CERTAIN LOGICAL IDEAS OF V. A. SMIRNOV AND MODERN LOGIC*

A. S. KARPENKO

Without any intention to depreciate the contributions of Vladimir Alexandrovich Smirnov to the methodology and philosophy of science (see [Anisov *1977*]), I would like to stress, that V. A. Smirnov was in the first place a logician, and a logician of the highest class. Besides, what is very uncommon for the logical community, he remained a *working* logician until the end of his life. The latter meant that he was always taking keen interest in the latest achievements in the field of modern symbolic logic and, what is most important, was striving to obtain new technical results in his selected areas of logic.

Without going into details of logical techniques, I will try to reproduce the atmosphere of some ideas of V .A. Smirnov, and in the first place the position that these ideas have occupied and now occupy in the contemporary world of logic.

There are only three works of V. A. Smirnov, that I will focus on in this study:

I. I. *Logical concepts of N. A.* ([Smirnov *1962*]);
II. *Formal Deduction and Logical Calculi* ([Smirnov *1972*]);
III. *Logical methods of analysis of scientific knowledge* ([Smirnov *1987*]).

The main ideas of these three works, spreading out in concentric circles and overlaying each other, shall lead us to the logical universe of V. A. Smirnov; ideas and developments that have influenced, still influence, and in many ways have even predicted, some trends in the development of present-day logic.

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I.

The Russian logician N. A. Vasil'ev (1880–1940) was written about before (see, e.g., V. A. Smirnov's "Preface" and the bibliography in [Vasil'ev *1989*]) but this work of V. A. Smirnov [Smirnov *1962*], even among all his other writings, happened to be the most lucky one for the simple reason that it was followed by a thorough review in English ([Comey *1965*]), and not somewhere other than in the leading international journal on logic.

The ideas of the possibility of construction of non-Aristotelian logics emanated in the beginning of this century in the work by L. Brouwer on the unreliability of the law of excluded middle [Brouwer *1908*], and simultaneously in 1910 in works of J. Lukasiewicz ([Lukasiewicz *1910*]) and N. A. Vasil'ev ([Vasil'ev *1910*]), who independently from each other had come to the conclusion that the revision of the basic laws of Aristotelian logic (especially such as the law of non-contradiction: *the same judgment can not be both true and false*, and the law of excluded middle: *of two contradicting judgments either the first one or the second must be true*) should arrive at construction of a non-Aristotelian logic, with both scholars referring to the example of the creation of non-Euclidean geometry. But N. A. Vasil'ev's ideas were much more vivid and broad (see also [Vasil'ev *1912*; *1912-1913*]), and it was their profundity that V. A. Smirnov noted in his article.

The review [Comey *1965*] of the mentioned article on Vasil'ev by V. A. Smirnov [Smirnov *1962*] attracted attention at once and, on the one hand, N. A. Vasil'ev already appeared as one of the forerunners of many-valued logic in the monograph by N. Rescher [Rescher *1969*] on many-valued logic, while on the other hand N. A. Vasil'ev became one of the forerunners of paraconsistent logic (see [Arruda *1983*; *1984*] (in such logics the principle of non-contradiction does not always work). At the next International Congress an invited report about N. A. Vasil'ev was delivered by V. A. Smirnov ([Smirnov *1987a*]; see also [Smirnov *1989*]). Finally in 1989 V. A. Smirnov arranged and published the repeatedly mentioned here selected works of N. A. Vasil'ev [Vasil'ev *1989*] with a large supplement, including also his own article about N. A. Vasil'ev [Smirnov *1989a*].

The logical ideas of N. A. Vasil'ev had a great influence on V. A. Smirnov, and until his last days Smirnov developed these ideas in various directions. Thus appeared the idea of *combined logics*, where operations on situations were introduced that played the role of internal logical signs, while ordinary logical signs acted as external logical signs, and this part of

logic was an abstract logic. From V. A. Smirnov's point of view a dual approach to non-classical logics is acceptable. Either the abstract part of logic (logic of truth) is not varied, and the internal, ontological part may differ from classical (for example, by changing of ontological conditions), or its ontological part remains unchanged, and the abstract part is modified (cognitive conditions revised). A combination of these two approaches is also possible, where non-classicality appears through the revising of both the ontological and the cognitive conditions (see [Smirnov *1987a*; *1985*; *1989b*]). It should be mentioned that the idea of the dividing of logical operations within one system into internal (object-language) and external (meta-language) is very productive and came independently to different logicians. The work by D. A. Bochvar [Bochvar*1938*] should be specially mentioned here in which the first three-valued logic of nonsense is constructed for the decision of some paradoxes of set theory. In their turn, the ideas of D. A. Bochvar were developed by V. K. Finn and his followers, which led to unique and efficient methods of axiomatization of different classes of finitely valued predicate logics (see, e.g. [Anshakov & Rychkov *1984*]). However, V. A. Smirnov's approach is notable for its exceptional breadth.

Another idea of V. A. Smirnov, namely the idea of *multidimensional logics*, goes back to the division of logical principles into the two levels suggested by N. A. Vasil'ev: internal and external, abstract and empirical. The first level depends on our cognitive position, it does not vary — it is the logic of falsehood and truth. On this level the principle of non-contradiction and the principle of excluded middle are true. The second level depends on ontological assumptions about the knowable world, where experience in the "one-dimensional" world gives only positive atomic assertions, and negative assertions are not atomic, but a result of inference. V. A. Smirnov investigated the two-dimensional case using the example of the double algebras of Bauer [Smirnov *1993*]. Initially V. A. Smirnov suggested the axiomatics of N-dimensional logics in the form of syllogistic [Smirnov *1987b*]. Later he suggested the construction of logic of N-dimensions in form of the algebra of classes [Smirnov *1991*; *1993b*], intending to compare it later with N-dimensional logics in form of syllogistic.

The main idea of multidimensional logics is that experience gives us atomic assertions "of many types", which in its turn leads us to "multidimensional worlds". Such worlds have a logic of their own. It may be assume, that V. A. Smirnov was close to the idea of generalization of logical semantics of so-called "possible worlds". Particularly interesting cases start to appear in modern works. For example, according to A. N.

Prior [Prior *1968*], in each possible world there is a three-valued logic of Lukasiewicz , and this determines the semantics for the "logic of contingent existence". R. Routley [Routley *1984*] suggested semantics for relevant and paraconsistent logics, where in each possible world it is not the Boolean algebra, but the De Morgan algebra, that works; V. L. Vasyukov [*1993*] introduced ternary relation for worlds structured with Chang's *MV*-algebras of special form; thus the exact model for *discrete* infinitely-valued logic of Lukasiewicz was constructed, etc.

Unfortunately, V. A. Smirnov had no time to complete his numerous ideas on multidimensional logics.

II.

The book by V. A. Smirnov, *Formal Deduction and Logical Calculi* [*1972*], which is his doctorate thesis, is remarkably rich in completely new ideas and definitely is his intellectual apogée. Ideas presented and developed in this book were in many respects ahead of their time, and what is more important, they are being intensively developed these days in the international logical literature. I would dwell upon only two items that deserve special attention with respect tothe modern development of logic. But first I have to mention that the book not only was not translated into English, but there were even no reviews of it in any international journal, so that this brilliant work of V. A. Smirnov remains unfamiliar to foreign readers.

The book *Formal Deduction and Logical Calculi* marked the beginning of studies of logical systems without contraction (Chapter 5). The principle of contraction allows us to avoid repetition of the same formula, and this attribute of the logical system appears to be associated with the decision problem of the calculus itself, i.e., logicians are naturally interested in the fact that that for each correctly constructed formula of a particular calculus the problem is decided of whether this formula is a theorem or not.

V. A. Smirnov constructed such a sequential calculus, the result of extending of which by adding two structural rules of contraction (from the left-hand side and from the right-hand side) was a sequential version of the classical logic of predicates. It was proven that the propositional part of the calculus was equal to the propositional part of classical logic and the decision problem was decidable for it. There were also other results obtained with respect to this calculus.

It should be remarked in all fairness that at the same time and independently of V. A. Smirnov there appeared the brief theses of V. N. Grishin [*1972*], whose attention was attracted by the works on applications of the many-valued logics of Lukasiewicz (it should be mentioned that Lukasiewicz's logic was chronlogically first where the principle of contraction existed) to the set theory. It was the works by V. N. Grishin [Grishin *1974*; *1976*] that became available to foreign experts and attracted attention.

In 1985 a fundamental work was published by Japanese scholars ([Ono & Komori *1985*]) on logics without contractions, followed by quite a number of pure logical works in this field, and then the famous work by J. Girard [*1987*] which marked a separate direction in the application of logic without contraction to computer sciences. But no foreign papers had any references to works of V. A. Smirnov (see especially [Kiriyama & Ono *1991*]).

Another idea of V. A. Smirnov that is presented and developed in the book, in my opinion, is his main scientific achievement. First, V. A. Smirnov had constructed a predicate logic system, which he called *absolute*, and which was the foundation for the entire hierarchy of logical systems. The absolute system is a system of relevant logic (see [Popov & Dolgova]), and its implicative fragment is equal to the "weak positive implication" of Church [Church *1951*]. So the implicative fragment of the relevant logic **R** was discovered independently of A. Church. (V. A. Smirnov once told the author of this article that when he had been in his post-graduate course he had to get *special* permission to borrow any foreign literature from the library. No wonder that most of western scientific works were unavailable to him).

Beginning in late '80s a number of works appeared where various hierarchies of logical systems were constructed (see Došen *1988*; *1989*], [Wansing *1990*], and especially [Ono *1990*]). Here the full calculus of syntactic categories of Lambek [Lambek *1958*] stands as an initial logical system. But the main purpose of V. A. Smirnov was to formulate *a classification of logical calculi*. The book presents a classification of singular sequential calculi which is in its turn based on the classification of rules for the inclusion and removal of logical signs from the left and from the right. Such an approach to classification I would call *external*. Another approach is suggested, an internal one, which is based upon the logical connective of implication, "if . . . , then . . .", that is very natural for logical calculi, and then the question is brought up of classification of implicative logics, i.e., such logics in which the only logical sign is the

implication. For this approach, two methods of classification are clearly pointed out:

1) as formal deductions differ in their structure, correspondingly the theorem of deduction takes different forms. The latter allows to classify implicative logics according to which form of the theorem of deduction is present;
2) classification may be based on structural rules depending on correspondence between these rules and implicative formulas.

The subject of the classification of implicative logics was developed by V. A. Smirnov in another paper ([Smirnov *1979*]), where a serious problem is touched, namely that both methods of classification do not cover classical logic. In the first method, the deduction theorem, which is true for intuitionist logic, is also true for classical, and in this case does not distinguish one from the other. In the second method there is no such structural rule that could be responsible for a transition from intuitionist implication to classical implication. In Hilbertian calculi the transition from intuitionist implication to classical implication is usually done by adding Peirce's law, but there is no structural rule corresponding to this principle.

There may be a quite different approach to the classification of implicative logics, employing the attributes of basic (initial) *combinators* **I, B, C, W, K and S**, first introduced by M. Shönfinkel [*1924*], and then by H. Curry (see [Curry & Feys *1958*]). It turned out that between the combinators and the implicative formulae there was a one-to-one correspondence. Based on this correspondence (which is also called the *Curry-Howard isomorphism*) implicative logics may be classified using combinators and *visa versa* (see [Gabbay & de Queiros *1992*]).

However, this classification as well as V. A. Smirnov's classification does not cover classical implicative logic, because there is no such combinator that would correspond to Peirce's law and, in general, to any non-intuitionist implicative formula. For that reason in the reviewed work a **P** "combinator" is constructed in a rather sophisticated way, that would correspond to Peirce's law.

So we face the following initial problem (let us call it *V. A. Smirnov's problem*): to find a common basis for classification of implicative logics that would also include the classical implication.

The solution of this problem was suggested by the author of this article (see [Karpenko *1993*; *1993a*; *1997*]) and is based upon classification of

independent axiomatics of implicative logics by means of a finite lattice. As a result a picture of the relationship between different non-classical logics is obtained, the natural ways of extension of calculi for classical logic are discovered, and many other problems are raised and resolved.

Certainly, any *universal* classification of logics is impossible, because the world of logic is too diverse and even continual in its essence. But still the construction of various hierarchies of related logical systems and the classification of particular classes of calculi attracts the growing attention of specialists.

III.

The next book by V. A. Smirnov, that is *Logical Methods of Analysis of Scientific Knowledge* [Smirnov *1987*], was a long-suffering one. It appeared with a long delay following a hard struggle (late '70s and first half of '80s) in the Sector of Logic in the Institute of Philosophy of the Academy of Science. Though the leading role of V. A. Smirnov as a logician was indisputable, the administration of the Institute in those times supported the opposing party.

Again I would like to dwell on just two subjects from the book, namely the results in the field of modal-tense logics and the comparison of theories. The book (Chapter 5, §2) sums up the results of work on modal-tense logics. The first paper was published in 1978 [Smirnov *1978*], and at the same time independently (how often this happens in the history of science) a number of works on the same subject by J. Burgess [*1978*] appeared. Initial ideas of logics with modal-tense operators as common logical operations (like "it possibly will be that . . .") were first expressed by A. N. Prior (see especially [Prior *1967*]). And again in this connection he also introduced tense structures with linear time to past and branching to future.

A. N. Prior proceeded from purely philosophical problems, and it was in the mentioned work of V. A. Smirnov (*Logical Methods of Analysis of Scientific Knowledge*) where a very interesting solution was suggested for the well-known Aristotelian problem of the sea battle[1] by introducing metric modal-tense operators (see also [Smirnov *1984*]). Such an approach

[1] For more details on the fatalistic argument of Aristotle and logical reconstructions of this argument, which led also to appearance of modal-tense logics, see [Karpenko *1990*].

does not require the introduction of an intermediate truth value as was done by J. Lukasiewicz.

Another important idea expressed here by V. A. Smirnov is a new understanding of *conjunction* between past and future. In ordinary tense logics between past and future there is a mirror symmetry, or as A. N. Prior [*1958*] suggested, operators of the future may be three-valued, and in this way some fatalistic assertions were disproved. V. A. Smirnov (see [Smirnov *1983*]) suggested a principle according to which *what was realized, was possible in the past, but not necessarily in the indefinitely remote past.* Naturally, questions at once arose on the embedding of known modal logics into new temporal systems, the solution to which V. A. Smirnov always paid attention.

Studies of modal-tense logics then became more and more technical because of the need to resolve the problems of completeness and decidability of logical systems for which tree-like structures were models[2]. However, the contribution of V. A. Smirnov to philosophic logic is unquestioned. The problem of comparison of various theories, primarily axiomatic theories, occupies a considerable place in the book. Actually V. A. Smirnov was interested in this problem over the entire period of his mature scientific activity. As a matter of fact this subject continues his studies on *definability*, particularly the definability of descriptive terms. The results obtained in this field he reported (together with V. N. Sadovskii) to the Fifth International Congress on Logic, Methodology and Philosophy of Science in 1975 (see [Sadovskii & Smirnov *1977*]).

Several works were written on logical relations between theories (see [Smirnov *1986*]), and to show what nice results may be obtained here I would give an impressive example in the field of comparing of algebraic theories.

It is known that the theory of groups initially emanated as a theory of finite groups of substitutions (C. Jordan, 1870). But very soon it was realized that it had nothing to do with substitutions, and the main thing was the study of attributes of binary operations without assumption on finiteness of the set of elements and without any assumptions on the nature of the group elements. This approach was first formulated as a separate area in mathematics in 1916 when the book by O. Y. Schmidt, *The Abstract Theory of Groups*, was published. At the same time a three-valued logic of Lukasiewicz started to form as a result of "the struggle to liberate the human spirit" (see [Lukasiewicz *1918*]). In 1929 this logic was generalized

[2] Starting from 1985 a number of works by A. Zanardo appear (see [Zanardo *1985*]); see also [Gurevich & Shelah *1985*].

to the infinitely-valued case (see [Lukasiewicz & Tarski *1930*]), and in the middle of the century the algebraization of Lukasiewicz's infinitely-valued logics occured in the form of Chang's *MV*-algebras ([Chang *1958*]), i.e., as it was in the case of group theory, so there was a complete abstraction from the nature of elements. At the same time the theory of groups was enriched with a lattice order and started its vigorous development as a separate field of mathematics in form of the theory of lattice-ordered groups (see [Kopytov *1984*]).

Finally in 1986 a fundamental work by M. Mundici was published ([Mundici *1986*]), proving the equivalence between a number of algebraic theories that emerged on different bases at different times, and Chang's *MV*-algebras, including the proof of the equivalence of lattice-ordered groups (with certain limitations) to *MV*-algebras (see also Nola & Lettieri *1994*]).

There are other interesting examples of equivalence of some very dissimilar theories, but all of them are special cases, and V. A. Smirnov's approach to the problem of comparison of theories was much more comprehensive, i.e., he worked on *the theory* of comparing of theories. He introduced the notions of the inessential extension of theory, and the translatable extension, and employed them to analyze logical relations between theories, formulated in different languages and based on different logics. He examined a whole range of different types of relations between theories — embedding operations, enclosability of one theory into another, recursive equivalency, relative equivalency — and proved a number of theorems describing their attributes. Later on V. A. Smirnov repeatedly used these methods in his studies of relations between different theories. One of his last results was the proof of the equivalence of Lesniewski's ontology and Ockhamian syllogistics ([Smirnov *1993a*; *1993c*]).

Certainly, here we have not mentioned all of the logical ideas of V. A. Smirnov (see [Karpenko *1997a*]) but only those, as it was stressed above, that are of special interest for the modern world of logic. Maybe something was missed, but I would take the liberty to remark that having worked with V. A. Smirnov for nearly a quarter of a century (first as his student, post-graduate student, and then as a colleague in the Sector), that the main credit of my Teacher to logic was not his achievements, but that he managed to create a fantastic atmosphere of friendship of logicians not only in this country but also with logicians from other countries. This atmosphere encouraged work, exchange of ideas at numerous conferences and getting new results.

His numerous students now are all over the world, and they gratefully

remember and talk about Vladimir Alexandrovich Smirnov. And when the time comes for personal recollections of his students, then out would come some outstanding features of his character as not only a logician but as a *personality*.

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