# Comment

## John D. Spurrier

Professors Hedayat, Jacroux and Majumdar are to be congratulated for the excellent job they have done of summarizing the vast amount of research that has been done over the last several years in the problem of comparing several experimental treatments with a control or a standard treatment. They have, in my opinion, been very thorough and even-handed in their treatment of the several different approaches to the problem. I hope that one major contribution of their article is to make applied researchers aware of the need for special designs in the treatment versus control problem. As they discuss, designs which are optimal for comparing a set of experimental treatments are not usually optimal in the treatment versus control problem. One needs to observe the control more often than the individual experimental treatments, because the control is involved in all contrasts of interest and the individual experimental treatments are involved in only one contrast.

I have chosen to comment on two aspects of the problem. These comments reflect the facts that I was attracted to the problem from a background in simultaneous inference rather than a background in classical experimental design and that I have a continuing interest in nonparametric statistics.

## CHOICE OF OPTIMALITY CRITERION

The authors point out that the choice of an "optimal" design depends upon the optimality criterion that is used. Thus, in order to choose an optimal design, the applied researcher must first select an optimality criterion. The authors are correct that the majority of the work in this area has been done using the A-optimality and MV-optimality criteria. The next most popular optimality criterion is that of minimizing the widths (or maximizing the coverage probability) of the simultaneous confidence intervals for the $v$ differences between the experimental treatment means and the control mean.

It is my experience in treatments versus control experiments that the applied researcher is generally either wishing to give simultaneous confidence bounds on the treatment versus control differences or attempting to declare treatment means to be signifi-

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cantly different from (better than) the control mean. Thus, from my viewpoint, the natural optimality criterion for the problem is that of minimizing the sum of the expected widths of the simultaneous confidence intervals. With this criterion one is looking for a design $d$ which minimizes the product of a design dependent, multivariate $t$ probability point and the sum of the standard errors of the $v$ estimators of the treatment versus control differences. In the spirit of MV-optimality, it is also possible to define a criterion of minimizing the width of the widest confidence interval.

There are probably three reasons why these criteria have been less popular in the literature than A-optimality and MV-optimality. First, they are notions of simultaneous inference and not of classical experimental design. Second, because there is no closed form for the multivariate $t$ probability point, it is more difficult to establish that a design is "optimal." In addition to the articles that the authors referenced, Dunnett (1955, 1964) discussed "optimal" designs using these criteria for the zero-way elimination of heterogeneity under the assumption of equal sample sizes for the experimental treatments. Nizam (1987) has established some results for the case of $v = 2$ experimental treatments using these criteria for the zero-way elimination of heterogeneity with possibly unequal sample sizes for the experimental treatments. Third, because the probability point is a function of the simultaneous confidence level $(1 - \alpha)100\%$, it is possible that for different values of $\alpha$ there will be different "optimal" designs.

Although the applied researcher may be confused by the fact that different optimality criteria produce different "optimal" designs, it is comforting to note that in my experience the "optimal" design under one criterion is generally close to optimal under the other criteria.

## DESIGNS FOR NONPARAMETRIC ANALYSIS

Almost all of the work in this area of treatment versus control experiments has been done with an implicit or explicit assumption of normality. Clearly, the assumption of normality is not appropriate in all treatment versus control experiments. The notable exceptions to the assumption of normality are discussed in Steel (1959a, 1959b) which present nonparametric analyses of the treatment versus control

problem for the zero-way and one-way eliminations of heterogeneity. He does not discuss the problem of optimal design. Spurrier (1988) presents a method of finding asymptotic "optimal" designs for the one-way elimination of heterogeneity problem based on a variation of asymptotic relative efficiency.

It is interesting to note that at times one gets different "optimal" designs for normal theory and nonparametric analyses. This fact suggests that the question of the normality assumption needs to be addressed at the design stage. I feel that there is much more research to be done in the nonparametric approach to these problems.

## ADDITIONAL REFERENCES

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# Comment

## R. J. Owen

I would like to commend the authors for this broad and valuable review of the literature on optimal designs for comparing treatments with controls.

My first comments refer to the case where there is more than one control. An important situation where this happens is when an experimental treatment is tested against a placebo, the current commercial product and some of the competitors products. In such cases, interest in the different controls may not be the same and it would be useful to have results which take account of this using suitable asymmetric design criteria.

The second point I would like to make concerns the use of prior information. Although those who do not fully accept the Bayesian position often feel uncomfortable with using prior information in the analysis of an experiment, they are less inhibited in its use in choosing a design. Indeed experimenters readily accept that effort should be concentrated where the uncertainty is greatest. The material outlined in Section 7 therefore has potential appeal outside strictly Bayesian circles. In comparing treatments with controls our experiment would often be part of a sequence of several similar experiments. In such cases the information from previous experiments should influence the current design.

When experiments are part of a sequence the choice of design of a single experiment considered in isolation, or even considered posterior to the preceding

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experiments, is no longer necessarily the appropriate question. Rather it may be better to consider the global design of the whole sequence of experiments. Of course the potential information of one experiment influences the designs of the other experiments. This may be illustrated in the context of continuous designs, one control and one-way elimination of heterogeneity as follows.

Assume the model formulation of Theorem 7.1 and consider two distinct cases: (a) each treatment may appear in each experiment and (b) different experiments can only have the control in common.

In each case let the prior distribution for $(\theta', \gamma')$ take the form stated in Section 7.1 immediately after the error distribution. In case (a) assume that all treatment contrasts with the control $(\theta_1, \theta_2, \cdots \theta_v)$ are exchangeable with respect to their *a priori* dispersion (as in Theorem 7.1). In case (b) this type of exchangeability is also assumed, but now only for the contrasts within an experiment whereas between experiments *a priori* independence of the nonoverlapping sets of treatment contrasts is assumed. This formulation could be appropriate when each experiment is believed to be dealing with a different type of treatment. In both cases consider the criterion of Bayes A-optimality for the sequence of experiments considered as a whole.

In case (a) the overall optimal allocation is given by Theorem 7.1 and its associated algorithm. This still leaves some freedom of choice for individual experiments and it is clear that each experiment may have a design which is exchangeable with respect to the noncontrol treatments. Moreover the first experiment, when considered in isolation, may be taken to be