Madame Pompadour, alleged mistress of Louis XV, was Poisson. All the Poissons at the meeting claimed some descent from her but would argue about her relationship with the king.

I cannot let Good's comments on the pronunciation of Poisson pass without comment. While spoken English at the movies, theater, radio, and TV might indicate that Brooklynese for "person" would lead to the pronunciation "Poyson," he might find that pronunciation more common in his region of the country. In fact, I recall hearing more than one prominent U.S. Senator from the South producing those sounds.

Comment

C. C. Heyde

Good's matter of fact treatment of some of his topics suppresses the human interest and controversy surrounding much of Poisson's work. Poisson was ambitious and competitive and often provoked violent debate. Accounts of the controversy over his work on probability and the law and over the law of large numbers appear in Heyde and Seneta (1977), Chapters 2 and 3, respectively.

The curious title of Poisson's major work in probability and statistics (1837), not all of which is quoted by Good, bears some explanation. Poisson's motivation is evident from the title although from a modern point of view the work contains so much important preliminary material of a purely theoretical kind that it should be regarded as a treatise on probability with applications to the legal system.

The background to Poisson's investigation of the jury law is interesting. This was a subject which had already received considerable attention, dating back to the work of Condorcet (1785), which had apparently been stimulated by the administrator Turgot, and flowing from the concern for justice which was widespread at the time. Laplace had been a major contributor and results in the first supplement, added in 1816, to his treatise on probability (1812) became a source of argument after the jury system was changed in 1831. Ammunition for this argument was readily available since the publication by the government of annual statistical reports, *Comptes généraux de l'administration de la justice criminelle*, had commenced in 1825.

In the period from 1825–1830, criminal trials were heard before a jury of 12 members and a majority of at least 7 to 5 was required for conviction. This was changed to 8 to 4 in 1831 and these arrangements were strongly, but ultimately unsuccessfully, defended in the Chamber of Deputies in 1835 by the famous astronomer Arago, then Deputy from Pyrénées-Orientales. In this argument, Arago drew heavily on a formula of Laplace in which (under questionable conditions) the probability of a correct verdict is given as a function of the majority obtained. Nevertheless, the law of 9 September 1835 returned the required majority to at least 7 to 5.

Laplace had been Poisson's teacher and patron but the use of his formula was sharply criticized by Poisson (1835) in a paper read to the Academie des Sciences on 14 December 1835, and it was here that he first applied his law of large numbers as described in Section 4 of Good's paper. This did not settle the matter, however, and Poisson in turn came in for serious criticism from Poinsot, Dupin, Navier, and others as the argument continued. Some of this is reported in the *Comptes Rendus* following the papers Poisson (1836a, 1836b) (read, respectively, on 11 April and 18 April 1836) which, together with the 1835 paper cited above were the basis of his work of 1837. A detailed account of the changes to the jury law and the contributions of probability to the debate, including the work of Poisson, can be found in Cournot (1984), Chapter XVI and editorial notes thereon.

The first paragraph of Section 6 of Good's paper, which concerns the Poisson distribution, is not entirely accurate. Poisson limits of the first few binomial probabilities were established in the context of a particular gambling problem by de Moivre (1967, Problem V, p. 45) and he states that "... the law of continuation of these equations is manifest." For further background on the history of the Poisson approximation to the binomial see Seneta (1983). Finally, a useful biographical account of Poisson appears in Costabel (1978).

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Comment

Nozer D. Singpurwalla

In the introduction to this admirably written paper, Professor Good states that his focus is on influences that Poisson's work has had on statistics and probability "interpreted in a broad sense." The author then highlights three topics: (i) the law of large numbers and the distinction between kinds of probability, (ii) the Poisson summation formula, and (iii) the Poisson distribution. In what follows I shall direct my comments to (i), mainly because this topic is of current interest to me. However, before doing this, I would like to give the following additional information pertaining to Poisson's work on statistics and probability, which appears to have escaped Professor Good's mention, but which may be of historical interest to many of the readers of this journal.

According to Sheynin (1981) it is Poisson who introduced the concept of a *random quantity* and a *cumulative distribution function.* Poisson's influence on Chebychev, the originator of the Russian school of probability (whose most prominent representatives are Markov, Voroni, Lyapunov, Steklov, and Kolmogorov), is beyond any question. It is also of interest to note that Poisson qualitatively connected his law of large numbers with the existence of a stable mean interval between molecules (Gillispie, 1963, p. 438). If the above is true then is it possible that it was Poisson who paved the way for Einstein and von Smoluchowski (see Maistrov, 1974, p. 225) to develop in 1905, probabilistic arguments for a theory of Brownian motion? If such be the case then a proper eponymy for Brownian motion could be *Poissonian-Brownian motion.* After all, it was only 1827, 17 years after Poisson (as Editor for *Mathematics of the Bulletin of the Philomatic Society*) was involved in probability theory (see Bru, 1981), that the English botanist Robert Brown observed the phenomenon named after him. Another noteworthy aspect of Poisson's interest in statistics and probability, and one which appears to have escaped Professor Good's notice (also see Good, 1983a, Part V), is his use of the calculus of probability to clarify Hume's notion of *causality* (see p. 163 of Poisson, 1837). Incidentally, Bru (1981) regards the material on page 163 of Poisson (1837) as a "strengthening of the 'philosophical probability' of the theory of chances and its applications to nature." By "philosophical probability" I take it to mean *logical probability* or *credibility*, and if this be so, then Bru's view would lend support to Professor Good's interpretation that Poisson's concept of probability was that of logical probability.

In Section 2 of the paper under discussion, Professor Good states that "The empirical evidence that gives some support for the existence of logical probabilities, or at least multipersonal probabilities, is that, for many pairs (A, B) the judgments of $P(A \mid B)$ by different people do not differ very much." Recognizing that the existence of logical probabilities is controversial, I would all the same, like to add a supplement to the above statement. With the recent work by DeGroot (1974) on reaching a consensus, and by Lindley et al. (1979) on the reconciliation of probability judgments, it appears to me, by analogy with Good (1983a, p. 197), that *insofar as logical probabilities can be measured, they can be done only in terms of subjective probability.*

Professor Good's remark about quantum mechanics and Einstein's statement that "God does not play dice" prompted me to do some searching about the physicists' view of probability, and what may have prompted Einstein to make the above, now famous, comment. For this I found the book by Pagels (1983) most informative and fascinating to read. My understanding of the material there, particularly that in

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