GÖDEL'S UNPUBLISHED MANUSCRIPTS, 1930 – 1970: THE OFFICIAL EDITION*

Review of Kurt Gödel, *Collected Works. Volume III. Unpublished essays and lectures.* Editor-in-chief Solomon Feferman. New York and Oxford: Oxford University Press, 1995. xvii + 532 pp.

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I

This book is the third of the important series that the Association for Symbolic Logic is devoting to critically editing Gödel's works and correspondence, published and unpublished. Yet there is an important novelty now; while the two former volumes contained all the published writings and reviews,[1] the present one includes most of the unpublished essays and lectures (in English, and also in the original German when relevant) which have been found in Gödel's *Nachlass,* most of which appear here for the first time.[2] Another important change is that some members of the editorial team have been replaced by other scholars. In particular, the late Jean van Heijenoort (deceased in 1986), the late Stephen C. Kleene (deceased in

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* For the preparation of the manuscript of this review resources provided by the grant DGICYT PS93-0220 are gratefully acknowledged. Also, I am grateful to Francine Abeles for her help with the English of a first version of this review.

[1] See my essay-reviews; THIS JOURNAL, **3** (1992), 58–74 and **4** (1994), 318–327.

[2] Another book containing unpublished materials by Gödel appeared in the same year: K. Gödel, *Unpublished Philosophical Essays* (Basel/Boston/Berlin: Birkhäuser, 1995), edited by this writer. The Spanish edition appeared one year before, K. Gödel, *Ensayos inéditos* (Barcelona, Mondadori, 1994).

1994), and Gregory H. Moore (who left to devote himself full-time to the Russell project) have been replaced by Warren Goldfarb and Charles Parsons.

As a result of being mainly a volume containing unpublished materials, other minor changes in the general pattern of former volumes have taken place as well. Thus, a preface by Solomon Feferman (the editor-in-chief), a brief overview of the Gödel *Nachlaß* by John W. Dawson (pp. 1–5) and a report on Gödel's Gabelsberger shorthand by Cheryl A. Dawson (pp. 7–12) have been included. A further consequence has been the considerable expansion of the textual notes section (39 pp., against 13 pp. in vol. II). Unfortunately, the editors continue their policy of including only an index of names, but not one of concepts, which, in dealing with unpublished materials, would have been particularly useful here.

The Gödel materials in this volume can be divided into two categories: technical and philosophical, although some of the technical writings include philosophical remarks. Yet while the technical are mostly expository and extend mainly from 1930 to 1941, the philosophical are clearly at the beginning of the volume and are a clear sign that from the early forties to the end of his life philosophical reflection was the most important intellectual activity for Gödel. Accordingly, I have organized this review by devoting the first section to the technical materials and the remaining sections to the four main philosophical writings appearing here. When I have something to say on the corresponding introductory notes, I say it in the appropriate section.

II

Most of Gödel's technical materials appearing in this edition are the texts of lectures given with the aim of explaining technical results or discussing possible ways of solution to certain problems. Thus, we can find, among others manuscripts dealing with more specialized points, lectures on the completeness of elementary logic, on the celebrated incompleteness theorems, on the consistency of the continuum hypothesis, on the sense in which intuitionistic logic is constructive, and — rather surprisingly — on some possibilities for continuing Hilbert's program in a revised form (whose original epistemological value is strongly defended).

The text of an invited lecture of 1933 in Cambridge, Massachusetts, in what was Gödel's first trip to the US, is particularly useful to provide us with a global view of "The present situation in the foundations of mathematics" (1933) by that time. In particular, Gödel, in a rather Russellian way, describes the foundational problem as one divided into two parts: first to reduce the actual methods of proof to a minimum number of axioms and rules of inference; second to give a justification of these axioms. According to him, the first part can be easily solved by a simple theory of types without certain restrictions. The second is more difficult, for although we can deal with the formalism as if it were a symbolic game, it is when we attempt to attach some meaning to those symbols that serious problems arise: the notions of existence, of class, and the axiom of choice. The amazing point is that after discussing these difficulties, Gödel goes on to say that if we interpret the axioms in a meaningful way, then we are presupposing a kind of Platonism, "which cannot satisfy any critical mind and which does not even produce the conviction that they are consistent" (p. 50). Even after reading Feferman's analysis of this statement (pp. 39–40) the reader does not know what to do with it, although to say the least, it should be a sign of Gödel's constant difficult relationships with "public" and "private" Platonism.

Another interesting lecture is the one of 1949 on rotating universes. The text is more or less the same as the published one (1949), and it contains new solutions to Einstein's field equations of gravitation, but as is usually the case with Gödel's manuscripts prior to publication, it contains more material than the one actually published. An interesting example is Gödel's assertion that he arrived at the new solutions while he was working on the similarities between Kant's philosophy and relativity physics "insofar as in both theories the objective existence of a time in the Newtonian sense is denied. On this occasion one is led to observe that in the cosmological solutions known at present there does exist something like an absolute time. [. . .] So one is led to investigate whether or not this is a necessary property of all possible cosmological solutions." (p. 274). If I understand this well, then it could be interpreted as a new argument in favour of a general thesis on the relations between Gödel's technical work and his global philosophical motivation of it.[3]

Another remarkable technical work by Gödel appearing here (although not for the first time) is his celebrated and much discussed logico-ontological proof of the existence of God, which is well-known at least since the early seventies, thanks to Dana Scott's first presentation in a seminar, after he was shown the proof by Gödel himself. The text is the strict presentation —in hardly one and half pages, pp. 403–404 — of the proof in logical symbols, whose general theme is the transition from the essence of a being possessing all positive properties to her necessary existence. The introductory note, by R. M. Adams, is very useful, as it contains some historico-technical background, as well as a good discussion of Gödel's concepts and proof-steps, and a review of the relevant literature

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[3] The present writer has been maintaining this general thesis in several publications on Gödel. See, for instance, my essay-review of vol. II of the Gödel series in THIS JOURNAL (4, (1994), 318–327), especially concerning Gödel's work on rotating universes (where I already conjectured that Gödel's reasons for working on new solutions to Einstein's equations of gravitation must have been philosophical), as well as my introductory essay to the book cited in footnote 2.

which appeared between 1970 and 1991. Yet I find that the not especially interested reader is going to miss not having a simple, intuitive presentation of the general structure of the proof.[4]

Finally, a still more famous text appears in this book: Gödel's never before published legendary paper entitled "Some considerations leading to the probable conclusion that the true power of the continuum is $\aleph_2$" (together with two other closely related writings, including an unsent letter to Tarski). This is the text which Gödel sent in 1970 to Tarski for submission to the *Proceedings of the National Academy of Sciences* that was never published. The paper is very short (not even three printed pages, pp. 420–422), but the introductory note by R. M. Solovay offers to the reader a full analysis of the axioms given by Gödel as well as of the alleged — but failed — proof, and some discussion of further possible work on similar lines and some open questions. Solovay was the referee who turned back the paper to Tarski saying that, "if the author were anyone but Gödel, I would certainly recommend that the manuscript be rejected" (p. 405), so his final remark providing some explanation for Gödel's errors is worth citing: "Ill health both made it seem urgent that his ideas be communicated to the world and made it impossible for him to carry out his usual scrupulously careful presentation and checking of the details" (p. 420).

III

In the 1949 Schilpp volume on Einstein, Gödel published a short paper entitled "A remark about the relationship between relativity theory and idealistic philosophy". It was devoted to defending a particular kind of idealism (close to Parmenides and Kant) according to which relativity theory is not incompatible with the thesis that change is not objectively real. Among Gödel's papers, five manuscripts written between 1946 and 1949 have been found dealing with somewhat similar matters; two of them appear in this volume under the title "Some observations about the relationship between theory of relativity and Kantian philosophy." Yet they are not at all mere preparatory drafts of the Schilpp paper, which is more cautious and less extended in scope, but contain bolder and more extended arguments. According to them, although there is something important in Kant's philosophy which has been confirmed by relativity theory, some reinterpretation of that philosophy must be undertaken as a result of Einstein's theory.

[4] See, for instance, the one contained in Hao Wang, *Reflections on Kurt Gödel* (Cambridge, Mass., The MIT Press, 1988), p. 195.

I think the following passage by Gödel can be regarded as a good summary of his global position in these papers on the point that relativity theory is opposed to Kant (p. 244):

> A real contradiction between relativity theory and Kantian philosophy seems to me to exist only in one point, namely, as to Kant's opinion that natural science in the description it gives of the world must necessarily retain the forms of our sense perception and can do nothing else but set up relations between appearances within this frame. This view of Kant has doubtless its source in his conviction of the unknowability (at least by theoretical reason) of the things in themselves, and at this point, it seems to me, Kant should be modified, if one wants to establish agreement between his doctrines and modern physics; i.e., it should be assumed that it is possible for scientific knowledge, at least partially and step by step, to go beyond the appearances and approach the world of things.

On the other hand, Gödel was convinced that relativity theory has confirmed Kant's doctrine that the natural world picture is essentially "subjectivistic" (p. 245). So our view of the world and science should be a compromise between this confirmation and that rejection. The introductory note by H. Stein is clear and detailed, offering the reader both useful technical context and philosophical comments.

Two points seem to me worth recalling in connection with these two manuscripts by Gödel, one related to the content of Gödel's position, the other to his method of work. For one thing, it seems to be clear that Gödel is here maintaining a position very close to the one he was maintaining in his philosophy of mathematics. According to that position, while the objects of mathematics are objective, we do possess an intellectual intuition of them which, with some cultivation, is able to let us *see* their deep nature. Thus, Kant's philosophy should be amended in order to release itself of the limits of intuition based strictly on sense perception.

Secondly, the fact that Gödel wrote up to five extended and bolder manuscripts, when finally he published only a short, cautious and more general paper, seems to be due to his usual fear of making public his actual philosophical beliefs. Something similar took place with his paper for the Schilpp volume on Russell.[5] But this style tending to introversion, fear of controversy and the consequent increasing self-censorship was at its height in the series of papers on Carnap, as we shall see in the next sections.

[5] See my review of vol. II cited in footnote 3.

IV

"Some basic theorems on the foundations of mathematics and their philosophical implications" was the title of Gödel's Gibbs Lecture, given at Brown University in 1951. It seems that Gödel had some doubts about publishing it because the essay, which is fully philosophical, contains a strong attack against Carnap's nominalism-conventionalism. So Gödel finally decided not to publish it, probably because he was not completely satisfied with his own arguments, and was fearful of the controversy that would ensue.

The kernel of the lecture is the showing that mathematics is inexhaustible (through the exposition of the iterative concept of set and of Gödel's incompletability results), followed by the drawing of some philosophical consequences which, according to Gödel, can be adduced against Carnap's conventionalistic viewpoint, then in favour of some sort of a platonistic philosophy of mathematics.

The arguments against Carnap's view that mathematics is a vast tautology are basically that the *explanation* that this is so cannot be given without resorting to the axioms of mathematics themselves, and that the corresponding *proof* would be also a proof of the consistency of mathematics, which is not allowed by Gödel's second incompleteness theorem. As for the arguments in favour of Platonism, they are mainly arguments against the view that mathematics is our own creation. For Gödel this cannot be so: first, because if mathematics were our creation we should see its inner more difficult intricacies with perfect clarity, which is by no means the case; second, because it is obvious that we cannot arbitrarily create the truth of the theorems; finally, because (p. 314):

> if mathematical objects are our creations, then evidently integers and sets of integers will have to be two different creations, the first of which does not necessitate the second. However, in order to prove certain propositions about integers, the concept of set of integers is necessary. So here, in order to find out what properties *we* have given to certain objects of our imagination, we must first create certain other objects — a very strange situation indeed!

The text of the lecture is the accurate and painstaking reconstruction of the handwritten manuscript by Charles Parsons. This is a genuine editor's nightmare, for the original text is an almost inextricable tangle of revisions, deletions, crossed-out passages, footnotes (again revised, deleted, etc.) and additional notes to the text and to the footnotes (again revised, deleted, etc.), many of which cannot be properly located in the main text. The final result is admirable, although the editor does not give us all his criteria of reconstruction in the textual notes at the end of the volume. Yet a study of the final rendition of the text in comparison with the manuscript leads us to believe that he has kept everything which possibly could be inserted or linked to the main text. It is obviously a perfectly legitimate choice, although not the only one.[6]

The introduction, by George Boolos, is a model of clarity and usefulness for any serious reader. After a brief historical information and overview, he offers us a summary of Gödel's main arguments, together with some explanation of some of his concepts, and even some very convincing criticisms of Gödel's position. Yet those readers interested in locating Gödel's ideas of this essay in a wider historico-philosophical context will have to look elsewhere.

V

Two years after having read the Gibbs Lecture, Gödel accepted the Schilpp invitation to contribute to the Carnap volume. Perhaps that may be why he decided not to publish the former lecture, for an important part of the kernel of the 1951 text contained a strong criticism of Carnap's position, and he decided to concentrate on a full development of those criticisms. From 1953 to 1959 Gödel wrote up to six versions of an essay, entitled "Is mathematics syntax of language?", which was finally not submitted. The editors of this volume offer us versions III and V, adducing that III is the richest in content and that V is the clearest version of Gödel's final position. I think this is true, but other arguments could be adduced to have chosen other versions.[7]

I think the best way to briefly describe Gödel's endeavours in the essays appearing here is by quoting from his own summary in version V. He says there that the kernel of the position to be refuted can be expressed through three assertions: "I. Mathematical intuition . . . can be replaced by conventions about the use of symbols and their applications. II. . . . there do

[6] Another choice, the one followed by this writer in his edition of this manuscript (see footnote 2), is trying to keep absolutely everything which can be actually read in the manuscript, including crossed-out passages and footnotes and additional notes which cannot be linked to any particular passage of the main text. For the reader to have some idea of both results in an objective way, I can say that Parsons' whole reconstruction (main text, footnotes, loose notes) has over 10,000 words, and my own 13,000.

[7] In my edition of Gödel's essays (see footnote 2) I chose versions II and VI. Among other things, I found version II a more finished one and thought that version VI, being the last of the series, should be published at any rate, especially if complemented with some comparative study of V, which I did in the footnotes. Fortunately, this has lead to a situation in which four versions of the Carnap essay are now published, and the two versions omitted are only I, which is just the handwritten version of II, and IV, which seems to be only the beginning of an attempt of reduction conducing to V and VI.

not exist mathematical objects and facts. Mathematical propositions . . . are void of content. III. . . . the a priori validity of mathematics [is] compatible with strict empiricism." (p. 356).

On the contrary, Gödel says, "these assertions, for an adequate interpretation of the terms occurring in them (such as "content", disprove", "replace", etc.), turn out to be wrong. Moreover, I believe, it can be shown directly that the arguments which may be adduced in favor of these assertions, including the existence of actual elaborations of the syntactical scheme, are all fallacious." (p. 357). One of the most powerful arguments used by Gödel is based on his famous metamathematical results (instead of starting with them, as in the Gibbs Lecture). Thus, in connection to assertion I, Gödel says that although it is true that we can replace mathematical intuition by conventional rules about symbols, "What must be known is that the rules, by themselves, do not imply the truth or falsehood of any proposition expressing an empirical fact. Such rules may be called 'admissible'. Admissibility, for *our* mathematics, entails consistency. For from an inconsistency *all* propositions, the empirical ones included, could be derived," (p. 357). Thus the rules in question cannot be *really* conventional, for to prove that consistency "an intuition of the same power" is needed.

In the course of the analyses and the detailed discussions, Gödel often refers to a large number of items from the technical and philosophical literature (starting, of course, with Carnap's *Logical Syntax of Language*), which is not only mentioned in the footnotes, but sometimes even discussed in the main text. It is then not difficult to imagine the spectacular impression the essay wouild have caused had it been published during Gödel's lifetime. I think this expected outcome, and the consequent controversy, always feared by Gödel, may have been one of the main reasons he decided finally not to submit the paper for publication.

The introductory note by Warren Goldfarb is quite useful. It provides the reader with some information on the history of the several versions and the reasons for their abandonment, together with some clear expositions of Gödel's arguments and some criticisms of them. The historico-philosophical context offered, however, seems to this writer somewhat sparse, as well as the literature related to these topics, which is only partially cited.

VI

We come now to the last philosophical text to be referred to here, the one entitled "The modern development of the foundations of mathematics in the light of philosophy". It was written about 1961 as a lecture which was never actually given. It is a very curious philosophical manuscript in which Gödel places the development of the different conceptions of the philosophy of mathematics into a twofold general frame of world-views. He writes: "I believe that the most fruitful principle for gaining an overall view of the possible world-views will be to divide them according to the degree and the manner of their affinity to or, respectively, turning away from metaphysics (or religion). In this way we immediately obtain a division into two groups: skepticism, materialism and positivism stand on one side, spiritualism, idealism and theology on the other" (p. 375).

Later on, Gödel adds optimism and, so it seems, apriority to the second group, which seems to be the one he thinks to be correct. Thus, we have for the first time a clear scheme to more or less clearly insert his position on the foundations and philosophy of mathematics into a general view of the world. If this is so, a platonistic philosophy of mathematics should be the only one to give justice to an "optimistic", non positivistic *Weltanschauung*. Also, the general thesis of this writer that Gödel was mainly a philosopher in search of technical results to "verify" his philosophical doctrines seems now to be more plausible.

The second part of the text is devoted to defending the position that to gain insight into the problems related to the foundations of mathematics, i.e. the meaning of the concepts and the truth of the axioms involved, the most fruitful way is not Hilbert's formalism, but something like Husserl's phenomenology: "Here clarification of meaning consists in focusing more sharply on the concepts concerned by directing our attention in a certain way, namely, onto our acts in the use of these concepts, onto our powers in carrying out our acts, etc." (p. 383). Finally, Gödel insists on his position in former manuscripts that Kant's conception of mathematics also can be useful in this respect.

Føllesdal's introductory note is devoted to summarize Gödel's ideas and to relate them to Husserl's and also with the ideas of others appearing in the rest of the manuscripts edited here, mainly in connection to realism and intuition.

As a whole this volume is as indispensable as the two former ones for any serious student of Gödel's ideas and achievements, but in this case it is also indispensable for philosophers interested in logic and mathematics. The fourth (and last?) volume of this formidable series will be devoted to Gödel's correspondence, so we should look forward to having it to study.