lclcrcl} \partial [ks & & (q^{-1})(q^{+}+1) & \otimes M(0) & \otimes M(0) \\ e^{i}\partial (s & (q^{-1})(q^{+}-1) & \otimes M(0) & \otimes M(0) \\ e^{i}\partial (s & (q^{-1})(q^{+}-1) & \otimes M(0) & \otimes M(0) \\ e^{i}\partial (s & (q^{-1})(q^{+}-1) & \otimes M(0) & \otimes M(0) \\ e^{i}\partial (s & (q^{-1})(q^{+}+1) & \otimes M(0) & \otimes M(0) \\ e^{i}\partial (s & (q^{-1})(q^{+}+1)) & \otimes M(0) & \otimes M(0) \\ e^{i}\partial (s & (q^{-1})(q^{+}+1)) & \otimes M(0) & \otimes M(0) \\ e^{i}\partial (s & (q^{-1})(q^{+}+1)) & \otimes M(0) & \otimes M(0) \\ e^{i}\partial (s & (q^{-1})(q^{+}+1)) & \otimes M(0) & \otimes M(0) \\ e^{i}\partial (s & (q^{-1})(q^{+}+1)) & \otimes M(0) & \otimes M(0) \\ e^{i}\partial (s & (q^{-1})(q^{+}+1)) & \otimes M(0) & \otimes M(0) \\ e^{i}\partial (s & (q^{-1})(q^{+}+1)) & \otimes M(0) & \otimes M(0) \\ e^{i}\partial (s & (q^{-1})(q^{+}+1)) & \otimes M(0) \\ e^{i}\partial (s & (q^{-1})(q^{-1})) & \otimes M(0) \\ e^{i}\partial (s & (q^{-1})(q^{-1}) & \otimes M(0) \\$			είαε		$\times (q^3 - 1)$			99	αδζμ		$(q+1)^2(q^2-1)$)2
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			дүкв		$(q-1)(q^3+1) \times (q^3-1)$			8⇔10	αβδζ		$(q+1)(q^{2}-1) \times (q^{3}-1)$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			εĩõs		$(q^2-1)(q^4-1)$				αελζ		$(q^{8}-1)^{2}$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			εκαε		$(q-1)(q^{2}-1) \times (\alpha^{3}+1)$				1985		$(q^2-1)(q^4-1)$	(
$\begin{array}{lcl} aeric \varepsilon & (q^2-1)(q+1)^* & re \beta \delta \\ rac \beta & (q^3-1)(q+1)^* & re \beta \delta \\ e\beta r s & (q^3-1)(q^3+1)^3 & e\beta r q \\ e\beta r s & (q^3-1)(q^3+1)^3 & re \beta c \\ e\beta ac s & (q^3-1)(q^3+1)^3 & a\beta r r c \\ e\beta ac s & (q^3+1)(q^3-1) & a\beta r r c \\ e\beta r s & (q^3+1)(q^3-1) & re \rho r r \\ er r c r c r c & (q^3+1)(q^3-1) & re \rho r r \\ er r c r c & (q^3+1)(q^3+1) & a\beta r r r \\ er r r c r c & (q^3+1)(q^3+1) & a\beta r r r \\ er r r c r c & (q^3+1)(q^3+1) & a\beta r r r \\ er r r c r c & (q^3+1)(q^3+1) & a\beta r r r \\ er r r c r c & (q^3+1)(q^3+1) & a\beta r r r \\ er r r c r c & (q^3+1)(q^3+1) & a\beta r r r \\ er r r c r c & (q^3+1)(q^3+1) & a\beta r r r \\ er r r c r c & (q^3+1) & a\beta r r r \\ er r r c r c & (q^3+1) & a\beta r r r \\ er r r c & (q^3+1) & a\beta r r r \\ er r r c & (q^3+1) & a\beta r r r \\ er r r c & (q^3+1) & a\beta r r r \\ er r r & (q^3+1) & a\beta r r r \\ er r & a r r & a r r r \\ er r r & (q^3+1) & a r r r \\ er r & (q^3+1) & a r r r r \\ er r & (q^3+1) & a r r r r \\ er r & (q^3+1) & a r r r r \\ er r & (q^3+1) & a r r r r r \\ er r & (q^3+1) & a r r r r r \\ er r & (q^3+1) & a r r r r r r \\ er r & (q^3+1) & a r r r r r r r r r r r r r r r r r r$			Υκα 8		$(q-1)^2(q^4+1)$				7εδζ		$(q-1)(q^{5}-1)$	
$\begin{array}{lclcl} & & & & & & & & & & & & & & & & & & &$			αεζξ8		$(q^2-1)(q+1)^4$				τεβδ		$(q-1)(q^2-1) \times (a^3+1)$	
$ \begin{array}{llllllllllllllllllllllllllllllllllll$			γ αεβ s		$(q^{3}-1)(q+1)^{3}$				γεδμ		$(q-1)^{2}(q^{2}+1)$)2
$\begin{array}{llllllllllllllllllllllllllllllllllll$			e BT KB E BAKO		$(q+1)^{2}(q^{4}-1)$ $(a^{2}-1)(a^{2}+1)^{2}$				αβεγζ		$(q+1)(q^2+q)$	+1)
$\begin{array}{llllllllllllllllllllllllllllllllllll$			church		$(q+1)(q^2-1)$				αδΥζμ		$(q+1)^2(q^4-1)^{-1}$	(1
$\begin{array}{lclcrcl} \kappa \kappa \cos s & q^{\circ} - 1 \\ \delta \tilde{c}^{\dagger} \kappa s & (q^{2} + 1)(q^{+} - 1) \\ \tilde{c}^{\dagger} \tilde{c} \kappa \alpha s & (q^{2} + 1)(q^{+} - 1) \\ \tilde{c}^{\dagger} \kappa \alpha s & (q^{2} - 1)(q^{+} + 1) \\ \tilde{c}^{\dagger} \kappa \alpha s & (q^{2} - 1)(q^{+} + 1) \\ \tilde{c}^{\dagger} \kappa \alpha s & (q^{2} - 1)(q^{+} + 1) \\ \tilde{c}^{\dagger} \tilde{c} \kappa \alpha s & (q^{2} + 1)^{2} \\ \tilde{c}^{\dagger} \tilde{c} \kappa s & (q^{2} + 1)^{3} \\ \tilde{c}^{\dagger} \tilde{c} \kappa s & (q^{2} + 1)^{3} \\ \tilde{c}^{\dagger} \tilde{c} \kappa s & (q^{2} + 1)^{3} \\ \tilde{c}^{\dagger} \tilde{c} \kappa s & (q^{2} + 1)^{3} \\ \tilde{c}^{\dagger} \tilde{c} \kappa s & (q^{2} + 1)^{3} \\ \tilde{c}^{\dagger} \tilde{c} \kappa s & (q^{2} + 1)^{3} \\ \tilde{c}^{\dagger} \tilde{c} \kappa s & (q^{2} + 1) \\ \tilde{c}^{\dagger} \kappa \delta s & (q^{2} + 1) \\ \tilde{c}^{\dagger} \kappa \delta s & (q^{2} + 1) \\ \tilde{c}^{\dagger} \kappa \delta s & (q^{2} + 1) \\ \tilde{c}^{\dagger} \kappa \delta s & (q^{2} + 1) \\ \tilde{c}^{\dagger} \kappa \delta s & (q^{2} + 1) \\ \tilde{c}^{\dagger} \kappa \delta s & (q^{2} + 1) \\ \tilde{c}^{\dagger} \kappa \delta s & (q^{2} + 1) \\ \tilde{c}^{\dagger} \kappa \delta s & (q^{2} + 1) \\ \tilde{c}^{\dagger} \kappa \delta s & (q^{2} + 1) \\ \tilde{c}^{\dagger} \kappa \delta s & (q^{2} + 1) \\ \tilde{c}^{\dagger} \kappa \delta s & (q^{2} + 1) \\ \tilde{c}^{\dagger} \kappa \delta s & (q^{2} + 1) \\ \tilde{c}^{\dagger} \kappa \delta s & (q^{2} + 1) \\ \tilde{c}^{\dagger} \kappa \delta s & \tilde{c}^{\dagger} \kappa \delta s \\ \tilde{c}^{\dagger} \kappa \delta s & \tilde{c}^{\dagger} \kappa \delta s \\ \tilde{c}^{\dagger} \kappa \delta s & \tilde{c}^{\dagger} \kappa \delta s \\ \tilde{c}^{\dagger} \kappa \delta s & \tilde{c}^{\dagger} \kappa \delta s \\ \tilde{c}^{\dagger} \kappa \delta s & \tilde{c}^{\dagger} \kappa \delta s \\ \tilde{c}^{\dagger} \kappa \delta s & \tilde{c}^{\dagger} \kappa \delta s \\ \tilde{c}^{\dagger} \kappa \delta s & \tilde{c}^{\dagger} \kappa \delta s \\ \tilde{c}^{\dagger} \kappa \delta \delta s & \tilde{c}^{\dagger} \kappa \delta s \\ \tilde{c}^{\dagger} \kappa \delta \delta s & \tilde{c}^{\dagger} \kappa \delta s \\ \tilde{c}^{\dagger} \kappa \delta \delta s & \tilde{c}^{\dagger} \kappa \delta s \\ \tilde{c}^{\dagger} \kappa \delta \delta s & \tilde{c}^{\dagger} \kappa \delta s \\ \tilde{c}^{\dagger} \kappa \delta $			enudo		$\times (q^3+1)$				<i>τεδζμ</i>		$(q+1)(q^{5}-1)$	
$\begin{array}{lclcrcrc} 0^{6l.6.8} & (q^{-+1})(q^{1}) & \beta^{le}\delta^{\prime}_{c} \\ \varepsilon^{\prime}\delta\alpha & (q^{+1})(q^{+}-1) & \beta^{\prime}\varepsilon\delta^{\prime}_{c} \\ \varepsilon^{\prime}\kappa\alpha & (q^{+1})(q^{+}+1) & \gamma^{\prime}\varepsilon\delta^{\prime}_{c} \\ \varepsilon^{\prime}\beta^{\prime}\varepsilon^{\prime}\delta & (q^{-1})(q^{+}+1) & \alpha^{\prime}\delta^{\prime}\mu^{\prime}_{c} \\ \varepsilon^{\prime}\beta^{\prime}\varepsilon^{\prime}\delta & (q^{-1})(q^{+}+1) & \alpha^{\prime}\delta^{\prime}\mu^{\prime}_{c} \\ \varepsilon^{\prime}\beta^{\prime}\delta^{\prime}\delta & (q^{+1})(q^{+}+1) & \alpha^{\prime}\delta^{\prime}\mu^{\prime}_{c} \\ \varepsilon^{\prime}\beta^{\prime}\delta^{\prime}\delta & (q^{+1})^{\prime}(q^{+}+1) & \alpha^{\prime}\delta^{\prime}\mu^{\prime}_{c} \\ \varepsilon^{\prime}\beta^{\prime}\kappa\alpha & (q^{+1})^{\prime}(q^{+}+1) & \alpha^{\prime}\delta^{\prime}\mu^{\prime}_{c} \\ \varepsilon^{\prime}\beta^{\prime}\kappa\alpha & (q^{+1})^{\prime}(q^{+}+1) & \alpha^{\prime}\delta^{\prime}\mu^{\prime}_{c} \\ \varepsilon^{\prime}\kappa\alpha\delta & (q^{+1})^{\prime}(q^{+}+1) & \alpha^{\prime}\delta^{\prime}\mu^{\prime}_{c} \\ \varepsilon^{\prime}\kappa\alpha\delta & (q^{+1})^{\prime}(q^{-}+1) & \alpha^{\prime}\delta^{\prime}\mu^{\prime}_{c} \\ \varepsilon^{\prime}\kappa\alpha\delta & (q^{+1})^{\prime}(q^{-}+1) & \alpha^{\prime}\delta^{\prime}\mu^{\prime}_{c} \\ \varepsilon^{\prime}\lambda^{\prime}\delta^{\prime} & (q^{+1})^{\prime}(q^{-}-1)^{\prime} & \delta^{\prime}\mu^{\prime}_{c} \\ \varepsilon^{\prime}\lambda^{\prime}\delta^{\prime} & \gamma^{\prime}\delta^{\prime}_{c} \\ \varepsilon^{\prime}\lambda^{\prime}\delta^{\prime} & \gamma^{\prime}\delta^{\prime} \\ \varepsilon^{\prime}\delta^{\prime}\delta^{\prime} & \gamma^{\prime}\delta^{\prime} \\ \varepsilon^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime} & \gamma^{\prime}\delta^{\prime} \\ \varepsilon^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime} & \gamma^{\prime}\delta^{\prime} \\ \varepsilon^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime} & \gamma^{\prime}\delta^{\prime}\delta^{\prime} \\ \varepsilon^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime} & \gamma^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime} \\ \varepsilon^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime} & \gamma^{\prime}\delta^{\prime}\delta^{\prime} \\ \varepsilon^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime} & \gamma^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime} \\ \varepsilon^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime} & \gamma^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime}\delta^{\prime}\delta$			EK COS		q°-1 (~2 + 1)(~4 = 1)				αδζγε		q^6-1	
$\begin{array}{llllllllllllllllllllllllllllllllllll$			0EI K8 ETÃNO		$(d - \pm 1)(d_{2} - 1)$				βΓεδζ		$(q^2 - 1)(q^4 + 1)$	a
$\begin{array}{llllllllllllllllllllllllllllllllllll$			el kas		$(q^2-1)(q^4+1)$				γεδζυ		$(q-1)(q^2+1) \times (a^3+1)$	
$\begin{array}{lclcrc} \varepsilon\beta7\xi\kappa& (q^2+1)(q+1)^4 & & & & & & & & & & & & & & & & & & &$			κĩ α õ 8		$(q-1)(q^{5}+1)$				αβεγμζ		$(q^2+q+1)^3$	
$ \begin{array}{cccccc} \kappa^{10ep8} & (q+1)(q^{+}+1) \\ \varepsilon\beta\gamma\delta\kappas & (q+1)(q^{+}+1) \\ \varepsilon\beta\gamma\kappa\alphas & (q+1)^{2}(q^{+}+1) \\ \varepsilon\beta\gamma\kappa\alpha\deltas & (q+1)^{2}(q^{+}+1) \\ \varepsilon\gamma\kappa\alpha\deltas & q^{5}+1 \\ W(E_{6})\times Z_{2} & 1 & A_{2}(q) & (q-1)^{6} \\ Z_{2} & \gamma & (q+1)(q-1)^{6} \\ s & 1 \rightarrow -1 & \gamma \\ s & (q+1)^{2}(q-1)^{4} \\ \end{array} \right) \qquad \begin{array}{c} \alpha\delta\gamma\gamma\varepsilon\beta \\ \alpha\delta\beta\gamma\gamma\varepsilon\beta \\ \beta\omega\delta\gamma\zeta \\ \beta\omega\delta\gamma \\ \beta\omega\delta\gamma\zeta $			εβ7ξ κ8		$(q^2+1)(q+1)^4$				αδζ μ] ε		$(q+1)(q^2+q$	+1)
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			K106P8		(q ⁻ +1) ⁻ (a+1)(a ² +1)						$\times (q^{\circ}+1)$	
$\begin{array}{cccc} \varepsilon\beta\gamma\kappa\alpha s & (q+1)^2(q^4+1) & \alpha\delta\rho\beta\gamma\zeta \\ \varepsilon\gamma\kappa\alpha\delta s & q^6+1 & & & & & & & & \\ W(E_6)\times Z_2 & 1 & A_2(q) & (q-1)^6 & & & & & & & & & \\ Z_2 & \gamma & & (q+1)(q-1)^5 & & & & & & & & & & \\ s: \ 1 \rightarrow -1 & \gamma c & & (q+1)^2(q-1)^4 & & & & & & \gamma s & & & & & & & & & \\ \end{array}$			εβγδκε		$(\mathbf{q}^{3}+1)$ × $(\mathbf{q}^{3}+1)$				αδζγεβ		$(q^{*}+q+1)$ $\times (q^{4}-q^{2}+1)$	
$\begin{array}{c cccc} w(E_{0}) \times Z_{2} & q^{\circ} + 1 & & & \\ W(E_{0}) \times Z_{2} & 1 & A_{3}(q) & (q-1)^{6} & & & & \\ Z_{2} & & & & & & & \\ & Z_{2} & & & & & & & \\ & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & &$			εβγκα8		$(q+1)^2(q^4+1)$				αδρβγζ		q^6+q^8+1	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	4.	$\mathbf{U}(E_{i}) \times \mathbf{I}$	εί καό 8 Ζ. 1	4 _(<i>n</i>)	$q^{\circ}+I$				βεωδυζ		$(q^{3}+q+1) \times (n^{2}-n+1)$	63
8: $1 \rightarrow -1$ γ_{ε} $(q+1)^2(q-1)^4$ $\gamma_{\mathcal{B}}$	8, 10}	Z2	1 22	1/6/2171	$(q+1)(q-1)^{5}$				80	$ ^{2}A_{2}(q^{2}) $	$(q+1)^6$	
	E	↑			$(q+1)^{2}(q-1)^{4}$				78		$(q^2-1)(q+1)$)4

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Δ,	Ω_J	[<i>w</i>]	$ (M^g)_o $	[(S ⁹) _a]	Condition for occurrence	d_J	Ω_J	[m]	$ (M^g) _\sigma$	$ (S^{q})_{\sigma} $	Condition for occurrence
		Tes		$(q^2-1)^2(q+1)^2$		{1, 3, 10,	2↔5	τΓ		$(q-1)^2(q^3-1)$	_
		7 ð 8		$(q^3+1)(q+1)^3$		16, -17	3→3	τφĩ		$(q-1)(q^4-1)$	
		lebs		$(q^2 - 1)^3$			<u>1</u> →7	ατφω		$(q^2 - 1)(q + 1)^3$	
				$(q+1)(q^2-1)$			$10 \rightarrow 10$	ταφ		$(q+1)(q^2-1)^2$	~
		<i>ا</i> در&		$\times (q^3+1)$			16↔	γφτα		$(q^2 - 1)(q^3 + 1)$	
		Teôs		$(q+1)^2(q^4-1)$			-17	τφικ		$(q-1)(q^2+1)^2$	2
		αδζμε		$(q-1)^2(q^2-1)^2$		9	s_2 : 1 \rightarrow 16 \rightarrow			$(q+1)(q-1)^4$	
		αβδζε		$(q-1)(q^{z}-1) \ ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~$			-171	ад		$(q-1)(q^2-1)^2$	8
		nelle		$(0^3+1)^2$			3→3	τφλ		$(q+1)(q^2-1)^2$	
		r AST e		$(a^2 - 1)(a^4 - 1)$			$2 \rightarrow 7 \rightarrow$	τγλ		$(q^2 - 1)(q^3 - 1)$	
		redra		$(n+1)(n^5+1)$			5→2	τφιλ		$(q+1)(q^4-1)$	
		0500 I		(1 - 2)(2 - 1)			$10 \rightarrow 10$	ατφωλ		$(q+1)^{5}$	
		Γεβδε		$(q^{1}-1)(q^{2}-1)$				ταφλ		$(q^2-1)(q+1)^3$	
		γεδυς		$(q+1)^2(q^2+1)^2$				γφταλ		$(q+1)(q^3+1)$	
		αβεγζε		$(q^2 - 1)(q^2 - q + 1)^2$	-1)2			τφζκλ		$(q+1)(q^2+1)^2$	61
		adths		$(q-1)^2(q^4-1)$				81	$ A_1(q^2) A_1(q$	$ A_1(q^2) A_1(q) (q+1)(q-1)^4$	
		τεδζμε		$(q-1)(q^{5}+1)$				τs_1		$(q^2+1)(q-1)^3$	e 9
		αδζγες		q^6-1				ξ81		$(q-1)(q^2-1)^2$	8
		βγεδζε		$(q^2 - 1)(q^4 + 1)$				$\gamma \xi s_1$		$(q+1)(q^2-1)^2$	8
		Γεδζυβ		$(q^2+q+1)(q^4-$	-1)			rs_1		$(q-1)^2(q^3+1)$	_
		αβεγμζε		$(q^2 - q + 1)^3$				$\alpha \gamma s_1$		$(q^2 - 1)(q^3 - 1)$	_
		war inter		$(q-1)(q^2-q+1)$	1)			$\alpha \tau s_1$		$(q-1)(q^4-1)$	
		as infor		$\times (q^3 - 1)$				ακφ81		$(q-1)(q^4+1)$	
		αδζγεβs		(q^2-q+1)				aw931		$(q+1)(q^4-1)$	
		~		$\sqrt{q} - \frac{q}{r^3 + 1}$				781		$(q-1)(q^2-1)^2$	2
		auppi 58		q - q + 1				$\tau\lambda s_1$		$(q-1)(q^4-1)$	
		βεωδυζ s		$\times (q^4 + q^2 + 1)$				ξλsı		$(q+1)(q^2-1)^2$	63
$3A_1$	$W(F_4) \times Z_2$ 1	\mathbf{Z}_2 1	$ A_1(q) ^{8}$	$(q-1)^5$				7£281		$(q^2-1)(q+1)^3$	en
{2, 5, 7}	S,	α		$(q+1)(q-1)^4$				$r_{\tau\lambda s_1}$		$(q^2 - 1)(q^3 + 1)$	<u> </u>
D.+ 4.	1-1-1	40		$(\alpha - 1)(\alpha^2 - 1)^2$		-		~ . ~		/~ 1 1/2/~8 1/	

<i>A</i> ₂	Ω_J	[<i>m</i>]	$ {}^{o}(M_{o}) $	$ (S^g)_\sigma $	Condition for occurrence	d,	Ω_J	[m]	$ (M^g)_o $	(S ^q) _o	Condition for occurrence
		ατλ81		$(q+1)(q^4-1)$				À18		$(q+1)(q^2-1)^2$	
		ακφλ81		$(q+1)(q^4+1)$				29 8		$(q+1)^{2}(q^{3}+1)$	_
		αωφλ81		$(q^2+1)(q+1)^3$				Àŋe8		$(q-1)(q^2-1)^2$	8
		S 2	$ A_1(q^3) $	$(q-1)^2(q^3-1)$		-		де ζв		$(q^2 - 1)(q^3 + 1)$	
		αs_2		$(q-1)^{2}(q^{3}+1)$				2138		$(q+1)(q^4-1)$	
		$\alpha \omega s_2$		$(q^2 - 1)(q^3 - 1)$				let 98		$(a^2 - a + 1)(a^3 + 1)$	+1)
		ατωφ82		$(q^2-1)(q^3+1)$				X17e98		$(a-1)(a^4-1)$	ì
		r_{ts_2}		$(q-1)(q^4-q^2+1)$	1)			275.98		$q^{b}+1$	
		ξ φ8 2		$(q-1)(q^2-q+1)^2$	1) ²			Anel 98		$(q-1)(q^4+q^2+1)$	+1)
		$\pi \omega s_2$		$(q^{2}+q+1)(q^{3}-1)$	-1)	A_{3}	$W(B_b)$		$ A_3(q) $	$(q-1)^{5}$	•
		782		$(q^{2}-1)(q^{3}-1)$		{7, 8, 10}	Z_{2}	ø		$(q+1)(q-1)^4$	
		$\alpha \lambda s_2$		$(q^{2}-1)(q^{3}+1)$		$D_{ m s}$	s: 1→21	αδ		$(q+1)^2(q-1)^3$	
		αωλει		$(q+1)^2(q^3-1)$		{1, 2, 3, 4, 5}	§ 2→2	βε		$(q+1)^2(q-1)^3$	~
		ατωφλ82	23	$(q+1)^2(q^3+1)$			3→3	aĩ		$(q-1)^2(q^3-1)$	
		Γτλ8 ₂		$(q+1)(q^4-q^2+1)$	-1)		4-→4	βεγ		$(q-1)^2(q+1)^3$	
		ξφ λ8 2		$(q+1)(q^2-q+1)^2$	1) ²		5→5	αΓε		$(q^2 - 1)(q^3 - 1)$	-
•	1	$\pi \omega \lambda s_2$		$(q+1)(q^2+q+1)^2$	1) ²		7⇔10	βεδ		$(q-1)(q^4-1)$	
$A_2 + A_1$	$S_6 \times Z_2$		$ A_2(q) A_1(q) $	$(q-1)^{5}$			8→8	αδγ		$(q-1)(q^4-1)$	
{I, Z, 3}	Z ³	~		$(q+1)(q-1)^4$				αωβε		$(q-1)(q+1)^4$	
A ₅	8: 1↔3	luy .		$(q-1)(q^2-1)^2$				βεαγ		$(q+1)^3(q^3-1)^3(q^$	•
{0, 0, 7, 8, 40)	7 7→27			$(q-1)^2(q^3-1)$				βεδτ		$(q+1)(q^4-1)$	
10T	5 → - 5 ,			$(q-1)^2(q+1)^3$				βεδυ		$(q-1)(q^2+1)^2$	21
	9 - ← 9			$(q^2 - 1)(q^3 - 1)$				<i>τ</i> βεδ		$(q^2-1)(q^3+1)$	
	L→←L			$(q-1)(q^4-1)$				αδγε		$q^{5}-1$	
	8→ 8			$(q-1)(q^2+q+1)^2$	1) ²			τ βεδυ		$(q^{2}+1)(q^{3}+1)$	
	10 10	kne.g		$(q+1)(q^4-1)$				βεδαω		$(q^2+1)(q+1)^3$	8
	-10	Ant 9		$q^{5}-1$				αδβεγ		$(q+1)(q^4+1)$	
		Anel I			-1)			8	$ {}^{2}A_{3}(q^{2}) $	$(q^2-1)(q-1)^3$	8
		°,	$ {}^{z}A_{2}(q^{2}) A_{1}(q) $	$(q+1)^{5}$				7 8		$(q-1)(q^2-1)^2$	-
		48		$(q^2-1)(q+1)^3$	·			as		$(q^2+1)(q-1)^3$	

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(Continued)

d _J	Ω	[<i>w</i>]	$ {}^{o}(M){}^{o} $	$ (S^g)_\sigma $	Condition for occurrence	d,	$arOmega_J$	[w]	$ (M^g)_\sigma $	$ (S^g)_\sigma $	Condition for occurrence
		eľ s		$(q+1)(q^2-1)^2$			3→3	8382	$ A_1(q^3) A_1(q) \ (q-1)(q^3-1)$	$(q-1)(q^{3}-1)$	
		εβs		$(q+1)(q^2-1)^2$			5↔7	$\beta s_3 s_1$		$(q-1)(q^3+1)$	
		ð7 s		$(q^{2}-1)(q^{3}-1)$			6←6	K 8381		$(q+1)(q^3-1)$	
		εαε		$(q-1)(q^4-1)$			$10 \rightarrow 10$	$\beta \kappa s_3 s_1$		$(q+1)(q^3+1)$	
		αĩs		$(q-1)^2(q^3+1)$			13⇔14	828381	$ A_1(q^4) $	q^4-1	
		7εβs		$(q^2 - 1)(q + 1)^3$				$\beta s_2 s_3 s_1$		q^4+1	
		eĩðs		$(q+1)(q^4-1)$		$[4A_1]''$	H_2	1	$ A_1(q) ^4$	$(q - 1)^4$	2 q-1
		εðαs		$(q^2+1)(q^3-1)$		(2, 5, 7, 10)	S_4	a		$(q-1)^2(q^2-1)$	2 q-1
		εβαs		$(q+1)(q^4-1)$		D_4	$s_1 \colon 1 {\rightarrow} 1$	τφ		$(q^2 - 1)^2$	2 q-1
		εαĩs		$(q^2 - 1)(q^3 + 1)$			2⇔5	<i>τ</i> 1		$(q-1)(q^3-1)$	2 q-1
		rads		$(q-1)(q^4+1)$		-17}	3→3	τφι		$q^4 - 1$	2 q-1
		εβαγε		$(q+1)^2(q^3+1)$			7→7	ατφω		$(q+1)^4$	2 q-1
		Τξεβs		$(q+1)^5$			$10 \rightarrow 10$	ταφ		$(q+1)^2(q^2-1)$	2 q-1
		εβαδε		$(q+1)(q^2+1)^2$			$16 \leftrightarrow -17$			$(q+1)(q^3+1)$	2 q-1
		αδεĩ s	:	$q^{5}+1$	-	Š	$s_2: 1 { ightarrow} 16 { ightarrow}$	τφικ		$(q^2+1)^2$	2 q-1
[4A1]/ I	$W(B_4)$		$ A_1(q) ^4$	$(q-1)^4$			-17-1		$ A_1(q^2) $	$(a-1)^2(a^2-1)$	2 a-1
(3, 5, 7, 10)	S4	Ø,		$(q+1)(q-1)^3$				ð1	$ imes A_1(q) ^2$	(T A) (T A)	The
$4A_1 s_1$: 2↔13	₿Ę		$(q^2 - 1)^2$			$2 \rightarrow 7 \rightarrow 5 \rightarrow 2 \tau s_1$	181		$(q-1)^2(q^2+1)$	2 q-1
{2, 9, 13, 14}	3⇔5	βξπ		$(q-1)(q+1)^3$			$10 \rightarrow 10$	ξ8 <u>1</u>		$(q^2 - 1)^2$	2 q-1
	1←7	βξπκ		$(q+1)^4$		Š	s₃: 1→1	$r\xi s_1$		$(q+1)^2(q^2-1)$	2 q-1
	6←6	81	$ A_1(q^2) A_1(q) $	$A_1(q) ^2 (q-1)^2(q^2-1)$			2↔5	rs_1		$(q-1)(q^3+1)$	2 q-1
	$10 \rightarrow 10$	βs_1		$(q-1)^2(q^2+1)$			3-→3	$\alpha T s_1$		$(q+1)(q^3-1)$	2 q-1
	14→14	πs_1		$(q^2 - 1)^2$			7↔10	$\alpha \tau s_1$		q^4-1	2 q-1
S 2	: 2→2	$\beta \pi s_1$		$q^4 - 1$			$16 \rightarrow 16$	ακφδι		q^4+1	2 q-1
	3→3	$\pi \kappa s_1$		$(q+1)^2(q^2-1)$			17→−17 awys1	$\alpha \omega \rho s_1$		$(q+1)^2(q^2+1)$	2 q-1
	5→5							S 2	$ A_1(q^3) A_1(q) $	$(q-1)(q^3-1)$	2 q-1
	7↔10	βπκει		$(q+1)^2(q^2+1)$				αs_2		$(q-1)(q^3+1)$	2 q-1
	9⇔14	8281	$ A_1(q^2) ^2$	$(q^2 - 1)^2$				$\alpha \omega s_2$		$(q+1)(q^3-1)$	2 q-1
	13→13	$\beta s_2 s_1$		q^4-1				$\alpha \tau \omega \varphi s_2$		$(q+1)(q^3+1)$	2 q-1
88	8₀: 2→2	$\beta\pi s_{*}s_{1}$		$(0^{2}+1)^{2}$				Ire.		$(n^4 - n^2 \pm 1)$	01. 1

					Condition						Condition
d _J	Ω_J	[m]	$ (M^{o})_{o} $	(S ^g) ^o	for occurrence	d,	Ω	[m]	$ ^{o}(M^{a})^{o} $	$ (S^{q})_{\sigma} $	for occurrence
		$\xi \varphi 8_2$		$(q^2 - q + 1)^2$	2 q-1		$10 \rightarrow -10$	1981		$(q-1)(q^3+1)$	
		$\pi \omega 8_2$		$(q^2+q+1)^2$	2 q-1		8₂: 2→2	l9k81		$(q-1)^2(q^2+1)$	
		S ₃	$ A_1(q^2) ^2$	$(q-1)^{4}$	2 q-1		3→3	S 2	$ {}^{2}A_{2}(q^{2}) A_{1}(q) ^{2}$ $(q+1)^{4}$	$ ^{2}$ $(q+1)^{4}$	
		αs_3		$(q-1)^2(q^2-1)$	2 q-1		$4 \rightarrow -(\epsilon_2 + \epsilon_5) \lambda_{82}$	λ82		$(q+1)^2(q^2-1)$	
		t483		$(q^2-1)^2$	2 q-1		5↔6	дк 82		$(q^2-1)^2$	
		$\alpha \varphi s_s$		$(q^2 - 1)^2$	2 q-1		8→−8	2.982 2.982		$(q+1)(q^3+1)$	
		$\tau I S_8$		$(q-1)(q^3-1)$	2 q-1		6-←6	1.9K82		q^4-1	
		$\tau \varphi f 8_3$		$q^4 - 1$	2 q-1		$10 \rightarrow -10 \ 8_{18_{2}}$	8182	$ {}^{2}A_{2}(q^{2}) $	$(q-1)^2(q^2-1)$	
		$\alpha I \pi s_3$		$q^4 - 1$	2 q-1				× A1(q ⁻)		
		ατφω83		$(q+1)^{4}$	2 q-1			λ8 <u>1</u> 82		$(q^{z}-1)^{z}$	
		ταφ83		$(q+1)^2(q^2-1)$	2 q-1			λ κ 8182		$(q+1)^{2}(q^{2}-1)$	
		γφτα8 ₃		$(q+1)(q^3+1)$	2 q-1			λ98182		$(q+1)(q^3-1)$	
		TOTES		$(a^2+1)^2$	2 q-1			λ9κ8182		$(q+1)^2(q^2+1)$	-
		828183	$ A_1(q^4) $	$(q-1)^2(q^2-1)$	2 q-1	$2A_{2}$ (;	$2A_2 (S_3 \times \mathbb{Z}_2) \wr \mathbb{Z}_2$	1	$ A_2(q) ^2$	$(q-1)^4$	
		TS2818 3		$(q-1)^2(q^2+1)$		(5, 6, 8, 10)	$\{5, 6, 8, 10\}$ $W(B_2)$	β		$(q-1)^2(q^2-1)$	
		K 828183		$(q^{2}-1)^{2}$	2 q-1	$2A_2$	$s_1:1{ ightarrow}{ ightarrow$	μβ		$(q-1)(q^3-1)$	
		Γ κ 828183		$(q+1)^{2}(q^{2}-1)$	2 q-1	{1, 2, 3, 11}	+ 2→2	βα		$(q^2 - 1)^2$	
		TT828188		$(q-1)(q^3+1)$	2 q-1		3→-3	μβα		$(q+1)(q^3-1)$	~
		<i>\$</i> 7828183		$(q+1)(q^3-1)$	2 q-1		5⇔10	μβγα		$(q^2 + q + 1)^2$	
		<i>p</i> t 828183		q^4-1	2 q-1		6↔8	81	$ A_{2}(q^{2}) $	$(q^2 - 1)^2$	
		¢ξα828183		q^4+1	2 q-1		11→11	βs_1		$(q+1)^2(q^2-1)$	(1
		φΓα828183			2 q-1			$\mu\beta s_1$		$(q+1)^2(q^2+q+1)$	+1)
$A_{2}+2A_{1}$	$S_4 imes (Z_2)^2$	7	$ A_2(q) A_1(q) ^2$	$(q-1)^4$				α81		$(q-1)^2(q^2-1)$	
(2, 3, 5, 6}	$(\mathbb{Z}_2)^2$	۲						$r_{\alpha s_1}$		$(q-1)^2(q^2-q+1)$	+1)
A_3 8	81:2↔3	<i>l</i> k		$(q^2-1)^2$				$\beta \alpha s_1$		$(q^2-1)^2$	
{8, -9, 10}	4-→4	2 .9		$(q-1)(q^3-1)$				$\beta \mu \alpha s_1$		$(q+1)(q^{3}-1)$	
	5→5	29ĸ		q^4-1				$\beta T \alpha s_1$		$(q-1)(q^3+1)$	
	99	81 -	$A_2(q) A_1(q)^2 $	$(q+1)^2(q^2-1)$				βμγα81		q^4+q+1	
	8→-8	λ8 ₁		$(q^2 - 1)^2$				S ² 22	$ {}^{2}A_{2}(q^{2}) ^{2}$	$(q+1)^4$	
	6 - + 6	λκ81		$(q-1)^2(q^2-1)$				Bes		$(n+1)^2(n^2-1)$	

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(Continued)

d _J	Ω_J	[m]	$ {}^{o}(M){}^{o} $	$ (S^g)_\sigma $	Condition for occurrence	d,	Ω_J	[m]	$ (M^g)_\sigma $	(<i>S</i> ^g) _o	Condition for occurrence
		Bµs2		$(q+1)(q^3+1)$				TKÀS		$(q^2 - 1)^2$	
		$\beta \alpha s_2^2$		$(q^2 - 1)^2$				$T\alpha\lambda s$		$(q+1)(q^3+1)$	
		$\mu\beta\alpha s_2^2$		$(q-1)(q^3+1)$				ĸľals		$q^4 - 1$	
		$\mu\beta\Gamma\alpha s_{2}^{2}$		$(q^2 - q + 1)^2$		A_4	$S_5 imes Z_2$	1	$ A_4(q) $	$(q-1)^{4}$	
		8182	$ {}^{2}A_{2}(q^{2}) A_{2}(q) $	$(q^2 - 1)^2$		{6, 7, 8, 10}	\mathbf{Z}_2	a a		$(q^2 - 1)(q - 1)^2$	
		$\beta s_1 s_2$		$q^4 - 1$		A_4	8: 1→-1	αδ		$(q^2 - 1)^2$	
		$\beta \mu s_1 s_2$		$q^{4} + q^{2} + 1$		(1, 2, 3, 4)	$2 \rightarrow -2$	αĩ		$(q-1)(q^3-1)$	
•		$\beta\mu\alpha s_1s_2$		q^4-1			3→-3	ατβ		$(q+1)(q^3-1)$	
		S 2	$ {}^{2}A_{2}(q^{4}) $	$(q^2+1)^2$			4→-4	atô		$q^4 - 1$	
	•	βs_2		$q^4 - 1$			6⇔10	ws RT		$q^4 + q^3 + q^2$	
		$\beta \mu s_2$		$q^{4} - q^{2} + 1$			OT. O	idon		+q+1	
		$\beta\mu\alpha s_2$		q^4-1	-		7↔8	8	$ {}^{2}A_{4}(q^{2}) $	$(q+1)^4$	
A_3+A_1	$\mathrm{S}_4 imes (\mathbf{Z}_2)^2$	Ţ	$ A_{s}(q) A_{1}(q) $	$(q-1)^{4}$				αs		$(q+1)^2(q^2-1)$	
{2, 5, 6, 7}	\mathbb{Z}_2	ν		$(q-1)^2(q^2-1)$				αдε		$(q^2 - 1)^2$	
	s: 1→1	T.c.		$(q^2 - 1)^2$	<u>.</u>			αĩs		$(q+1)(q^3+1)$	
{1, 3, 9, 10}	2-→2	τα		$(q-1)(q^3-1)$				αĩ βs		$(q-1)(q^3+1)$	
	$3 \rightarrow -3$	кľа		$q^4 - 1$				αĩδs		$q^4 - 1$	
	5↔7	Y		$(q-1)^2(q^2-1)$				aňBľs		$q^4 - q^3 + q^2$	
	9←9	αλ		$(q^2 - 1)^2$						-q+1	
	9-→6	TKR		$(q+1)^2(q^2-1)$		D_4	$W(F_4)$	-	$ D_4(q) $	$(q-1)^4$	
	$10 \rightarrow 10$	ζαλ		$(q+1)(q^{3}-1)$		(2, 3, 4, 5)	S.	μ		$(q-1)^2(q^2-1)$	
		Klah		$(q+1)^2(q^2+1)$		D_4	s₁: 2→2	ŋĸ		$(q^2 - 1)^2$	
	•		$ {}^{2}A_{3}(q^{2}) A_{1}(q) $	$(q+1)^2(q^2-1)$		{7, 8, -9,	3↔5	n.9		$(q-1)(q^3-1)$	
		α		$(q^2-1)^2$		10}	4-→4	nr.g		$q^4 - 1$	
		TKS		$(q-1)^2(q^2-1)$			6-↔2	ηπκλ		$(q+1)^{4}$	
		$r_{\alpha s}$		$(q-1)(q^3+1)$			8↔8	ηπκ		$(q+1)^2(q^2-1)$	
		2,2		(q-1)			10>10	ηπκθ		$(q+1)(q^3+1)$	
		N 1 3		$\times (q^3 - q^2 + q - 1)$	(I		82: 2→3→5 7×9x	7×9x		$(q^2+1)^2$	
		λs	to t sever report of the several or news, were	$(q+1)^4$			75	81	$ {}^{2}D_{4}(q^{2}) $	$(q^2 - 1)(q - 1)^2$	
		αλε		$(q+1)^2(q^2-1)$			4→4	η81		$(q^2+1)(q-1)^2$	

(Continued)

CHEVALLEY GROUPS

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$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	<i>д</i> ,	Ω,	[<i>w</i>]	$ (M^g)_\sigma $	(S ^g), e	Condition for occurrence	d _J	Ω_J	[<i>m</i>]	$ (M^g)_\sigma $	(S ^g) ₀	Condition for occurrence
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		7→-9→	y 81		$(q^2-1)^2$			7-→7	$\pi 8_{1} 8_{3}$		$(q-1)(q^2-1)$	2 q-1
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		-14-→7	9y81		$(q^2-1)(q+1)^2$			9↔14	π K8 183		$(q+1)(q^2-1)$	2 q-1
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		8-→8	In81		$(q-1)(q^3+1)$			10→10	ξπ8183		$(q+1)(q^2-1)$	2 q-1
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		10→10	$\pi \vartheta s_1$		$(q+1)(q^{3}-1)$			13→13	ξπκ8183		$(q+1)^{3}$	2 q-1
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			$\pi\eta s_1$		$q^4 - 1$	3_	A_2+3A_1	$S_8 imes (\mathbf{Z}_2)^2$	_	$ A_2(q) A_1(q) ^3$	$(q-1)^{8}$	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			TXK81		$q^{4}+1$						$(q-1)(q^{2}-1)$	
$ \begin{array}{llllllllllllllllllllllllllllllllllll$			П 7 К 8 1		$(q^2+1)(q+1)^2$					$ A_2(q) A_1(q^2) $	$(a-1)(a^{2}-1)$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			82]($(q-1)(q^{3}-1)$					$\times A_1(q) $		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			$\pi 8_2$		$(q-1)(q^{8}+1)$						$(q+1)(q^{2}-1)$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			$\pi\lambda s_2$		$(q+1)(q^3-1)$					$ A_2(q) A_1(q^3) $	q^{8} —1	
$\begin{array}{lclcrcl} & & & & & & & & & & & & & & & & & & &$			πηλ κ8 2		$(q+1)(q^3+1)$						$(q+1)(q^2+q+$	(1)
$\begin{array}{lclcrcl} y_{\mathbf{k}\mathbf{s}\mathbf{s}} & (q^2-q+1)^3 \\ z^{2}\mathbf{s}_{\mathbf{s}} & (q^2+q+1)^3 \\ Z_{\mathbf{s}} & \xi & (q^2+q+1)^3 \\ S_{\mathbf{s}} & (q^2+q+1) & 2 q^{-1} \\ S_{\mathbf{s}} & \xi & (q^2) A_1(q) ^3 & q^3+1 \\ S_{\mathbf{s}} & g^3+1 & (q^2+1) & 2 q^{-1} \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) ^3 & q^3+1 \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) ^3 & q^{2}+1 \\ S_{\mathbf{s}} & g^{2}\mathbf{s}_{\mathbf{s}} & (q^2+1)(q^2+1) \\ S_{\mathbf{s}} & g^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)(q^2-1) \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)(q^2-1) \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)(q^2-1) \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)(q^2-1) \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)(q^2-1) \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)(q^2-1) \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)(q^2-1) \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)(q^2-1) \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)^3 \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)^3 \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)^2 \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)^2 \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)^2 \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)^2 \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)^2 \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)^2 \\ S_{\mathbf{s}} & \xi^{2}\mathbf{s}_{\mathbf{s}} & A_1(q^3) A_1(q) & (q^2-1)^2 \\ S_{\mathbf{s}} & A_1(q^2) A_1(q) & A_1(q^2-1) \\ S_{\mathbf{s}} & A_1(q^2) A_1(q) & A_1(q^2-1) $			97 82		$q^{4} - q^{2} + 1$				-	$ {}^{2}A_{2}(q^{2}) A_{1}(q) ^{5}$	$(q+1)(q^2-1)$	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			YK 82		$(q^2 - q + 1)^2$						$(q+1)^{8}$	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			$z\lambda s_2$		$(q^2+q+1)^2$					$ {}^{2}A_{2}(q^{2}) A_{1}(q^{2}) $	$ (a-1)(a^2-1) $	
$\begin{array}{lclcrcl} S_4 & \xi & (q-1)(q^2-1) & 2 q-1 \\ s_1: 2 \rightarrow 2 & \xi \pi & (q+1)(q^3-1) & 2 q-1 \\ 3 \rightarrow 3 & \kappa \xi \pi & (q+1)(q^3-1) & 2 q-1 \\ 5 \rightarrow 5 & s_1 & A_1(q^2) A_1(q) ^8 & (q-1)(q^2-1) & 2 q-1 \\ 7 \rightarrow 10 & \xi s_1 & +1)(q^3-(q-1)) & 2 q-1 \\ 9 \rightarrow 14 & \pi s_1 & (q-1)(q^3+1) & 2 q-1 \\ 13 \rightarrow 13 & \xi \pi s_1 & (q-1)(q^3+1) & 2 q-1 \\ 13 \rightarrow 13 & \xi \pi s_1 & (q-1)(q^3+1) & 2 q-1 \\ 13 \rightarrow 13 & \xi \pi s_1 & (q-1)(q^3+1) & 2 q-1 \\ 3 \rightarrow 3 & \xi s_1 s_2 & A_1(q^3) A_1(q) ^2 & q^3-1 & 2 q-1 \\ 8 \rightarrow 3 & \xi s_1 s_2 & A_1(q^3) A_1(q) & (q-1)(q^3-1) & 2 q-1 \\ 9 \rightarrow 9 & \xi s_1 s_2 & A_1(q^3) A_1(q) & (q-1)(q^3+1) & 2 q-1 \\ 9 \rightarrow 9 & \xi s_1 s_2 & (q-1)(q^3+1) & 2 q-1 \\ 9 \rightarrow 9 & \xi s_1 s_2 & (q-1)(q^3+1) & 2 q-1 \\ 9 \rightarrow 9 & \xi s_1 s_2 & (q-1)(q^3+1) & 2 q-1 \\ 10 \rightarrow 10 & \pi s_1 s_2 s_3 & (q-1)(q^3+1) & 2 q-1 \\ 13 \rightarrow 14 & \xi \pi s_1 s_2 s_3 & (q-1)(q^3+1) & 2 q-1 \\ s_1 & \xi s_1 s_3 & (q-1)(q^3+1) & 2 q-1 \\ 13 \rightarrow 14 & \xi \pi s_1 s_2 s_3 & (q-1)(q^3+1) & 2 q-1 \\ s_1 & (q^3) ^2 A_1(q) & (q-1)(q^3-1) & 2 q-1 \\ s_2 & \xi s_1 s_3 & (q-1)(q^3-1) & 2 q-1 \\ 3 \rightarrow 3 & \xi s_1 s_3 & (q-1)(q^3-1) & 2 q-1 \\ \end{array}$	$5A_1$	H_1	1	$ A_1(q) ^5$	$(q-1)^{3}$	2 q-1				$\times A_1(q) $		
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	{2, 3, 5, 7,	S,	ev		$(q-1)(q^2-1)$	2 q-1					$(q+1)(q^2-1)$	
$\begin{array}{lclcrcl} & 3 \rightarrow 3 & \kappa \not \in \pi & (q+1)^3 & 2 q-1 \\ & 5 \rightarrow 5 & s_1 & A_1(q^2) A_1(q) ^8 & (q-1)(q^2-1) & 2 q-1 \\ & 7 \leftrightarrow 10 & \xi_{31} & +1)(q^2(-q_1) & 2 q-1 \\ & 9 \leftrightarrow 14 & \pi s_1 & (q-1)(q^2+1) & 2 q-1 \\ & 3 \leftrightarrow 13 & \xi_{73} & (q-1)(q^2+1) & 2 q-1 \\ & 13 \rightarrow 13 & \xi_{73} & (q-1)(q^2+1) & 2 q-1 \\ & 8_2 : 2 \rightarrow 2 & s_1 s_2 & A_1(q^3) A_1(q) ^2 & q^3-1 & 2 q-1 \\ & 8_2 : 2 \rightarrow 2 & s_1 s_2 & A_1(q^3) A_1(q) ^2 & q^3-1 & 2 q-1 \\ & 8_2 : 2 \rightarrow 2 & s_1 s_2 & A_1(q^3) A_1(q) & q^3-1 & 2 q-1 \\ & 8 \rightarrow 3 & \xi_{31} & g_{32} & (q-1)(q^2+1) & 2 q-1 \\ & 9 \rightarrow 9 & \xi_{31} s_{28} & (q-1)(q^2+1) & 2 q-1 \\ & 9 \rightarrow 9 & \xi_{31} s_{28} & (q-1)(q^2+1) & 2 q-1 \\ & 10 \rightarrow 10 & \pi_{31} s_{28} & (q-1)(q^2+1) & 2 q-1 \\ & 10 \rightarrow 10 & \pi_{31} s_{28} & (q-1)(q^2+1) & 2 q-1 \\ & 10 \rightarrow 10 & \pi_{31} s_{38} & (q-1)(q^2-1) & 2 q-1 \\ & s_3 : 2 \leftrightarrow 5 & s_1 s_3 & (q-1)(q^2-1) & 2 q-1 \\ & s_4 : 2 \leftrightarrow 5 & s_1 s_3 & (q-1)(q^2-1) & 2 q-1 \\ & 3 \rightarrow 3 & \xi_{31} & (q-1)(q^2) ^2 A_1(q) & (q-1)^3 & 2 q-1 \\ & 3 \rightarrow 3 & \xi_{31} & (q-1)(q^2) ^2 A_1(q) & (q-1)^3 & 2 q-1 \\ & 3 \rightarrow 3 & \xi_{31} & (q-1)(q^2) ^2 A_1(q) & (q-1)^3 & 2 q-1 \\ & 3 \rightarrow 3 & \xi_{31} & (q-1)(q^2) ^2 A_1(q) & (q-1)^3 & 2 q-1 \\ & 3 \rightarrow 3 & \xi_{31} & (q-1)(q^2) ^2 A_1(q) & (q-1)^3 & 2 q-1 \\ & 3 \rightarrow 3 & \xi_{31} & (q-1)(q^2) ^2 A_1(q) & (q-1)^3 & 2 q-1 \\ & 3 \rightarrow 3 & \xi_{31} & (q-1)(q^2) ^2 A_1(q) & (q-1)^3 & 2 q-1 \\ & 3 \rightarrow 3 & \xi_{31} & (q-1)(q^2) ^2 A_1(q) & (q-1)^3 & 2 q-1 \\ & 3 \rightarrow 3 & \xi_{31} & (q-1)(q^2) ^2 A_1(q) & (q-1)^3 & 2 q-1 \\ & 3 \rightarrow 3 & \xi_{31} & (q-1)(q^2) ^2 A_1(q) & (q-1)^3 & 2 q-1 \\ & 3 \rightarrow 3 & \xi_{31} &$	10}	8₁: 2→2	ξπ		$(q+1)(q^2-1)$	2 q-1				$ {}^{2}A_{2}(q^{2}) A_{1}(q^{3}) $	$q^{3}+1$:
$ \begin{array}{lcccccccccccccccccccccccccccccccccccc$	$3A_1$	3→3	κξπ		$(q+1)^{3}$						$(q-1)(q^2-q+$	([
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	{9, 13, 14}	5→5		$ A_1(q^2) A_1(q) ^3$	$(q-1)(q^2-1)$		$2A_2 + A_1$	$S_3 imes (\mathbf{Z}_2)^2$		$ A_2(q) ^2 A_1(q) $	$(q-1)^{3}$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		7↔10	ξ81		$+1)(q^{2}-(q1)$	2 q-1					$(q-1)(q^2-1)$	
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$		9⇔14	$\pi 8_1$		$(q-1)(q^2+1)$	2 q-1					$q^{8}-1$	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		13→13	ξπ81		$(q+1)(q^2+1)$	2 q-1				$ A_2(q^2) A_1(q) $	$(q-1)(q^2-1)$	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		8₂: 2→2	8182	$ A_1(q^3) A_1(q) ^2$	$q^{3}-1$	2 q-1					$(q+1)(q^2-1)$	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		3→3	ξ8182		q^3+1	2 q-1					$(q-1)(q^2-q+$. 1)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		5↔7		$ A_1(q^4) A_1(q) $	$(q-1)(q^2-1)$					$ {}^{2}A_{2}(q^{2}) ^{2} A_{1}(q) $		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		6←6	ξ81828 3		$(q-1)(q^2+1)$	2 q-1					$(q+1)(q^2-1)$	
$\begin{array}{c ccccc} [4 & \xi \pi s_1 s_2 s_3 & (q+1)(q^2+1) & 2 q-1 \\ & s_1 s_3 & A_1(q^2) ^2 A_1(q) & (q-1)^3 & 2 q-1 \\ & \xi s_1 s_3 & (q-1)(q^2-1) & 2 q-1 \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_3 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \begin{array}{c} [4 & \xi \pi s_1 & (q-1)(q^2-1) \\ \end{array} \right \left. \left. \begin{array}{c} [4 & \xi \pi s_1 $		10→10	$\pi 8_{1} 8_{2} 8_{8}$		$(q+1)(q^2-1)$	2 q-1					$q^{3}+1$	
$egin{array}{llllllllllllllllllllllllllllllllllll$		13↔14	ξπ81828 3		$(q+1)(q^2+1)$	2 q-1				$ A_2(q^2) A_1(q) $	$(q-1)(q^2-1)$	
$\xi_{3_13_3}$ $(q-1)(q^2-1)$ $2 q-1 $		8s: 2↔5	8183			2 q-1					$(q+1)(q^2-1)$	
		3 →3	ξ8183		$(q-1)(q^2-1)$	2 q-1					$(q+1)(q^2+q+$	-1)

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4J 34J	o(M)o	$ (S^g)_\sigma $	Condition for occurrence	J J	Ω_J	$ (M^{\sigma})_{\sigma} $	$ (S^q)_{\sigma} $	Condition for occurrence
$[A_3+2A_1]' W(B_2) \times Z_2$	$ A_{3}(q) A_{1}(q) ^{2}$	$(q-1)^3$ $(q-1)(q^2-1)$		$A_3 + A_2$	$W(B_2) imes Z_2$	$ A_3(q) A_2(q) $	$(q-1)^3$ $(q-1)(q^2-1)$	
	12 4 1~211 4 1~112	$(q+1)(q^2-1)$					$(q+1)(q^2-1)$	
	$ ^{-A_{3}(q^{-})} _{A_{1}(q)} ^{-} (q-1)(q^{2}-1)$	$(q-1)(q^{2}-1)$ $(q+1)(a^{2}-1)$				$ {}^{*}A_{3}(q^{*}) A_{2}(q) $	$(q-1)(q^2-1)$	
		$(q+1)^3$				$ A_{a}(a) ^{2}A_{a}(a^{2}) $	$(1-1)(a^2-1)$	
	$ A_3(q) A_1(q^2) $	$(q+1)(q^2-1)$				1/ 5/2++ 11/5/6++1	$(q+1)(q^2+1)$	
		$(q-1)(q^2+1)$				$ {}^{2}A_{3}(q^{2}) $	$(\alpha - 1)(\alpha^2 - 1)$	
	$ {}^{2}A_{3}(q^{2}) A_{1}(q^{2}) \ (q+1)(q^{2}-1)$	$(q+1)(q^2-1)$				$ imes {}^2A_2(q^2) $	(T_ b)(T_b)	
		$(q+1)(q^2+1)$					$(q+1)(q^2-1)$	
$[A_3+2A_1]'' S_4 \times (\mathbb{Z}_2)^2$	$ A_3(q) A_1(q) ^2$	$(q-1)^{3}$	2 q-1				$(q+1)^{3}$	
		$(q-1)(q^2-1)$	2 q-1	$A_4 + A_1$	$S_3 \! imes \! Z_2$	$ A_4(q) A_1(q) $	$(q-1)^{3}$	
		$(q+1)(q^2-1)$	2 q-1				$(q-1)(q^2-1)$	
		$q^{3}-1$	2 q-1			$ {}^{2}A_{4}(q^{2}) A_{1}(q) $	q^{3} —1	
		$(q+1)(q^2+1)$	2 q-1				$(q+1)^3$	
	$ {}^{2}A_{3}(q^{2}) A_{1}(q) ^{2}$	$(q+1)^{3}$	2 q-1				$(q+1)(q^2-1)$	
		$(q+1)(q^2-1)$	2 q-1				$q^{3}+1$	
		$(q-1)(q^2-1)$	2 q-1	A_5	$S_3 imes ({oldsymbol{Z}}_2)^2$	$ A_5(q) $	$(q-1)^{3}$	÷
		$q^{3}+1$	2 q-1				$(q-1)(q^2-1)$	
		$(q-1)(q^2+1)$	2 q-1				$q^{3}-1$	
	$ A_3(q) A_1(q^2) $	$(q+1)^{3}$	2 q-1				$(q-1)(q^2-1)$	
		$(q+1)(q^2-1)$	2 q-1				$(q+1)(q^2-1)$	
		$(q-1)(q^2-1)$	2 q-1				$(q+1)(q^2+q+1)$	1)
		$q^{8}+1$	2 q-1			$ {}^{2}A_{5}(q^{2}) $	$(q+1)(q^2-1)$	
		$(q-1)(q^2+1)$	2 q-1				$(q-1)(q^{2}-1)$	
	$ {}^{2}A_{3}(q^{2}) A_{1}(q^{2}) $	$(q-1)^{3}$	2 q-1				$(q-1)(q^2-q+1)$	1)
		$(q-1)(q^2-1)$	2 q-1				$(q+1)^{3}$	
		$(q+1)(q^{2}-1)$	2 q-1				$(q+1)(q^2-1)$	
		$q^{\circ}-1$	2 q-1				$q^{3}+1$	
		$(q+1)(q^2+1)$	2 q-1	$D_4 + A_1$	$W(B_{s})$	$ D_4(q) A_1(q) $	$(q-1)^{8}$	

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Δ,	Ω,	$ \langle M^g \rangle_{\sigma} $	(S ^g),	Condition for occurrence	dr.	Ω_J	°(M)	$ (S^q)_\sigma $	Condition for occurrence
			$(q-1)(q^2-1)$ $(q+1)(q^2-1)$				<u> </u>	$a^{2} (q+1)^{2}$ $q^{2}+q+1$	2 q-1 2 q-1
		$ {}^{2}D_{4}(a^{2}) A_{1}(a) $	$(q+1)^3$ $(q-1)(q^2-1)$	_ 	$2A_{2}+2A_{1}$	$W(B_2)$	$ A_2(q) ^2 A_1(q) ^2$ $ A_8(q) ^2 A_8(q^2) $	$(q-1)^2$	
			$(q-1)(q^2+1)$				$\times [A_1(q^2)]$	q"1	
			$(q+1)(q^{2}-1)$ $(q+1)(q^{2}+1)$				$ A_{2}(q^{-}) A_{1}(q^{1}) ^{-}$ $ ^{2}A_{2}(q^{4}) A_{1}(q^{2}) $	q^{2} -1 q^{2} +1	
		$ {}^{s}D_{4}(q^{s}) A_{1}(q) $	$q^{3}-1$				$ {}^{2}A_{2}(q^{2}) {}^{2} A_{1}(q) {}^{2}$	$(q+1)^2$	
			$q^{3}+1$		$3A_2$	$(S_8)^2 imes Z_2$	$ A_{2}(q) ^{8}$	$(q-1)^{2}$	$\frac{3 q-1}{2}$
$D_{\rm b}$	$S_4 imes Z_2$	$ D_{6}(q) $	$(q-1)^{3}$				·	$q^{i}-1$ $a^{2}+a+1$	3 9-1 3 1-1
			$(q-1)(q^{z}-1)$				9 <u>A_(n</u> 2) 2 <u>A_(n</u> 2)	ч тчт⊥ (n_1) ²	304-1 30+1
			$(q+1)(q^{2}-1)$				1/ KV2E7 11/ KV2E71	q^2-1	3 q+1
			$(a+1)(a^2+1)$					$q^{2}+q+1$	3 q+1
		$ ^{2}D_{b}(q^{2}) $	$(q+1)^3$				$ ^{2}A_{2}(q^{2}) ^{3}$	$(q+1)^2$	3 q+1
		-	$(q+1)(q^2-1)$					$q^{2}-1$	3 q+1
			$(q-1)(q^2-1)$					q^2-q+1	3 q+1
			$q^{3}+1$				$ A_2(q^2) A_2(q) $	$(q+1)^2$	3 q-1
			$(q-1)(q^2+1)$					q^2-1	3 q-1
A.+4A.	$S_L \times Z_c$	$ A_{s}(q) A_{1}(q) ^{4}$	$(q-1)^{2}$	2 q-1				$q^{2}-q+1$	3 q-1
Tere - 2	a	$ A_2(q) A_1(q^2) $		9 4 - 1			$ A_{2}(q^{3}) $	$(q-1)^{2}$	3 q-1
		$\times A_1(q) ^2$	T T	-				q*1	3 4-1
		$ A_{2}(q) A_{1}(q^{3}) $	$q^{2}+q+1$	2 q-1				q ² +q+1 /2 1/2	3 q-1 9 2-1
		$(\alpha^2) _{A_{\bullet}(\alpha)} _{A_{\bullet}(\alpha^2)} _{2}$	$(a-1)^{2}$	2 a-1			[72]/2H2	(I + I) 22 - 1	14-718 214-11
		$ A_{2}(a) A_{1}(a^{4}) $	0 ² -1	2 q-1				4 1 0 ² -0+1	30+1
		$ {}^{2}A_{2}(q^{2}) A_{1}(q) ^{4}$		2 q-1	A 12A	(7.)8	A ₂ (<i>a</i>) A ₂ (<i>a</i>) ³	$\frac{4}{(n-1)^2}$	219-1 210-1
		$ {}^{2}A_{2}(q^{2}) A_{1}(q^{2}) $	$q^{2}-1$	2 q-1	1		1/5/1-211/5/6-21	q ² 1	2 q-1
		$ {}^{2}A_{2}(q^{2}) A_{1}(q^{8}) $	$a^{2}-a+1$	2 a-1			$ {}^{2}A_{8}(q^{2}) A_{1}(q) ^{8}$	$(q+1)^2$	2 q-1
		$\times A_1(q) $	- - -					т _	T 517

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d_J	Ω_J	$ {}^{o}(M_{g}) $	(S ^g) _o	Condition for occurrence	d ,	Ω	°(sW)	$ (S^g)_\sigma $	Condition for occurrence
		$ A_{3}(q) A_{1}(q^{2}) imes A_{1}(q) $	$(q+1)^2$ $\alpha^2 - 1$	2 q-1	A_4+2A_1	$(Z_2)^2$	$\frac{ A_4(q) A_1(q) ^2}{ ^2A_4(q^2) A_1(q^2) }$	$(q-1)^2$ q^2-1	
		$ {}^2A_3(q^2) A_1(q^2) imes A_1(q) ^2 imes A_1(q) ^2$	$(q-1)^2$	2 q-1 2 a-1	A_4+A_2	$(\mathbf{Z}_2)^2$	$ {}^{-}A_4(q^*) A_1(q) ^{2} A_1(q^2) A_1(q^2) A_1(q^2) A_1(q^2) A_2(q) A_2(q)$	$egin{array}{c} (q+1)^{*} \ q^{2}-1 \ (q-1)^{2} \end{array}$	
$A_{3}+A_{2}+$	$A_3 + A_2 + A_1 ({f Z}_2)^2$	$\begin{array}{l} A_{3}(q) A_{2}(q) \\ \times A_{1}(q) \end{array}$	$(q-1)^2$	I I I			$ ^{2}A_{4}(q^{2}) ^{2}A_{2}(q^{2}) $	$\begin{array}{c} q^2 - 1 \\ (q + 1)^2 \\ s \\ s \\ s \end{array}$	
		$ {}^{2}A_{3}(q^{2}) {}^{2}A_{2}(q^{2}) imes A_{1}(q) imes A_{1}(q) $	$q^{2}-1$ $(q+1)^{2}$		$[A_5+A_1]'$	$({oldsymbol{Z}}_2)^2$	$ A_{\mathfrak{b}}(q) A_1(q) $	$\begin{array}{c} q^{2}-1 \\ (q-1)^{2} \\ q^{2}-1 \end{array}$	
[2A ₃]'	$W(B_2)$	$ A_3(q) ^2$	$q^{2}-1$ $(q-1)^{2}$				$ {}^{2}A_{5}(q^{2}) A_{1}(q) $	$q^{2} - 1$ (q+1) ²	
		$ {}^{2}A_{3}(q^{2}) A_{3}(q) A_{3}(q) A_{3}(q^{2}) A_{3}(q^{2}) $	q^2-1 q^2-1	<u> </u>	$[A_5+A_1]''$	$S_3 imes Z_2$	$\left A_{5}(q) ight \left A_{1}(q) ight $	$(q\!-\!1)^2$ $q^2\!-\!1$	2 q-1 2 q-1
[2A ₃]''	$(\boldsymbol{Z}_2)^2 \wr \boldsymbol{Z}_2$	$ ^{z}A_{3}(q^{z}) ^{z}$ $ ^{2}A_{3}(q^{4}) $ $ A_{3}(q) ^{2}$	$egin{array}{c} q^2 - 1 \ (q+1)^2 \ (q-1)^2 \end{array}$	2 q-1			$q \ ^2 A_5(q^2) A_1(q) $	$egin{array}{c} q^2 + q + 1 \ (q+1)^2 \ q^2 - 1 \end{array}$	2 q-1 2 q-1 2 q-1
		$\left ^{2}A_{3}(q^{2}) ight \left A_{3}(q) ight $	$egin{array}{c} q^2 - 1 \ (q+1)^2 \ q^2 - 1 \end{array}$	2 q-1 2 q-1 2 q-1	A_6	$(\boldsymbol{Z}_2)^2$		$q^2 - q + 1 \ (q - 1)^{2} \ a^{2} - 1$	2 q-1
		$ A_{s}(q^{2}) $	$\overset{\circ}{q}^2+1$ $(q-1)^2$	2 q-1 2 q-1			$\left {}^{2}A_{6}(q^{2}) ight $	$\begin{array}{c} {}^{4} & {}^{-} \\ (q+1)^{2} \\ q^{2}-1 \end{array}$	
			$q^{2}-1$ $q^{2}-1$	2 q-1 2 q-1	D_4+2A_1	$W(B_2) imes Z_2$	$ D_4(q) A_1(q) ^2$	$(q-1)^2$ a^2-1	2 q-1 2 q-1
		$ {}^{2}A_{3}(q^{2}) ^{2}$	$(q+1)^2$ $(q-1)^2$ a^2-1	2 q-1 2 q-1 2 a-1			$Q^{[D_4(q) A_1(q^2)]}$	$egin{array}{c} q^2+q+1 \\ q^2-1 \\ a^2+1 \end{array}$	219-1 219-1
		$ ^{2}A_{3}(q^{4}) $	$(q+1)^2$ q^2-1	2 q-1 2 q-1			$ {}^{2}D_{4}(q^{2}) A_{1}(q) ^{2}$	$\begin{array}{c} q^2 + 1 \\ q^2 + 1 \\ q^2 + 1 \end{array}$	219-1 219-1 219-1
			$q^{2}+1$	2 q-1			$ {}^{2}D_{4}(q^{2}) A_{1}(q^{2}) $	$(q+1)^2$	2 q-1

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d _J	Ω	$ ^{o}(_{a}M) $	$ (S^g)_\sigma $	Condition for occurrence	Ås	Ω_J	<i>0</i> (M)	(S ⁰) ₀	Condition for occurrence
			$q^{2}-1$ (<i>n</i> -1) ²	2 q-1 2 a-1	$egin{array}{c} A_8 + A_2 \ + 2A_1 \end{array}$	$(\boldsymbol{Z}_2)^2$	$egin{array}{c} A_{3}(q) A_{2}(q) \ imes A_{1}(q) ^{2} \ imes A_{1}(q) ^{2} \end{array}$	q-1	2 q-1
D_4+A_2	$S_8 imes Z_2$	$ D_4(q) A_2(q) $	$(q-1)^{2}$	r F			$ {}^{2}A_{8}(q^{2}) {}^{2}A_{2}(q^{2}) imes A_{1}(q) ^{2}$	q+1	2 q-1
		$ {}^{z}D_{4}(q^{2}) {}^{2}A_{2}(q^{2}) A_{2}(q) $					$ {}^{2}A_{3}(q^{2}) A_{2}(q) $ $\times A_{1}(q^{2}) $	q-1	2 q-1
		$ D_4(q) ^2 A_2(q^2) $	$(q+1)^2$				$egin{aligned} & A_{3}(q) ^{2}A_{2}(q^{2}) \ & imes A_{1}(q^{2}) \ & imes A_{1}(q^{2}) \end{aligned}$	q+1	2 q-1
		D4(q) A2(q) ³ D,(n ³) ² A ₆ (n ²)	$q^{2}-1$ $q^{2}-q+1$		$2A_3+A_1$	$(Z_2)^3$	$ A_{3}(q) ^{2} A_{1}(q) $	$q{-1}$	4 q-1
$D_{5}+A_{1}$	$(\mathbf{Z}_2)^2$	$ D_{b}(q) A_{1}(q) $	$(q-1)^2$					q+1	4 q-1
4	ì,						$ {}^{2}A_{3}(q^{2}) {}^{2} A_{1}(q) $	q-1	4 q+1
		$ {}^{2}D_{5}(q^{2}) A_{1}(q) $	$q^{2}-1$				$ A_{2}(\alpha^{2}) A_{2}(\alpha) $	9+1 0-1	4 9+1 4 0+1
1			$(q+1)^{2}$				1/1/10/11/ 1/80	4 - 1 1+0	4 0+1
D_6	$W(B_2)$	$ D_{6}(q) $	(q 1) ² 0 ² 1					₫-1 ₫-1	4 q-1
			4 - T (4+1) ²					q+1	4 q-1
		$ ^{2}D_{6}(q^{2}) $	q^2-1		$A_4 + A_2 + A_1$	$oldsymbol{Z}_{2}$	$egin{array}{l} A_4(q) A_2(q) \ imes A_1(q) \ imes A_1(q) \end{array}$	$q{-1}$	
E,	$S_s imes Z_s$	$ E_6(q) $	$q^{z}+1$ $(q-1)^{2}$				$ {}^{2}A_{4}(q^{2}) {}^{2}A_{2}(q^{2}) imes A_{1}(q) imes A_{1}(q) $	$q{+}1$	
ı	1		$q^{2}-1$		A_4+A_8	Z_2	$ A_4(q) A_8(q) $	q-1	
			$q^{2}+q+1$			1	$ ^{2}A_{4}(q^{2}) ^{2}A_{3}(q^{2}) $	q+1	t
		$ ^{2}E_{6}(q^{2}) $	(g+⊥)² n²-1		A_5+2A_1	Z_2	$ A_{5}(q) A_{1}(q) ^{2}$ $ ^{2}A_{-}(\alpha^{2}) A_{-}(\alpha) ^{2}$	q−1 a+1	2 q-1 2 n-1
			q^2-q+1		$A_{*}+A_{*}$	(Z ,) ²	$ A_{\rm s}(q) A_{\rm s}(q) $	4 - 1 q−1	$\frac{-3}{3 q-1 }$
$3A_2+A_1$	$S_8 imes \mathbf{Z}_2$	$ A_2(q) ^3 A_1(q) $	q-1	3 q-1		ì	-	q+1	3 q-1
		$ A_{2}(q^{2}) ^{2}A_{2}(q^{2}) \ \sub{A}_{2}(q^{2}) $	q-1	3 q+1			$ {}^{2}A_{5}(q^{2}) {}^{2}A_{2}(q^{2}) $	q-1	3 q+1
		$\sum_{a=1}^{2} A_2(q^2) a A_1(q) a $	q+1	3 q+1	-	Ŀ	A (m) A (m)	q+1 -1	3 q+1
		$ A_2(q^2) A_2(q) $	q+1	3 q-1	AtAl	2	$ ^{2}A_{6}(q^{2}) A_{1}(q) $	4 1 9+1	
		$ A_2(q^3) A_1(q) $	q-1	3 q-1	[A ₇]′	Z_{i}	$ A_r(q) $	q-1	
		$ {}^{2}A_{2}(q^{6}) A_{1}(q) $	q+1	3 q+1			$ {}^{z}A_{7}(q^{x}) $	q+1	

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(Continued)

$ (M^{\sigma})_{\sigma} = (S^{\sigma})_{\sigma} $ Condition for for occurrence	$q{-1}$	$\begin{array}{c} q+1\\ 1\end{array}$	$ {}^{2}A_{4}(q^{2}) ^{2}$ 1 $5 q-4$ $ {}^{2}A_{4}(q^{4}) $ 1 $5 q-4$		$\begin{array}{c c} A_{5}(q) A_{2}(q) \\ \times A_{1}(q) \end{array} = 1 \qquad 6 q-1 $	$ ^{2}A_{2}(q^{2}) = 1$ $ ^{2}A_{2}(q^{2}) = 1$	· -	4	q) 1 1 $3 q-1$	1	$ D_b(q) A_3(q) $ 1 4 q-1	$A_3(q^2) = 1$	1	$ E_{6}(q) A_{2}(q) $ 1 $3 q-1$		$ A_1(q) $ I $2 q-1$
Ω , ($oldsymbol{Z}_2 E_7(q) $	Z_4 $ A_4 $	² A4		Z_2 $ A_5(q) $ $\times A_1(q) $	$ ^{2}A_{6}(q^{2})$	$\mathbf{Z}_{\mathbf{s}}$ $ A_{\tau}(n) $		Z_2 $ A_8(q) $	$ ^{2}A_{8}(q^{2}) $	Z_2 $ D_b(q) $	$ {}^{2}D_{5}(q^{2})$		\mathbf{Z}_2 $ E_6(q) $	-	$\frac{1}{1}$
e dr	E,	2A4		4-4	A_1 + A_1		A_7+A_1		A_8		D_b+A_3	ſ	μ ^π	$L_6 + A_2$	R_+ 4	
Condition for occurrence	2 q-1 9 2-1	2 q-1	2 q-1 2 q-1	2 q-1	2 q-1	2 0-1	2 q-1	2 q-1	2 q-1			I	T-b12			
$ (S^g)_{\sigma} $	q-1 a+1	4 - 1 9 - 1	q+1 q-1	q−1 2 ± 1	4+1 0+1				q+1	q-1	q+1	4 T	4 - 1 0 - 1	a+1	a−1	- n+1
$ \langle M^{\sigma} \rangle_{\sigma} $	$ A_{7}(q) $	$ {}^{2}A_{7}(q^{2}) $	$ D_4(q) A_3(q) $	$ {}^{2}D_{4}(q^{2}) {}^{2}A_{3}(q^{2}) $	$ D_4(q) ^2 A_3(q^2) $	$ D_b(q) A_1(q) ^2$	$ {}^{2}D_{5}(q^{2}) A_{1}(q^{2}) $	$[{}^{2}D_{5}(q^{2}) A_{1}(q) ^{2}$	$ D_{b}(q) A_{1}(q^{2}) $	$ D_5(q) A_2(q) $	$ {}^{\pm}D_{5}(q^{\pm}) {}^{\pm}A_{2}(q^{\pm}) $	////	$ E_{a}(a) A_{i}(a) $	$ ^{2}E_{6}(q^{2}) A_{1}(q) $	$ D_r(q) $	$ {}^{2}D_{r}(a^{2}) $
Ω_J	$(\boldsymbol{Z}_2)^2$		$(\boldsymbol{Z}_2)^2$			$(Z_2)^2$			Ŀ	Z ²	'n	4	\mathbf{Z}_2		\mathbf{Z}_{2}	
$A_J \qquad \Omega_J$	$[A_7]''$		$D_4 + A_8 \qquad (Z_2)^2$			D_b+2A_1 $(\mathbf{Z}_2)^2$				$ u_5 \top A_2 $	$D_{6}+A_{1}$		$E_6 + A_1$		D_{7}	

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the tori which are obtained by twisting the maximal split torus T_0 by the elements of Ω_{ϕ} , where here Ω_{ϕ} is the whole Weyl group W. The conjugacy classes of W are known [3], therefore, the reader can have a complete list of the tori $(T_w)_{\sigma}$, $w \in W$, and their orders from the material of [3]. Thus we have not included in our tables the cases $J = \emptyset$.

We note that from the above tables one can obtain the degrees of Deligne-Lusztig [7] representations of the groups E_{τ} and E_{s} of adjoint type. In fact, these degrees are the p'-parts of $|G_{\sigma}|/|C_{G_{\sigma}}(x)|$, where G is a simply connected group E_{τ} or E_{s} and $C_{G_{\sigma}}(x)$ are the centralizers in G_{σ} of semisimple elements in G_{σ} .

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