

A SKETCH OF JEAN VAN HEIJENOORT'S VIEW OF THE HISTORY OF
MODERN LOGIC:
INTRODUCTION TO AN UNPUBLISHED PAPER

IRVING H. ANELLIS

Jean van Heijenoort was formally trained as a mathematician and specialized during his graduate student years in differential geometry. Most of his scholarly career, however, was devoted to logic, foundations of mathematics, and especially the history of logic, in which he was largely self-educated but deeply influenced by his close friend Georg Kreisel.

From Frege to Gödel was his most influential work and the primary basis for his reputation. Yet it was not his only important contribution: he edited, as well, a collection of Herbrand's logical writings, served on a number of editorial boards, including the *Journal of Symbolic Logic*, to which he contributed many reviews, and, at the time of his death, was one of the co-editors of Gödel's collected works; there were, besides, his shorter papers in the history and philosophy of logic, some published during his lifetime, others not, many posthumously also published in his *Selected Essays*. In addition, he did original work, published formally and informally through distribution to his students, in model-theoretic proof theory. The work in model-theoretic proof theory centered on his proofs of the soundness and completeness of the falsifiability tree method for both classical and nonclassical logics.

For van Heijenoort (1967, p. vii), "Mathematical logic is what logic, through twenty-five centuries and a few transformations, has become today." *From Frege to Gödel* documents the most crucial and profound of those transformations. Van Heijenoort's special interest within this perspective was the development of quantification theory as a family of formal systems providing different techniques for carrying out proofs in first-order logic. The principal contributors to this development were Frege and Hilbert, with their axiomatic systems and Hilbert's meta-

mathematics; Löwenheim, whose work helped to clarify the concepts of *satisfiability* and *validity* for Hilbert's system; Herbrand, whose development of his own quantification theory unified and advanced the work of Hilbert and Löwenheim; Gentzen, who both developed the sequent calculus and helped formulate the method of natural deduction out of which the falsifiability tree method eventually grew through the intermediacy of Beth and Hintikka; and Gödel, whose incompleteness theorems defined the limits of Hilbert's program even after Russell's theory of types was supposed to have insulated logic from the paradoxes. Van Heijenoort's booklet *El desarrollo de la teoría de la cuantificación* encapsulates this history.

Turning back from this perspective to the broad outline of the history of mathematical logic, van Heijenoort (1986) once wrote: "Let me say simply, in conclusion, that *Begriffsschrift* (1879), Löwenheim's paper (1915), and Chapter 5 of Herbrand's thesis (1929) are the three cornerstones of modern logic." Van Heijenoort's paper "Historical Development of Modern Logic," written in 1974, can be viewed as a detailed explanation of the meaning of this statement.

The paper is filed in Box 1 of the Jean van Heijenoort papers, 1946-1983, Archives of American Mathematics, University Archives, University of Texas at Austin. The paper is a 26-page typescript, with handwritten corrections inserted between the line or added to the lines of type or written in the margins, along with minor notations and printer's instructions inserted in the margins. In a number of cases, typed text has been deleted by van Heijenoort by blocking out; and in some of these cases, the blocked out text has been replaced by handwritten corrections. Where handwritten additions or corrections are legible, whether in the margin or elsewhere on the page, we have included them in the text. This paper was written before the deaths of Gödel and Tarski and these dates were not given in the manuscript; it is unclear why van Heijenoort did not complete the dates for Russell. We have without comment completed van Heijenoort's text by adding the appropriate dates. In the case of Löwenheim (on p. 6 of the typescript), van Heijenoort wrote "(1878-194?)". The year of Löwenheim's death has been given by Thiel (1975) as 1957 and we have as a consequence completed his dates using Thiel's information. Van

Heijenoort's other textual annotations are introduced as editorial comments and are given in numbered footnotes. Minor typographical, grammatical, and spelling errors in van Heijenoort's draft are corrected here without comment. Editor's footnotes are also used to call attention to other textual or factual discrepancies and for adding references.

References.

Thiel, C. (1957). *Leben und Werk Leopold Löwenheim (1878-1957)*, Jahresbericht der Deutschen Mathematiker-Vereinigung 77, 1-9.

van Heijenoort, J. (ed.) (1967). *From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931*, Cambridge, Mass., Harvard University Press.

van Heijenoort, J. (1986). *Logic as calculus and logic as language*, in J. van Heijenoort, (1986). *Selected Essays*, Naples, Bibliopolis (copyright 1985), 11-16; see p. 16.