ABSTRACTION AND NUMBER
IN MICHAEL DUMMETT'S *FREGE: PHILOSOPHY OF MATHEMATICS*

Review of Michael Dummett, *Frege: Philosophy of Mathematics*, Cambridge, Mass., Harvard University Press, 1991, xiii+331 pp.

by

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Only after writing my "Critical Remarks on Michael Dummett's *Frege and Other Philosophers*," did I have the opportunity of reading Dummett's *Frege: Philosophy of Mathematics*. The titles of the twenty-four chapters are so informative that to list them is the fastest way to provide a first insight into the more than three hundred pages of Dummett's latest book:

1. The significance of *Grundlagen*
2. The Introduction to *Grundlagen*
3. Analyticity
4. The Value of Analytic Propositions
5. Frege and Dedekind
6. Numerical Equations and Arithmetical Laws
7. What is Number?
8. Units and Concepts
9. Two Strategies of Analysis
10. Frege's Strategy
11. Some Principles of Frege's Strategy
12. Frege and Husserl
13. Frege's Definition of Cardinal Numbers
14. The Status of the Definition
15. Did Frege Refute Reductionism?

The dominating themes continue to be the same as in *Frege and Other Philosophers*: Frege's procedure in defining number; Husserl; and the context principle. One difference is that while the earlier book is largely polemical, *Frege: Philosophy of Mathematics* is not. Another difference is that some themes, for instance abstraction — just incipient in Frege and other philosophers — have grown, and are prominent in the new book.

There is a certain discrepancy between the title *"Frege: Philosophy of Mathematics"* and the contents of the volume, as Dummett himself explains. From the original project, started in 1973 (p. vii), and apparently intended to cover Frege's philosophy of mathematics at large, Dummett has "excised" in this volume a chapter on Frege's philosophy of geometry as well as "other discussions, unpublished and now suppressed, on topics of similar interest" (p. xi); the planned discussion of "everything in Frege's writings that bore on the philosophy of mathematics" has been replaced by one that "goes to the heart of Frege's philosophy of *arithmetic*" (p. xi; [emphasis mine]). In addition, a section on Frege's views on the consistency of mathematical theories has been left out, since it was already published in *Frege and Other Philosophers*.

I will concentrate my comments on the central business of Dummett's study, namely his discussion of Frege's answer to the question "What is number?", and on the main theme of this discussion: abstraction. My paper is divided into four parts that correspond to four "stations," as it were, in the long journey through the book.

Here is an outline of the stations and my disagreements in each of them. First we encounter a division of philosophers of arithmetic into "bad" and "good" depending on their using or not using, respectively, abstraction in their definitions of number; Dummett presents this classification together with an attack on abstraction that is, in my view, unacceptable — and should not be regarded as Fregean. Secondly Dummett argues that Frege is a champion of anti-abstractionism; again, I disagree. Properly, Frege champions the criticism of the misuse of abstraction by Cantor and others in their attempts to define number. In the third station we stop to consider Dummett's presentation of the heart of Frege's philosophy of arithmetic: the definition of number, as exhibited in §§62–69 of *Die Grundlagen der Arithmetik*. My difficulty here (as in the earlier book: *Frege and Other Philosophers*) is Dummett's apparent separation of Frege's procedure in §§62–69 into two methods, "contextual" and "explicit" definition — where Frege would discard the former and adopt the

latter — whereas I view Frege's work in §§62–69 as displaying one method in two stages. Perhaps this is what Dummett also thinks in the end, albeit not too clearly. Finally, Dummett thinks that Frege does, after all, practise abstraction — but in a new, better sense of the term, which Dummett specifies as "logical" abstraction, in contrast with the bad, "psychological" abstraction; here my disagreement is total: what Dummett calls "logical abstraction" is not abstraction.

1. First Station: Abstractionists and Anti-abstractionists. The central business of the book becomes visible for the first time in chapter 5 ("Frege and Dedekind"). The main point of this chapter is to compare Frege, in answer to the question "What is number?", not just with Dedekind (as the title, "Frege and Dedekind," suggests), but also with Cantor, Husserl, Russell, and some recent authors (such as Benacerraf).

The result of the comparison is quite clear: Frege and Russell are right, Cantor, Husserl, Dedekind are wrong (p. 52, *passim*). However, the "right" and "wrong" do not refer so much to the answers to the question "What is number?" as to the method used to reach an answer. In fact, primarily "wrong" means to believe that *abstraction* is a good ladder to catch number, and "right" means to believe that abstraction is not a good tool. Cantor, Husserl, and Dedekind are wrong for using abstraction. Russell is right for "obstinately refusing to recognise the role assigned by Dedekind to the process of abstraction" (p. 51), and even a "neo-Dedekindian" (Benacerraf, p. xii) is again right for having "jettisoned the doctrine of abstraction" (p. 52).

Indeed, in Chapter 5 everything said about abstraction is negative. For Dummett abstraction amounts to magic; he talks of the "magical operation of abstraction" (p. 52) and of the "creative powers" of abstraction: "one of the mental operations most frequently credited with creative powers was that of abstracting from particular features of some object or system of objects, that is, ceasing to take any account of them." The mind would by this means "create an object or system of objects lacking the features abstracted from, but not possessing any others in their place" (p. 50).

Now, this criticism of abstraction made by Dummett is unacceptable. It is theoretically wrong (since *that* is not abstraction) and historically targetless (since *that* is not the prevailing theory of abstraction in classical philosophy). In both features, Dummett's criticism resembles Locke's remarks on general ideas (*cf.* my [*1985*]).

It was a commonplace among the masters of abstraction — the scholastics — that *abstrahentium non est mendacium*: "those who do abstraction do not lie." To abstract, in considering a wooden wheel, from "wooden" is not to do magic or to exert creative powers. Disregarding ("abstracting from") certain properties of a set of automobiles does not mean that a new object is magically created: when we say "the car is the best transportation" we are not behaving like magicians. None of the scholastic theorists of abstraction, from the ancient authors till, say, the recent Santiago Ramírez, can be blamed for performing magic

or unjustified creativity. Abstraction, as understood in the history of philosophy at large, may have its hopeless problems, but magical creativity is not one of them.

It is not unlikely that the reader is led to think that Dummett's nasty remarks against abstraction are quite in the spirit of Frege. This would be a mistake. Frege was not, to be sure, a friend of abstraction, but he was not an enemy of it *à la* Dummett either.

In the Fregean texts on this issue (*cf.* my [*1984*]) two principal aspects may be distinguished. On the one hand Frege refers to abstraction — "ordinary abstraction" — as something harmless and possibly useful to obtain general concepts yet absolutely useless from the point of view of answering the question "What is number?" On the other hand, and this is the most prominent side of Frege's remarks on abstraction, he sarcastically criticizes, and ridicules, the "magical", even the "impossible", achievements that some authors — magicians indeed — claim to perform thanks to abstraction, such as keeping the units making up numbers equal and diverse at the same time. This association of abstraction with magic seems to be common to Dummett and Frege.

But this appearance is deceptive. It is one thing to make, with Frege, the very wise remark that abstraction is a dangerous tool in the hands of magicians, and that in fact it has been misused by them; but it is another to proclaim, with Dummett, that abstraction is — intrinsically — magic.

Aside from this unacceptable attack on abstraction, I want to mention the following aspects of Dummett's Chapter 5.

First, there is just one very minor hint in Chapter 5 at the possibility of a "good" abstraction in Dummett's terms; this is when he states that Russell "did not understand [Dedekind's belief in abstraction] because, very rightly, he had no faith in abstraction *thus understood*" (p. 52, emphasis mine). This suggests that abstraction is not entirely hopeless, and that there is a correct way of performing it. I will comment on this "good" abstraction later on.

Secondly, Cantor, Husserl, Dedekind not only use, according to Dummett, a bad method (abstraction): the final results they attain, concerning the question "What is number?" are equally bad. Husserl and Cantor say that number is the set of featureless units that is (supposedly) left over once all the differences among the members of the set are "abstracted from." Dummett refers to this as a "misbegotten theory" (p. 50), already subject to a "detailed and conclusive critique" by Frege in *Grundlagen* (p. 50). I fully agree with Dummett in this respect.

Thirdly, Dummett insists that a good answer to the question "What is number?" involves telling something about the *intrinsic* nature of numbers. This notion of "intrinsic," that plays an important role in Dummett's discussion in Chapter 5, seems to proceed from the Russell passage in *The Principles* quoted on p. 51; Russell writes, in connection with answers to this question: "If they [numbers] are to be anything at all, they must be *intrinsically* something; they must differ from other entities as points from instants, or colours from sounds...." Dedekind, who "applies the operation of abstraction to an arbitrary simply infinite system to obtain from it the system of natural numbers" (p. 50), does not end up

with any "misbegotten" set of featureless units, like Cantor or Husserl. But then his problem is that he says nothing at all — nothing "intrinsic" that is, about the nature of number, and views it in a purely "structural" fashion ("structure is all that matters," according to the neo-Dedekindian Benacerraf, p. 53): "Unlike Frege's, Dedekind's natural numbers have no properties other than their positions in the ordering determined by their generating operation, and those derivable from them" (p. 51).

In this third issue, I can only agree with Dummett's statements, but it remains to be seen what Russell, or Frege, manage to offer by way of an "intrinsic" account of the nature of number, and Dedekind's structural-looking account may in the end be better than a pseudo-intrinsic one.

Fourthly, the negation of the pure structuralist thesis may be formulated as the requirement that numbers be "specific objects," a phrase often used by Dummett. The phrase is acceptable, but only as part of a rejection of the pure structuralist thesis. Taken out of this particular context, however, the phrase "specific object" is quite unclear. What is a non-specific object? To make things worse, Dummett introduces, as it were, degrees in being a specific object. We have the category of "quite specific objects" (p. 53), we have the category of being "specific objects, but characterisable only as numbers" (p. 54). These notions require further explanations from Dummett.

Fifthly, at the very end of Chapter 5, Dummett suggests that the Fregean-Russellian project of defining numbers as specific objects may well end up in the self-defeating recognition that numbers must be understood, after all, à la Dedekind, i.e. structurally — an ominous final remark of chapter 5 whose necessity I fail to see.

Thus, the reader interested in the heart of Frege's philosophy of arithmetic leaves the first station: Chapter 5, carrying a mixed bag of dissatisfaction and open questions.

2. Second Station: Frege and Abstraction. Our next stop is a section of Chapter 8 titled "Abstractionism" (p. 83). Here Dummett claims that Frege "rejects the whole notion of abstraction" (p. 85) and that Frege "was one of the earliest and most vigorous opponents of the doctrine of abstraction" (p. 84), in opposition to Baker and Hacker who, according to Dummett, regarded Frege as "one of the chief nineteenth-century proponents of abstractionism" (p. 84).

Dummett is right in saying that to view Frege as abstractionist is "fatuous," but he is wrong in describing Frege as an anti-abstractionist *without qualifications*. As said above, Frege has nothing against abstraction *per se*, he is only furious at philosophers who think that abstraction can lead to number. (The eminent representatives of these philosophers criticized by Frege are Cantor and Husserl. The latter in particular may be seen as the culmination of the persistent classical attempt, mainly in the scholastic tradition, to view number as the result of an abstraction on sets. My judgement of this matter, as stated in my [*1984*, Section 7], is that the scholastics and Husserl put the abstraction in the wrong place,

namely "inside" each one of the individual elements of the set whose number is to be reached, and to this extent Frege's objections are justified; Frege's objections lose their force, however, as H. Weyl has already observed, if the abstraction is performed in the right place, namely on the set itself, as in [Lorenzen *1955*]).

It is interesting to compare, as I have done in my [*1984*], Frege's attitudes *vis à vis* each of the two important themes of abstraction and predication. Frege strongly complains about the misuse of the two notions, but in neither case are the complaints directed against predication and abstraction as such. Frege was deeply sensitive to predication, and in fact there is at least one text where he suggests that the term "predication" be kept, provided it is correctly used (namely just as the converse of "*x* falls under *y*," without including such foreign things as "subordination" between concepts). On the other hand he was not moved at all by abstraction, not even by the harmless, good old "ordinary" abstraction of logicians, so that his final recommendation may be taken to be that we dismiss abstraction from logic and pass the burden on to psychology.

Dummett describes three theses on abstraction, "in ascending order of strength" (p. 84), of which I will quote the second and third:

(2) the attainment of the new concept [...] is effected by abstracting from the properties differentiating the objects in question, i.e. by diverting the attention from them;

(3) the operation of abstraction referred to under (2) can also generate abstract mental constructions, that is, abstract objects or structures of objects that lack all those properties abstracted from and have no others in their place.

With regard to thesis (3) Dummett says that Frege never accepted it:

What, in *Grundlagen* and elsewhere, he was concerned to combat was thesis (3), which alone has a bearing on the philosophy of mathematics. His essential, and crucial, contention in *Grundlagen* was that abstraction is (at best) a means of coming to grasp certain general concepts: as a mental operation, it has no power to create abstract objects or abstract structures. (p. 85)

Dummett does not offer any textual reference to Frege's writings to justify this claim; the generic phrase "*Grundlagen* and elsewhere" is not good enough, and I believe that in fact there is no textual support at all for Dummett's claim.

Incidentally, the quoted passage offers, towards the end, two examples of pseudo-use of the word "abstract." I define a pseudo-use of "abstraction" and related terms as one in which no reference is made to the essential double feature of "leaving out (abstracting from)" something and "retaining" something. In the pseudo-uses so frequent in the logico-analytic tradition originating in Frege and Russell, the word "abstract" usually means the

same as "intangible", "non-physical" (*cf.* my [*1991*]), which is the only meaning one can attach to the word in Dummett's quoted passage.

Thesis (2) is what Frege calls "the ordinary abstraction of logicians." Dummett rightly observes that Frege "accepts thesis (2)" at the time of *Grundlagen*, but he is wrong in claiming that "subsequently, in the review of Husserl" (p. 85) Frege rejected that thesis. Neither the texts quoted on p. 85 from Frege's review, nor any other passage in the review, support this claim; moreover, thesis (2) reappears in later writings, for instance the piece on Schubert (*cf.* my [*1984*]).

3. Third Station: Frege's Definition of Number. Important as abstraction is in Dummett's book, albeit negatively, the central issue of Frege's philosophy of arithmetic is to find an answer to the question "What is number?" (As indicated already in the preface to the 1879 *Begriffsschrift*, Frege *expects* number to turn out to be a logical notion, but this does not make the search any less exciting.)

The search occurs in *Grundlagen*. Dummett devotes his entire chapter 1 ("The significance of *Grundlagen*") to the proof and discussion of the statement that *"Die Grundlagen der Arithmetik* is Frege's masterpiece." The chapter begins with the claim that that book "is his most powerful and most pregnant piece of philosophical writing, composed when he was at the very height of his powers" (p. 1), and ends with the slightly more cautious thesis that: "hence, despite some serious uncertainties, we may consider *Grundlagen* as expressing, with fair accuracy, Frege's mature philosophy of arithmetic, not merely a superseded phase of his thinking" (p. 9).

Aside from the sections of *Grundlagen* that are critical of other views on number, Frege's own view gradually emerges and includes the preliminary assertion that numbers are objects, not concepts. This is found by Dummett to be "wholly devoid of cogent justification" (p. 110); he refers to the "highly unsatisfactory passage from §55 to §61" (p. 111). Another preliminary Fregean finding is that numbers occur, so to speak, at the level of concepts, they "belong with" concepts, not with objects: 12 belongs with the concept "being apostle" rather than with any individual apostle.

The combination of these two features explains why Frege at a certain point refers to numbers by means of descriptive singular terms of the form "the number of the concept F."

In trying to determine the denotation of such singular terms one problem is that numbers are objects — but not physical objects. Dummett points that out in *Grundlagen*, after §61,

> Frege assumes that he has shown that numbers are objects, and must be treated as such. Since they are objects, he begins his new enquiry by posing the Kantian question, 'How are numbers given to us?' Kant's doctrine was, of course, that objects can be given only through sensible intuition. Frege has, however, already re-

jected the notion that number is any kind of perceptible feature of things, or that numbers are objects of which we can have intuitions. The problem is therefore an acute one... (p. 111)

The problem is discussed, and a solution is proposed, in sections §§62–69 of *Grundlagen*, sections that Dummett describes as "the most important for Frege's philosophy of mathematics, and, indeed, his philosophy generally" (p. 111). Dummett even believes that "of these inspired sections, §62 is arguably the most pregnant philosophical paragraph ever written" (p. 111) — a bit exaggerated, in my view.

Dummett's account of §§62-69 fails to be quite clear in the crucial respect of explaining exactly how Frege proceeds in his march towards the essence of number.

On the one hand the reader is told that Frege tries *first* a "contextual" definition of number, which he *then discards* and *substitutes* by an "explicit" definition. This "two methods" view of §§62–69 appears on p. 126 ("the contextual definition had a solution, but not a unique one; it had to be *replaced* therefore by an explicit definition"), p. 127 ("the third objection [to a contextual definition] [...] he sustains, and so adopts his explicit definition"), p. 159 ("Frege's motive for abandoning the attempt to give a contextual definition"), p. 165 ("rejecting the contextual 'definitions' ...in favor of explicit ones").

The contextual definition stipulates that the number of the concept F (briefly: $N'F$) = the number of the concept G ($N'G$) iff F and G are equinumerous, i.e. iff the objects falling under F and those falling under G can be put in a one-one correspondence, or "there are just as many Fs as Gs." This biconditional is referred to by Dummett as "the original equivalence" (p. 155). A phrase of the form "$N'F$" Dummett calls "the cardinality operator." Thus the contextual definition of the cardinality operator stipulates that the original equivalence is to hold (p.155) .

The explicit definition given by Frege stipulates that the denotation of "$N'F$" is identical to the extension of the concept "being equinumerous to F" (two concepts are "equinumerous" if their objects can be put in one-one correspondence).

On the other hand, Dummett speaks of "Frege's method" as follows:

> If we wish to introduce a new type of object, but not as a subspecies of some already familiar type, and can formulate the criterion of identity for objects of this new type as the obtaining of some equivalence relation between objects of some already known kind, this method enables us to identify the new objects as equivalence classes of the old ones under that equivalence relation (p. 167).

This method is neither the above mentioned contextual approach nor the explicit one. Is it a *third* procedure? Or is it a combination of the contextual and of the explicit definitional approaches? This is not quite clear in Dummett's exposition of the §§62–69 of *Grundlagen*.

The way out for the reader is to observe that what Dummett's describes as "Frege's method" has an obvious connection with the two approaches first mentioned by Dummett:

the equivalence relation is the equinumerosity of the contextual definition, the "identification" of the new objects as equivalence classes is an explicit definitional move.

This connection however is strong enough to create another problem with Dummett's presentation of §§62–69: instead of the relationship between "Frege's method" and the two definitional styles (contextual, explicit), the reader now wonders about the fact that the three items begin to look as if they express not three, but *just one global* procedure followed by Frege on his way to number — one procedure in which somehow the two approaches, presented by Dummett as independent, are unified and simultaneously present, rather than one of them being discarded in favor of the other.

This in fact defeats Dummett's own presentation of two separate, successive methods (contextual first, explicit second) in §§62–69. The truth behind the appearances of Frege's zig-zag moves and detours in §§62–69 is that there is *one* method, not two. Is this what Dummett describes as "Frege's method" in the above quoted text? Not quite: Dummett's description fails to be general enough, since equivalence classes are just one of the indefinitely many possible options. Dummett himself, just one page before the quoted passage, had hinted at this absolute freedom in the choice of objects, and had rightly said that "all that is demanded of it [the identification via an explicit definition] is that it respect the criterion of identity embodied in the original equivalence" (p. 166). Thus there is no reason, no necessity at all to choose the equivalence classes rather than any other entity equally respecting the "criterion of identity embodied in the original equivalence."

When the "absolute freedom" (within the limits imposed by the required compatibility with the original equivalence) is highlighted, one begins to appreciate the potentially wild conventionalism and arbitrariness of the method. Dummett acknowledges this fault in Frege's definition of number (pp. 177, 179) but he fails to make clear enough his feelings about it — sympathy? rejection?

As I have written in my [*1979*] and elsewhere, Frege offers the first instance of the philosophically regrettable procedure I have called "the looking-around method" or, alternatively and following an etymologically wise and witty suggestion from my colleague J. Nubiola, "circumspection."

4. Fourth Station: Dummett's Pseudo-abstraction. After having (wrongly) portrayed Frege as champion of the fight against abstraction, now Dummett (equally wrongly) proposes to present him as champion of a well-understood abstraction, something for which Dummett coins the phrase "logical abstraction." The adjective "logical" is intended to differentiate this good abstraction from what Dummett calls "psychological abstraction." Dummett characterizes these two abstractions in the following manner:

> Both types of abstraction aim at isolating what is in common between the members
> of any set of objects each of which stands to each of the others in the relevant equiva-
> lent relation: Frege's logical method by identifying the common feature with the

maximal set of objects so related to one another and containing the given objects; the spurious psychological operation by deleting in thought everything except that common feature (pp. 167–168).

Frege's procedure in defining number in *Grundlagen* §§62–69, or the looking-around or circumspection method in general, cannot be referred to as "logical abstraction" or as abstraction of any sort simply because *there is no abstraction in it*, except in the following remote, indirect sense.

The "circumspective" logicians, in their "scanning of surrounding objects" (to use the *Oxford English Dictionary* phrase for "circumspection") *are not really interested in this or that particular object they choose*. For instance, Frege, as he indicates in a footnote to his definition of number, does not attach particular importance to saying that number is an extension or a concept. All they pay attention to is *the compatibility of the chosen objects with the previously established condition*; naturally, each author may add extra requisites — Frege for instance would not accept a physical object as denotatum of "the number of the concept F," not even a piece of the Kantian pure intuition of space and time. But then of course this obscure desire for doing abstraction should be properly expressed, and the equivalence class could no longer be put forward by the "circumspective" authors as the denotatum; rather *the equivalence class-qua-compatible with the previously established condition* should be the denotatum. Now the denotatum of the singular term "the equivalence class-*qua*-compatible with the previously established condition" is quite a new entity, an abstractum in a genuine sense. A rigorous handling of this abstractum and of the corresponding abstraction would require, first, disassembling the looking-around technique, and secondly, reassembling it according to some abstraction method, of which I know only one model, namely the "modern abstraction" incipient in Peano, visible in H. Weyl, and fully clear in Lorenzen (*cf.* my [*1979*]).

Dummett claims that "Frege was fully aware that the device accomplished, in a legitimate way, what others attempted to accomplish by means of the operation of psychological abstraction" (my [*1979*]). This is a mysterious statement: Dummett does not offer any proof of it, and I believe it is plainly false. To be sure, if asked, Frege would have agreed that his definition replaced the wrongheaded Husserlian and other philosopher's efforts to grasp number *via* abstraction by something better. But the quoted statement is misleading in suggesting that Frege thought of himself as still doing abstraction, and a better one.

Dummett's phrase "logical abstraction" is a striking new example of how the word "abstraction" has lost, in contemporary philosophy, and very especially in the logico-analytic tradition, any connection with what I think is essential to abstraction: the "leaving out" something and the "retaining" something (*cf.* my [*1991*]). Moreover, Dummett's text quoted above is perhaps a unique item in the collection of pseudo-uses of "abstraction" of the past one hundred years. I do not know in fact of any other text where the operation of "deletion" (the "leaving out," essential to abstraction in my view) is *explicitly* excluded from the notion of abstraction. Dummett's passage represents a final blow to that notion.

Bibliography

ANGELELLI, Ignacio. 1979. *Abstraction, looking-around and semantics*, Studia Leibnitiana **8**, 108–123.

—. 1984. *Frege and abstraction*, Philosophia Naturalis **21**, 453–471.

—. 1985. *El triángulo universal de Locke: su inmerecida fama*, Revista Latinoamericana de Filosofía **XI**, 47–53.

—. 1991. *La abstracción en la filosofía contemporánea*, in *El hombre: inmanencia y transcendencia (xxv reuniones filosóficas, 1988)* (Pamplona, Universidad de Navarra), vol. 1, 167–180.

—. 1993. Critical remarks on Michael Dummett's *Frege and other philosophers*, Modern Logic **3**, 387–400.

DUMMETT, Michael. 1991. *Frege and other philosophers*, Oxford, Clarendon Press.

LORENZEN, Paul. 1955. *Einführung in die operative Logik und Mathematik*, Berlin-Verlag.

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GÖDEL'S LAST WORKS, 1938-1974: THE EMERGING PHILOSOPHY

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Review of

Kurt Gödel, *Collected Works*. Volume II. *Publications 1938-1974*. Editor-in-chief Solomon Feferman. New York and Oxford, Oxford University Press, 1990. xvii + 407.

I

This is the second volume of this impressive series of Gödel's works. Its general characteristics are the same as those of the first volume which appeared in 1986 with the same editorial team. Since I already reviewed those characteristics in my earlier essay-review (THIS JOURNAL, 3, (1992), 58–74), I will concentrate here on the particular content we are offered now. As before, my viewpoint will be that of a philosopher, so I will make comments mostly on the philosophical implications of (or the philosophical theses explicitly maintained in) this set of Gödel's works, as well as on the way in which those implications are taken into consideration in the corresponding introductory notes. The reasons for choosing this procedure, already stated in my former review, can be summed up here: while Gödel's technical works have been well studied and have exerted massive influence in the logico-mathematical development of the second two thirds of this century, his philosophical ideas have been rarely taken into consideration by philosophers. Also, I think that much of Gödel's technical results were obtained mostly in search of logico-mathematical support for his philosophical beliefs. In this connection this second volume of his published works is really fundamental, as it was only in this period that Gödel decided to make public some