

PEIRCE AND FREGE, A QUESTION UNANSWERED*

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Abstract. The apparent absence, in reference or allusion, of Peirce to Frege and of Frege to Peirce, to work of the one in work of the other, and *vice versa*, would suggest — probably or even plausibly that neither Peirce nor Frege were aware of the other's work. It nevertheless seems almost inconceivable that Peirce and Frege could have been unaware of the the other's work, with circumstantial evidence available that would favour some likelihood of such awareness. This paper collates and speculates about the circumstantial evidence that is available.

1. Peirce and Frege's work. Charles S. Peirce (1839 – 1914) and Gottlob Frege (1848 – 1925) have emerged within recent historical research, as two extraordinary figures in logic and mathematics of the nineteenth and early twentieth centuries. Martin [1976, 244] observes that

One does not approach his [Peirce's] work as though it were a century old; one reads him as though he had written yesterdsy. His ideas are for the most part so astonishingly modern that one is shocked to encounter antiquainted methods, inadequate formulations, and the like. It is a tribute to his greatness that his mode of writing solicits criticisms in accord with contemporary standards of rigor and in the light of contemporary knowledge. Peirce and Frege are unique in this respect among the logicians of the nineteenth century.

It is noteworthy, then, that Peirce and Frege are two mathematicians and logicians having critically appreciated, upon publication, the work of Georg Cantor (1845–1918). It is all the more interesting that filling their writings, as they did, with caustic — and in Frege's case [see Dipert 1990/1991, 124–125], even hostile — criticism of mathematical and logical work of their day. The work of Cantor is rather singular (see [Hawkins 1986, 67–68] and [Houser 1990/1991]), in Peirce's and Frege's appraisals.

The question is: were Peirce and Frege also aware of each other's work?

* The present paper (with revisions, deletions, and additions) comprises extracts of [Hawkins 1989], with permission of the editors, *Studies in the Logic of Charles S. Peirce*, where (see References) the paper, *in toto*, is forthcoming.

Peirce, preparing a 1903 review of Russell's *The Principles of Mathematics* [1903] (see [Peirce 1966 VIII, 131, n. 1; Hawkins 1989], could not have but noticed the extensive expansions and numerous citations of Frege's work; nor that scant and cursory are the references, which Russell [1903, pp. 23; 26; 203, n. †; 232, n. †; 320, n. †, 376; 387, n. *] culls from Peirce's work. The question here, whether or not Peirce duly noticed Russell's references to Frege, is intriguing. Peirce's annotated, extant copy of Russell [1903], with his marginalia never being too far from Russell's references to 'Peirce,' seems rather to indicate (see [Hawkins 1989]) that Peirce's was a 'reading' via the entries, under 'Peirce,' in Russell's index.

There are yet some grounds, if circumstantial, to speculate about Peirce's knowledge of Frege's work.¹ There is, for example, the copy (catalogue number BC 135.F8) of Frege's *Begriffsschrift* [1879] in the library at Johns Hopkins University. There are no annotations by Peirce in this volume, but its date of accession is April 5, 1881, concurrent with Peirce's ephemeral tenure at Johns Hopkins (1879–1884), as an instructor in logic (see [Fisch & Cope 1952]). There is also the copy of Frege's *Begriffsschrift* now in the Rare Book Room at Princeton University, a volume having been once owned by A. Marquand, a contributor to Peirce's *Studies in Logic* [1883], the collection of papers under Peirce's editorship. "These papers, the work of my students," Peirce [1883, iii] remarks, "have been so instructive to me, that I have asked and obtained permission to publish them in one volume." There are, then, the citations of Frege's *Begriffsschrift* and Schröder's [1880] review of Frege's *Begriffsschrift* that C. Ladd-Franklin [1883, 70–71] enters in the bibliography of her contribution to Peirce's *Studies*. There is the copy of Schröder's [1880] review of Frege's *Begriffsschrift* among a bound collection (catalogue number Phil. 5005.4; binder's title "Formal Logic, 1866–1906") of offprints from Peirce's library, now in the Widener Library at Harvard. The collection was almost certainly bound after its acquisition by Harvard (see [Fisch 1972, 488, n. 9]), Schröder's review being numbered 23 (not by Peirce) and marked (by Peirce) in green pencil "Formal Logic." The copy of Frege's *Begriffsschrift* in the Johns Hopkins Library, Marquand's copy of the *Begriffsschrift* in Princeton University's Rare Book Room, the citation of the *Begriffsschrift* and Schröder's [1880] review of *Begriffsschrift* by Ladd-Franklin, and the copy of Schröder's [1880] review among Peirce's collection of offprints do not, of course prove that Peirce read the *Begriffsschrift* or Schröder's review of it.

2. Peirce, Schröder and his review. The context of Peirce's offprint of Schröder's review of the *Begriffsschrift* is an opacity. There is neither evidence when nor from whom Peirce acquired the offprint.

The very nature and length of Peirce's acquaintance with Ernst Schröder (1841 – 1902) being uncertain (see [Houser 1990/1991, 206–210]), it is uncertain when, or if, Peirce received the offprint from Schröder. The earliest of Schröder's extant letters to Peirce is

¹ As matters now stand, speculation (see [Peirce 1966, VII, 89–165; 1985, II, pp. 705–865]) appears inescapable, when circumstantial evidence is available and reason would seem to dictate an account, with probable or even plausible scenari, where direct evidence of the facts end æ as Peirce [1985, II, 862] succinctly states, however, "all facts involve ideas."

dated February 1, 1890 (Robin catalogue ms. L392;² see [Houser 1990/1991, 211]), and begins: "Herewith I am trying to ascertain whether your address continues to be the same that I have known some years ago." Thus (see [Houser 1990/1991, 206–210]), a correspondence earlier than 1890 is obviously to be surmised. It may be, then, that Schröder sent the offprint of his review to Peirce before Peirce's [1883] *Studies* went to press in 1882.

It may well be, however, that (compare [Haack 1993, 34]; see [Dipert 1990/1991, 124–126] and [Houser 1990/1991, 206–210]) Ladd-Franklin read and subsequently drew Peirce's attention to Schröder's review in the 1880 *Zeitschrift für Mathematik und Physik* and the Peirce-Schröder correspondence began later.

The tantalising possibility (compare [Haack 1993, 34]) is that, having read Schröder's review, Ladd-Franklin read Frege's *Begriffsschrift* — either the copy in the Johns Hopkins Library or Marquand's copy — she also drew Peirce's attention to the *Begriffsschrift*.

There is every indication that Peirce and Schröder were professionally acquainted. Peirce includes Schröder's [1877] *Der Operationskreis der Logikkalkuls* in the list of texts for his courses of 1882–1883 at Johns Hopkins (compare [Peirce 1965, III, 125–126, n. 2]; see [Fisch & Cope 1952, 288] and [Houser 1990/1991, 206]). Schröder's review of the *Begriffsschrift*, includes references to four of Peirce's early papers, citing three papers of 1867 — "On an Improvement in Boole's Calculus of Logic" [1867], "On the Natural Classification of Arguments" [1867a] and "On a New List of Categories" [1867b] and one of 1870 — "Description of a Notation for the Logic of Relatives, Resulting from an Amplification of the Conceptions of Boole's Calculus of Logic" [1870]. Peirce yet writes in 1903 [Robin catalogue ms. 302; see Peirce 1976, III, 741, IV, 150]; also [Houser 1990/1991, 208–209]) that "I value Schröder's work highly & he was a highly sympathetic man whom it was impossible to know and not like even more than the great merit of his work justified" — which intimates their meeting and personal acquaintance.

The years 1870, 1875–1876, 1877, and 1883 mark Peirce's sojourns in Germany. There is also the trip he made to Europe in 1880; but before his reaching Germany, Peirce was called home by the last illness of his father, Benjamin Peirce (1809–1880). The likeliest time of their meeting, if meet they ever did, is the year 1883, concurrent with Peirce's instructorship in logic at Johns Hopkins, Schröder at that time giving the offprint of his review to Peirce, and thereafter Peirce or Schröder beginning their correspondence.³

² References to "Robin catalogue ..." are to Peirce's papers as catalogued in [Robin 1967]. As members of the Peirce Edition Project study the Peirce corpus, some of the papers are being reassigned new catalogue numbers to reflect the newest scientific findings about the chronology of Peirce's writings.

³ Professor Nathan Houser has only recently communicated privately (on April 7, 1993) certain points that would bear upon these 'speculations,' which the author here incorporates for the reader's benefit. Professor Houser states that "there is no clear evidence that Peirce ever met Schröder," though "Peirce did visit Europe five times and met many mathematicians and logicians — and, of course, scientists, for his primary purpose was always scientific." Professor Houser also states that his "guess is that Schröder sent Peirce his review of Frege. Schröder and Peirce began exchanging papers by 1879 it seems, for Schröder (see [Houser 1990/1991, 206] remarks in his published note in Vol. 2 of the *Reportorium* (1879) that he received certain of Peirce's papers 'from their author.' On 14 Dec. 1880 Peirce sent a note to the Johns Hopkins Metaphysical Club reporting that he had 'lately received papers' from a number of European logicians and psychologists and included in the list is Schröder. I expect that the review of Frege was among the papers Peirce had received from Schröder. Note that in

3. Peirce, Schröder, Frege, their connection. The writings of Schröder and Frege pose an extremely complicated textual connection with Peirce's work. There are various depictions of Schröder's work in the writings of Frege, while the writings of Schröder portray variously Peirce's and Frege's work.

Frege's writings are infused, as mentioned earlier, with hostile criticism, an especial butt being Schröder's work. Schröder certainly reciprocates Frege's criticisms (see [Dipert 1990/1991, 124–125]). If Schröder is mistaken about Frege, Frege has as often been misguided about or misconceived Schröder.

Frege [1967, 193–210] in a review of 1895, is preemptory of Schröder's *Vorlesungen* [1890–1895] (see [Sinisi 1969]; compare [Kneale & Kneale 1962, 443]). He asserts that Schröder's 'collective' sense of 'class' entails a contradiction for ' Λ '. He also contends that Schröder's 'class' comprising 'individuals,' a unit-class (*eine singuläre Klasse*), coincides with an individual (which, however, S. Leśniewski (1886 – 1939) later shows does not follow).

Schröder [1880, 81–87, 90–94; see Hawkins 1981, 381–386; Houser 1990/1991, 206], in his review of Frege's *Begriffsschrift*, cites four of Peirce's early papers (the three from 1867 and one from 1870 already mentioned; see [Peirce 1966, VIII, 261, 262]); one of these, "On an Improvement in Boole's Calculus of Logic," Schröder cites as anticipating "various results" (*verschiedene Ergebnisse*) in [Schröder 1877]. He also cites G.W. Leibniz (1646–1716) and, inaccurately, G. Boole (1815–1864) as obviating Frege's system of notation or 'concept-script' (*Begriffsschrift*).

Schröder [1898, 60–61; compare 47–48, 51–53, 56, 59] claims that Peirce's work is "fundamental" to pasigraphy, with Boole and A. De Morgan (1806–1871) being Peirce's "English precursors," and of the the "several isolated attempts" that

Herr Frege, who heedless of anything accomplished in the same direction by others, took immense pains to perform what had already been much better done and was therefore superseded from the outset, thus delivering a still-born child.

Schröder, that is, erroneously or not, alleges (see [Hawkins 1981; 1989]; compare [Frege 1964, 97–114; 1969, 9–59], [Peckhaus 1990/1991]; [Robin catalogue ms. L237 (August 29, 1891), ms. 499, pp. [1] – [5]]) that, whereas Peirce's work is continuous with the development of pasigraphy, Frege's concept-script is an isolated attempt.⁴

The interesting question is whether or not Frege espied in Schröder's writings any of the representations and developments of Peirce's work. It is almost inconceivable that Schröder, in his [1880] review of Frege's *Begriffsschrift*, with his references to four of Peirce's papers, would have escaped Frege's notice, with Schröder so subject as he is, to Frege's critical ire (see [Dipert 1990/1991, 124–125]). It is Schröder's *Vorlesungen* [1890–

C.L. Franklin's papers in *Studies in Logic*, a paper she had written perhaps as early as 1881, she lists Frege, showing that he had by then come to the attention of the JHU group."

⁴ Schröder's comparng of Frege with Boole, if with Peirce or De Morgan, seems to be something of a philosophical mare's-nest, although Frege (quoted by [Hawkins 1981, 385; see 381–389]; see also [Dipert 1990/1991, 124–125, 131–135], [Peckhaus 1990/1991, 174–175] expresses the wish to create, "in Leibniz's sense," a "*lingua characteristica*," not a "*calculus ratiocinator*," opposing the conception of Boole's notation.

1895] that, having been reviewed in 1895 by Frege (see [Frege 1967, 193–210]) that seems to have been Frege's more obvious secondary access to Peirce's work, containing as it does numerous discussions (prolix and laudatory though they may be) of Peirce's work. Even so, Frege's writings appear to be quite unaffected by Peirce's work (see [Hawkins 1975, 112, n. 2; 1981, 387]).

4. Peirce's awareness of Frege's work. Peirce [1965, IV, 89; see Hawkins 1975, 112, n. 2] poignantly complains in 1893 that "for all my life my studies have been cruelly hampered by my inability to procure necessary books." Peirce's words here are particularly significant, 1893 being the year in which volume 1 of Frege's *Grundgesetze* [1893] was published. The circumstances — the availability of the copy of Frege's *Begriffsschrift* in the Johns Hopkins library, of Marquand's copy of the *Begriffsschrift*, or of Peirce's offprint of Schröder's [1880] review of the *Begriffsschrift* — still seem to favour a distinct possibility that Peirce could not be unaware of Frege's work.

It appears ultimately, however, that Peirce's writings contain no mention of Frege (see [Hawkins 1975, 112, n. 2; 1981, 386–387; 1986, 67–68; 1989], although the polymath Peirce was generally attentive to references. Peirce ([Peirce 1965, III, 228]; cf. [1965, III, 211–212], [1975, 110–111], [1883, 72–106]; see [Beatty 1969], [Dipert 1990/1991, 135], [Hawkins 1975; 1981], [Houser 1990/1991, 208], [Martin 1976]), for example, seems entirely unaware of the 'quantification' in Frege [1879, §§9–12] when (1885) he renders a notational separation of the 'quantifying' and 'indexing' functions as a modification of O.H. Mitchell's 1883 system, so that " $\sum x_i$ means that x is true of some one of the individuals denoted by i or... $\prod x_i$ means that x is true of all these individuals."

"My analyses of reasoning," Peirce observes in 1903 ([Peirce 1965, V, 91]; see [1975, 112; 1981, 386–387; 1986, 64; 1989]), "surpasses in thoroughness all that has ever been done in print, whether in words or in symbols — all that De Morgan, Dedekind, Schröder, Peano, Russell, and others have ever done." The presence of 'Russell' here is as significant as the absence of 'Frege' in 1903, that being the year when Russell's *Principles* [1903] and the second volume of Frege's *Grundgesetze* [1903] were published or "in print," and the year of Peirce's review of Russell's *Principles*.

There are two instances where Peirce expressly gauges his "rank" as a 'logician'; where the absence of 'Frege' seems to further suggest Peirce was unaware of Frege. Thus, Peirce (in Robin catalogue ms. L387 (November 15, 1904); quoted in [Fisch 1972, 486]) writes that "I place myself somewhere about the real rank of Leibniz," who "had the advantage of coming to a field into which no reapers had come." Peirce (in Robin catalogue ms. L482 (June 18, 1909) or [Peirce 1976, IV, vi–vii]; also quoted in [Fisch 1972, 487]; see [Houser 1990/1991, 208]) also later writes, thinking that "the only writers known to me who are in the same rank as I are Aristotle, Duns Scotus, and Leibniz, the three greatest logicians in my estimation, although some of the most important points escaped each."

5. Peirce and Frege, an epilogue. The apparent absence, in reference or allusion, of Peirce to Frege and of Frege to Peirce, to work of the one in work of the other, and *vice versa*,

would suggest — probably or even plausibly, if the scenari were proffered (in the present paper, see footnotes 1 and 3) to the contrary — that neither Peirce nor Frege were aware of the other's work. It nevertheless seems almost inconceivable that Peirce and Frege could have been unaware of the the other's work, with circumstantial evidence available that would favour some likelihood of such awareness.

Peirce ([1976, IV, 152]; see [Murphey 1961, 1, n. 1]) refers, in 1903, to his logical work as "a rich vein," only "scattered outcroppings" of which are "in print," while Frege [1969, XXXIV] refers to "Gold" in his manuscripts. The "vein" and "Gold" are currently being worked, and perhaps someday will show that Peirce and Frege were aware of each other's work — for now a question unanswered.⁵

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⁵ Professor Nathan Houser (see footnote 3) has even more recently communicated provately (on June 29, 1993) further points that would bear upon these 'speculations,' which the author, likewise, incorporates for the reader's benefit. Professor Houser states: "it seems to me certain that Peirce and Frege knew of each other as co-workers in the field of logic, even though neither seems to have been motivated to look at the other's work. [Hawkins] mentions several of the links. But in general it must be said, I think, that no one who read the logic books and periodicals of the last quarter of the nineteenth century could have escaped hearing of Peirce and Frege. Each man was mentioned too often to be missed, and their work was discussed (well or poorly) and their papers and books were reviewed. In some cases, as in Venn's paper in the 6 Dec. 1880 *Proceedings of the Cambridge Philosophical Society*). It may also be worth noting that it was not only from Schröder's review of the *Begriffsschrift* that Peirce may (and probably did) forma a negative opinion of Frege's system, but from other reviews such as that of Venn in the April 1880 *Mind*, where Venn writes that 'it does not seem to me that Dr. Frege's scheme can for a moment compare with that of Boole' and 'I must confess that it [Frege's system] seems to me cumbrous and inconvenient.' Note also that Peirce sent (or had sent) a copy of his 1880 "Algebra of Logic" to the University of Jena Library, for which he received a receipt. Finally (for the moment), I ... recall that a correspondent of Frege's wrote to him mentioning Peirce's *Studies in Logic* in a way that obviously supposed that Frege knew of that work. The editor (Irving Anellis) adds that Louis Couturat, in his 6 Jan. 1901 letter to Frege, see p. 9, Frege (Gabriel, et al., eds.), *Philosophical and Mathematical Correspondence* (University of Chicago Press, 1980), CSP's *Studies* is not mentioned, but Couturat does there compare MacColl's $A : B$, CSP's $A \multimap B$, & Schröder's $A \neq B$."

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