JAN ŁUKASIEWICZ
ON THE LIAR PARADOX, LOGICAL CONSEQUENCE, TRUTH, AND INDUCTION

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Abstract. This paper presents Jan Łukasiewicz's ideas on the Liar paradox, logical consequence, truth, and induction.

Jan Łukasiewicz is well known for his investigations of many-valued logic, his contributions to propositional calculi and his work on the history of Stoic and Aristotelian logic.[1] Łukasiewicz achieved these results in 1918-1956, when he was almost entirely involved with mathematical logic. His earlier work, for instance, on the law of contradiction in Aristotle or induction, were of a more philosophical than a formal-logical character. Nevertheless, Łukasiewicz also worked in logic in 1902-1918 and produced several interesting ideas, particularly on the Liar paradox, the concept of logical consequence, truth, and induction.

1. Łukasiewicz on the Liar paradox and logical consequence. The simplest version of the Liar paradox is captured by a famous sentence of Savonarola: *"Hoc est falsum"* (what I am now saying is false). A simple examination of this sentence leads to the result that it is true if and only if it is false. Tarski in his famous essay on truth gives a version of the Liar which he attributes Łukasiewicz.[2] However, Tarski gives no reference to any published work of Łukasiewicz.

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[1] See [Sobociński *1956*], [Borkowski and Słupecki *1958*], [Kotarbiński *1958*], and [Woleński *1989*] for comprehensive surveys of Łukasiewicz's research in logic.

[2] [Tarski *1933*; see *1956*, 157-158].

Łukasiewicz's version of the Liar is included in his paper on the concept of on science published in 1915.[3] Explaining the Liar, Łukasiewicz proceeds as follow:

> There are mental constructions which seem to contain an inevitable contradiction. For example, the sentence: line 13 on p. XXXV of this book contains a false sentence, is a construction of this kind.
> This sentence contains a contradiction, because, observing that this sentence contains itself in the line 13 on p. XXXV of this book, it is easy to prove that its truth entails its falsity as well as that its falsity entails its truth. [Łukasiewicz *1915*, XXXV]

Then Łukasiewicz offers a solution of the Liar. His proposal is this:

> [...] every logical principle contains v a r i a b l e s .
> [...] These variables, like variables in mathematics, can have various values. Now there is a logical law which says that all logical principles concern only t h o s e objects which can be v a l u e s o f v a r i a b l e s . One can show that the above sentence containing the contradiction cannot be value of a variable. Hence logical principles do not apply to this sentence; this construction is outside logic. [Łukasiewicz *1915*, XXXV].

So far Łukasiewicz. Tarski's statement of the Liar in his essay can be taken as a strict formalization of the version proposed by Łukasiewicz. However, there is something more. Łukasiewicz's own solution of the Liar consists in excluding the Liar sentence from the domain of logic. Thanks to Tarski, Łukasiewicz's version of the Liar became standard. But although he proposed a way out from the Liar paradox, he did not explain why the Liar sentence does not adhere to the principles of logic. This was done by Leśniewski and Tarski, who argued that so-called "closed" languages (languages that contain own metalanguages) violate principles for constructing correct formalized languages and thereby must be ruled out of the province of logic.[4]

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[3] This essay is a general introduction to the whole volume, which contains many other interesting papers, in particular by Zygmunt Janiszewski, Wacław Sierpiński and Stefan Mazurkiewicz.

[4] See [Tarski *1933*, chapter 1] and [Tarski *1944*, sect. 8]. These two works contain references to Leśniewski's view on sources of the semantic paradoxes and ways of overcoming them. Unfortunately, Leśniewski had never published anything on semantic paradoxes. It is known that he prepared an extensive monograph on this topic. The sole copy, written in pencil, was destroyed during the Warsaw uprising in 1944.

It is a widespread opinion that Łukasiewicz stated his version of the Liar in an oral discussion.[5] This situation probably stems from the fact, which I have already mentioned, that Tarski gave no textual reference to Łukasiewicz. We can ask why [Łukasiewicz *1915*] is almost entirely overlooked (see [Łukasiewicz *1912*]).

The history of this paper is interesting. In 1912, Łukasiewicz published a paper on creativity in science.[6] Then he was invited to write an introductory essay for *A Guide for Autodidacts*, and he decided to extend his [*1912*] by adding remarks on the Liar paradox and logical consequence. The additions do not form new sections but are inserted into the old text. This may be main reason that references to Łukasiewicz's paper are mostly made to the version published in 1912 which became very influential in Polish philosophy, particularly for Łukasiewicz's classification of arguments given there for the first time.[7] When a selection of Łukasiewicz's logical papers was being prepared, its editor, Jerzy Słupecki, included the 1912 version and then it was translated into English.[8]

Łukasiewicz's paper of 1915 also contains a definition of logical consequence in the semantic sense:

> We say that a sentence *b* follows from a sentence or a group of sentences *a* if *b* must be true provided that *a* is true. [Łukasiewicz *1915*, XXII]

As far as I know this one of the first (perhaps even the very first) correct definitions of logical consequence in modern logic since Bolzano.[9] In Poland, it was rediscovered by Ajdukiewicz in 1923 and generalized by Tarski in 1936. For Ajdukiewicz [*1923*, 161]:

> A formula $f(x)$ formally entails a formula $\phi(x)$, if for any possible substitutions for *x*, either $f(x)$ is false or $\phi(x)$ is true.

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[5] I heard this opinion on several occasions from many Polish logicians and philosophers, including persons who had direct contact with Łukasiewicz, Tarski, and their students. Unfortunately, no written record confirms this fact.

[6] This situation seems paradoxical, for [Łukasiewicz *1915*] was reprinted twice in Poland in the interwar period.

[7] See [Giedymin *1985*] for general comments on the importance of [Łukasiewicz *1912*] for Polish philosophy and [Woleński *1991*] for Łukasiewicz's classification of types reasoning and its reception.

[8] See [Łukasiewicz *1961*] and [Łukasiewicz *1970*]. Słupecki as well as Borkowski (the editor of [Łukasiewicz *1970*]) probably forgot about additions in [Łukasiewicz *1915*]. In any case, there is no other explanation their editorial choices with respect to the paper in question.

[9] I base this assertion on my own examination of numerous books in logic published in the period under consideration. My evaluation of the matter was also confirmed by several people interested in the history of logic. Of course, this basis does not preclude the possibility that I am mistaken.

From this definition it follows that, if A formally (logically) implies B, then B cannot be false, provided that B is true.[10] For Tarski [*1936*; q.v. *1956*, 417]:

> The sentence X follows logically from the sentences of the class **K** if and only if every model of the class **K** is also a model of the sentence X.

Tarski's formulation is commonly regarded as a modern statement of Bolzano's idea of logical consequence. However, the essential point of this important idea was anticipated, at least in Poland, by Łukasiewicz.

2. Łukasiewicz on the concept of truth. Łukasiewicz, influenced by Aristotle and the Brentanist tradition, accepted the classical (correspondence) theory of truth.[11] His statement of this theory is this:

> A sentence is true or false only if it states that something exists or does not exist. [Łukasiewicz *1910*; 2nd (1987) edition, 14]

> A judgment is true if it ascribes to an object a property which belongs to this object or denies a property which does not belong to it. [Łukasiewicz *1911*, 86]

> If truth consists in conformity of thought to reality, we may say that those propositions are true which conform to [...] reality. [Łukasiewicz *1957*, 208]

These quotations illustrate Łukasiewicz's general position on truth. However, we also can find in [Łukasiewicz *1913*] an idea that remains in Tarski's semantic theory of truth.

For Łukasiewicz, truth and falsity are unconditional and absolute properties of sentences.[12] In particular, probability cannot be used for the logical evaluation of sentences. On the other hand, formulas in which free variables occur ("indefinite proposition" is Łukasiewicz's term) can be characterized by their logical probability. Let **D** be a domain consisting of a finite number of objects and let $F(x)$ be a formula with x as a

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[10] A similar definition can be extracted from [Ajdukiewicz *1934*].

[11] See [Woleński and Simons *1989*] for comments on Łukasiewicz's philosophy of truth. Łukasiewicz was a student of Twardowski in Lvov and Meinong in Graz, both of whom were in turn distinguished students of Brentano.

[12] For Łukasiewicz (and similarly for Tarski), sentences are always objects equipped with meaning. I do enter here into a discussion on whether this conception of sentences is sound or not. I note this point because most English commentators on Tarski's work on truth ask whether sentences (as purely syntactic items) may be true or false. The "Polish" answer is simple: yes, because sentences are meaningful items.

free variable. Now assume that there are n-many objects in D and m-many objects that satisfy $F(x)$. Then the ratio $\frac{m}{n}$ expresses the probability of $F(x)$. Moreover, Łukasiewicz says:

> Indefinite propositions are true if they yield true judgements for all values of the variables.
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> Indefinite propositions are false if they yield false judgements for all the values of the variables. ([Łukasiewicz *1913*, 16; page number is to the *1970* English translation])

To see how close Łukasiewicz was to the semantic definition of truth, let me recall Tarski's condition for quantified (universal) sentences: $\forall x F(x)$ is true if and only if $F(x)$ is satisfied by all values of a variable x. The right part of this biconditional is equivalent to Łukasiewicz's definition of truth for indefinite propositions. According to Tarski, a sentence is true if it holds (is satisfied) for all objects. Tarski's heuristic strategy is to regard sentences as particular cases of open formulas and truth as a special case of satisfaction. On the other hand, Łukasiewicz was not interested in defining truth *via* satisfaction, probably because he assumed that a gap between sentences and indefinite propositions is so essential that the former cannot be special cases of the latter. Hence, truth cannot be a special case of satisfaction. For Łukasiewicz, the truth of open formulas was nothing more than an auxiliary idea used by him to develop the theory of logical probability.

3. Łukasiewicz on induction. Induction was a favourite subject of Łukasiewicz before he concentrated on mathematical logic. He devoted his doctoral dissertation to the problem of induction (published as [Łukasiewicz *1903*]), and he delivered three talks on induction before the Polish Philosophical Society in Lvov in 1906-1909 ([Łukasiewicz *1906, 1907* and *1909*] are abstracts of these lectures). Induction is also briefly considered in [Łukasiewicz *1912*] and [Łukasiewicz *1915*]. Finally, he discussed induction in the last part of [Łukasiewicz *1929*].[13] At first, Łukasiewicz tried to develop the inverse theory theory of induction proposed by Jevons and Sigwart in the nineteenth century. On other hand, from the beginning of his interests in induction, Łukasiewicz was rather

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[13] This section (§11) is omitted in the second Polish edition (see [Łukasiewicz *1958*]) as well as in the English edition (see [Łukasiewicz *1963*]). Arthur Prior complains of this omission in his [*1968*] review of [Łukasiewicz *1963*]. The German translation of §11 is included in [Pearce and Woleński *1988*]. Note that the problem of induction is not touched upon in [Łukasiewicz *1913*]. One might be surprised that there is a chapter on induction in a very advanced textbook on mathematical logic. However, it was a tradition in Poland to speak on induction at the end of courses in mathematical logic.

sceptical about evaluating inductive conclusions by probability. He expressed this view in his dissertation. There is also an interesting fragment of his
letter to Kazimierz Twardowski (August 31, 1902):

> In order to reject (I would like to do this) that view [that experience provides the devices for the estimation of the probability of the inductive conclusions - J. W.] one must prove that particular propositions, independently of their number and kind, cannot serve as the logical basis for the probability of a generalization [...] I am eager to solve this question by admitting that there is only mathematical probability.

An argument against the probabilistic theory of induction is given in [Łukasiewicz *1909*]. The argument is this. Assume that H is a hypothesis which is tested by induction. At first sight, we we could apply Laplace's rule $p = \dfrac{n+1}{m+2}$. According to this rule, p is the probability that the $(n+1)^{st}$ event has a property, say P, provided that it is established in advance that n events have P. Since this rule applies only to particular events, it is not applicable to genuine inductive generalizations. Laplace's rule is a special case of a more general formula: $p = \dfrac{n+1}{n+m+1}$, where m is the cardinality of the domain of events to which H refers and n measures the basis of induction, i.e. the number of events already observed. Now m is always greater than n, so p cannot be greater that 2 and, what is more important, if m approaches infinity, p approaches 0. Thus, no finite amount of data acquired by experience is sufficient for confirming any general inductive hypothesis. Łukasiewicz also expresses his anti-inductivism in his papers of 1912 and 1915. The last section of [Łukasiewicz *1929*] contains perhaps his most negative evaluation of induction:

> [...] inductive logic has no scientific value. (p. 95)

> [...] so-called inductive reasoning has neither a scientific value no any application in the science. On the other hand, deductive reasoning plays an essential role in the science. (p. 196).

Clearly, Łukasiewicz anticipated basic tenets of Popper's anti-inductivism. Some commentators see general similarities in the two authors.[14] However, the most interesting point is perhaps that Łukasiewicz had Popper's celebrated mathematical argument against induction.

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[14] This point is stressed in [Prior *1968*].

4. Conclusions. As I have already noted, Łukasiewicz's version of the Liar paradox became standard *via* Tarski's work on truth. Thus, it is clear that Łukasiewicz ideas on the Liar began the development which culminated in the Leśniewski-Tarski account and solution of the paradox. No evidence is available as to whether or not Łukasiewicz's definition of the concept logical consequence influenced Tarski or Ajdukiewicz. This is also true of [Łukasiewicz 1913] and its influence on Tarski's conception of truth. Polish philosophers of science were mostly inductivists and, in general, they were not particularly attracted by Łukasiewicz's criticism of induction as a method of confirmation of empirical generalizations.[15] Thus, if published references provide evidence of how someone's ideas influence the history of logic, we must conclude that the ideas of Jan Łukasiewicz concerning truth, the concept of logical consequence, and induction did not exert any influence even in his own country. However, perhaps one remark is in order. It is known that many important ideas circulated among Polish logicians in conversation.[16] This was an important factor In doing logic in the Warsaw School. So it is quite possible that Łukasiewicz, who was the principal leader of the Warsaw School of Logic, communicated his ideas on truth and logical consequence in informal debates with his students and colleagues and thereby influenced their way of thinking on related topics.

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[15] See [Giedymin *1985*] for general comments on this matter.

[16] One can find evidence in footnotes to [Tarski *1933*].

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