Review of В.А. Бажанов "Николай Александрович Васильев", Москва, Наука, 1988, by Charles Duffy.

The author is the principal authority on the life of Nikolai A. Vasil'ev (henceforth referred to as Vasil'ev) and a member of the Lobachevskii Bicentennial Committee which is planning in the summer of 1992 to honor the memory of Kazan's greatest scientist and geometer, Nikolai I. Lobachevskii.

Bazhanov's small book is informative and readable. The author's intent is to give an overview of the life and work of Vasil'ev. His book is not a technical work on logic written for professional logicians. Its 144 pages are divided unevenly into eleven chapters. Following the main text, brief notes for each chapter make use of previously unpublished documents discovered by the author in his research on Vasil'ev. Following the notes, an appendix gives the author's Russian translation of the English precis of Vasil'ev's Imaginary Logic, contained in the Acts of the Fifth International Congress of Philosophy held in Naples in 1924. The appendix is followed by the one page review of the works of Vasil'ev on mathematical logic, written on January 4th, 1927 by Professor N. Luzin of Moscow University. On page 138, the author comments upon the review of Luzin. Excerpts taken from the poetry of Vasil'ev appear on page 139.The bibliography on pages 140-144 lists the published and unpublished works of Vasili'ev, archival material, and literature concerning the work of Vasil'ev.

Chapter one deals with the genealogy of Vasil'ev's family. Both on its maternal and paternal sides, his family came from a solid intellectual tradition. Kazan and its university were intimately connected with that tradition. Vasil'ev's paternal grandfather, Vasilii Vasil'ev, was a well known orientalist, member of the St Petersburg Academy of Science, and professor of Mongolian and Chinese at Kazan University. He married Sofie Ivanova Simonova, daughter of the rector of Kazan University, Ivan Simonov. Simonov was a well known astronomer, highly regarded in western Europe, and a participant in two trips around the world with admirals Bellinsgauzen and Lazarev(1819-1821). Simonov was a lifelong colleague of Nikolai Lobachevskii and replaced him as rector of Kazan University in 1846. The oldest son of Vasilii Vasil'ev, Aleksandr (1853-1929), the father of Vasil'ev, had an extraordinary gift for mathematics. Among the mathematicians he met were Darboux, Hermite, Hilbert, F.Klein, Kovalevskaia, Beppo Levi, Lie, Mittag-Löffler, Poincaré, Weyl, and Whitehead. Aleksandr published several articles and books on the life and work of Nikolai Lobachevskii. One such book, Nicolai Ivanovich Lobachevsky (AVV), however, received a very poor review by Charles Sanders Peirce in the 4 April 1895 issue of the magazine The Nation (Peirce). Aleksandr also played a leading role in promoting a Lobachevskii prize granted by Kazan University for excellence in mathematics. Sophus Lie, well known to Aleksandr, received the first Lobachevskii prize in 1897. Along with V.V. Bobynin, Aleksandr Vasil'ev was instrumental in promoting the study of the history of mathematics in Russia. Nikolai Vasil'ev, Aleksandr's brother and uncle of Vasil'ev, led the life of a political exile. Gifted and endowed with revolutionary fervor, Nikolai was acquainted with Karl Marx and became an associate of the father of Russian Marxism, Georgii Plekhanov. In discussing the position and teachings of the Menshevik faction of Russian Marxists, Lenin repeatedly mentioned the name of Nikolai Vasil'ev.

The maternal side of the family of Vasil'ev was also noted for outstanding educators and intellectuals. Pavel Maksimovich, his maternal grandfather, was a well known educator. He organized schools for women throughout Russia and was active in the formation of local public schools in the province of Tver (Kalinin). He also wrote a primer for reading that was universally acclaimed in educational circles. Maksimovich's son, Vladimir, the brother of Vasil'ev's mother, was a fine mathematician and a close friend of Sofia Kovalevskaia and P.S. Poretskii. Platon Poretskii was the first to teach mathematical logic in Russia. He did so at the University of Kazan.

The second chapter gives an account of the life of Vasil'ev himself. Born on June 29th, 1880, in St. Petersburg, he spent his youth in a world of adult conversation. His gift for foreign languages was great; remarkable was the depth and the range of the books he read as a child. From his earliest days Vasil'ev kept a diary in which he composed poetry and recorded his reflections on the nature of reality and the meaning of man. Strange as it may sound, Vasil'ev studied medicine at Kazan University because of his great interest in psychology, philosophy, and logic. He completed the medical faculty program in 1904 and prepared to practice medicine. In that same year, Vasil'ev published a book of poems entitled "Тоска по Вечности" (Longing for Eternity). Shortly after graduation from medical school he married Ekaterina Zab'yalova. She loved to sketch and had a deep interest in the theater. Shortly after their marriage, they became separated. He became a country doctor. She remained in Kazan. In 1906 Vasil'ev gave up the practice of medicine and began to study philosophy and psychology at Kazan University. His diary testifies that he continued to compose poetry and record his reflections during his studies.

The years 1907-1908 mark the transition from Vasil'ev the poet-philosopher to Vasil'ev the professional philosopher. As a poet-philosopher, Vasil'ev may be called a symbolist. His poetry and reflections written in his diary repeatedly refer to the world as a "forest of symbols." The whole universe is one cohesive system of symbols. Everything is a reflection of something else. Vasil'ev quoted Göthe's **Faust** when he wrote: " Alles vergängliche ist nur ein Gleichnis." The poetry of Algernon Swinburne and Emile Verhaeren are frequently cited by Vasili'ev in his diary. Vasil'ev found in these poets the realization of the emotional value of mere sounds which mystically unite the art of poetry and music. Dr Bazhanov points out that Vasil'ev's later development of Imaginary Logic was foreshadowed in his poetry.

In 1908 Vasil'ev attended the Third International Congress of Philosophy in Heidelberg. Subsequently Vasil'ev, while he continued to write articles on Swinburne and Russian literature, prepared himself seriously for a career as a professional philosopher and began to formulate his ideas concerning Imaginary Logic. On May 18, 1910, in a trial lecture given at Kazan University, Vasil'ev for the first time expressed his thoughts concerning a non-Aristotelean (Imaginary) logic. In October of 1910, Vasil'ev was appointed private-docent at Kazan University and began to give lectures on philosophy and logic. On January 13th, 1911, Vasil'ev addressed a session of the Physics-Mathematical Society of Kazan University. His theme: "Non-Euclidean Geometry and Non-Aristotelean Logic." In 1912-1913, Vasil'ev published two large articles entitled **Imaginary Logic** (NAV 1912) and **Logic and Metalogic** (Nav 1912a). In these articles, Vasil'ev makes it clear that the "imaginary geometry" of Lobachevskii inspired him to conceive of the possibility of an "imaginary logic." These articles and a few others he wrote prior to the outbreak of World War 1 contain his ideas concerning what is now know as multi-valued logics. Nothing that he wrote subsequently added significantly to his thought on the subject.

In October 1914, Vasil'ev was mobilized for war. He served as a doctor in the army. In 1915 he was decorated for outstanding bravery at the front. In 1916 he suffered severe depression, war related, and was released from military service. In December 1917 he was named docent at Kazan University and became a full professor at the university in October 1918. Then came a second, severe attack of depression. The counter-revolutionary Whites and a legion of former Czech war prisoners occupied Kazan in the summer of 1918. Vasil'ev has gone to Kainki (near Kazan) with his family to escape the occupation but the civil war followed Vasil'ev to Kainki. In the ensuing battles which raged around him, Vasil'ev became very ill and was committed to a hospital. In 1921-1922 Vasil'ev had sufficiently recovered from his depression to head a commission in Kazan to study the psychology of young people. He was particularly active during this period of his life in helping the blind of Kazan. Then he suffered yet another emotional crisis: his commission was disbanded and the institute which supported it was abolished. Vasil'ev then retired from public service and remained more or less incapacitated for the rest of his life.

Vasil'ev was diagnosed as a manic-depressive in 1922. At that time, Vasil'ev declared that his depression was due to the effect of a house fire upon him when he was a very young boy. Later, upon the prompting of his father Aleksandr, Vasil'ev sent the manuscript **Imaginary Logic** to the Fifth International Congress of Philosophy held in Naples in 1924. In 1925 Vasil'ev became estranged from his wife, who became "enraptured with religion." Thus, when well, Vasil'ev did not return home to his wife and spent the next eleven years of his life struggling to regain sanity and normalcy. He died on December 31, 1940, in Kazan.

Chapter 3 discusses Vasil'ev's book of poems, "Longing for Eternity." Vasil'ev's poetry was philosophical in tone. There are numerous references to infinity, eternity , and imaginary worlds where different rules of thought prevail. Chapters 4, 5, and 6 are very short. They describe the transition of Vasil'ev from poet to philosopher, and depict his growing desire to create an alternative to traditional logic. "If there is a geometry of curved space, why not a curved logic?", he asked. In chapter 7 the author fixes precisely the date of creation of Vasil'ev's new logic. Vasil'ev's trial lecture on May 18, 1910, was the basis for an article published in the Scientific Notes of Kazan University and later printed in the form of a brochure. In this article, Vasil'ev considers the formal aspects of his new logic. He did not employ the concept "imaginary logic" and did not analyze the law of contradiction, as he did later. The brochure was entitled "On Particular Judgements, On the Triangle of Opposition, On the Law of the Excluded Fourth." In this article Vasil'ev replaces the traditional square of opposition with his triangle of opposition. Vasil'ev observes that the traditional square of opposition expresses general and indefinite propositions and judgements (indefinite judgements are equivalent to particular judgements). Vasil'ev states that the particular proposition containing the word "some" must be understood as neither affirming nor denying that "some" means "only some." He further states that sub-contrary propositions are not opposed to one another, that is, sub-contrary propositions can not be advanced against one another. Thus, I and O propositions (referring to the traditional "four forms" of propositions called A, E, I, and O propositions in a symmetrical square of opposition) can be parts of one judgement called "M". In that case, the pairs (A,E), (A,M) and (E,M) both can not be true but both could be false, in which case the Law of the

Excluded Middle fails. Vasil'ev likewise observes that an A and an O proposition can both be false. He cites the example of a triangle with 360 degrees. "All triangles have 360 degrees; some triangles do not have 360 degrees." Both propositions are contradictories but both are false, for the O proposition contains the sense that some triangles have 360 degrees. If "some are not" contains the sense that "some are", then the traditional square of opposition does not hold. A "triangle of opposition" holds. Furthermore, the traditional square of opposition will not hold if the categorical propositions in question are analytic sentences or if an existential assumption is not made. In Chapter 8, Vasil'ev notes that the existence of many geometries suggests the existence of many logics. Vasil'ev conceives a body of metalogical principles, valid in all possible worlds, and a body of logical laws which vary from one imaginary world to another, depending upon the ontological reality of that imaginary world. The Law of Contradiction and the Law of the Excluded Middle belong to the ontological realm of logic. They may fail in some imaginary worlds. The Law of Non-Self Contradiction belongs to the metalogical realm, true whenever and wherever one thinks logically. As there are three kinds of lines in the imaginary geometry of Lobachevskii, so there are three kinds of judgements in imaginary logic in which the status of a proposition may be positive, negative, or indifferent (an indifferent proposition contains a positive and negative proposition). Vasil'ev's Law of the Excluded Fourth is applicable in a three state world. Chapter 10 briefly discusses logical and historical methods in ethics. Chapter 11 traces and stresses Vasil'ev's role as forerunner in the development of multi-valued logics (cf. Rescher) and paraconsistent logics (cf. Arruda). Mention is made of the work of A.I. Arruda and N. Da Costa in constructing a logical system possessing the basic features of Vasil'ev's logic. Pages 131-133 in the Notes present such a system. Bazhanov concludes his work with the poetic lines of Emile Verhaeren. These lines were quoted on more than one occasion by Vasil'ev:

Сегодня всему наступает пора, что чуть ли бредом казалось вчера.

(Today all has come about that seemed impossible yesterday)

An Analogy: Geometry and Logic

Like his father before him, Vasil'ev was deeply impressed by the work and thought of Nicholas Lobachevskii. Lobachevskii as is well know, created one of a number of new geometries, different from Euclid's, but just as consistent, complete, and productive. Lobachevskii's new hyperbolic geometry inspired Vasil'ev to seek a "new logic" and to consider alternatives to the classical logic of his age. As he developed his logic, Vasil'ev made an explicit comparison between the existence of many geometries and the possibility of many logics. He was not alone in doing this. Lukasiewicz and Post also likened the diversity of logical systems with the plurality of geometrical systems; thus, for example, Lukasiewicz wrote that "the relationship of many-valued logics to the two-valued logic reminds one of the relationship of non-Euclidean geometries to the geometry of Euclid. Like the non-Euclidean geometries, so too the many-valued logics are internally consistent, but different from the two-valued logic." Vasil'ev firmly believed that the analogy of many geometries and many logics was appropriate and instructive, and that there is a close connection between the methods of logic and geometry. Whatever disanalogies there are - and there will always be disanalogies - are not such as to seriously weaken the analogy and to cast doubts upon the value of its consequences.

A critical and historical analysis of Vasil'ev's claims begins with a determination of what constitutes a system of logic. This determination is important in judging the suitability of the analogy which Vasil'ev draws between logic and geometry. There are a number of questions to consider: In what sense is one system of logic different from another? Then, what is a system of geometry? In what sense is one system of geometry different from another? A brief historical survey of logic and geometry precedes answering these questions.

In antiquity, the question "What is logic?" was debated by the Stoics and the Peripatetics. The Stoic logicians developed the informal eristic or argumentative logic of debate (dialectics) and asserted that logic was a branch of philosophy. The Peripatetic logicians, who followed Aristotle in developing syllogistic from the standpoint of formal proofs, claimed that logic was a preliminary to philosophical studies, as it had always been. But both the proponents of the "new" Stoic logic and the Peripatetic adherents of the "old" logic agreed that logic dealt with the patterns of arguments, with the relations of universals, and with the relations of propositions (one of the Stoic contributions to the study of logic, who investigated the relations of propositions in terms of inference schemata). In the beginning, the study of logic also involved the study of syntax and semantics. and of language in general. The correct use of words, classification of different kinds of sentences, and systematizing types of knowledge were all regarded as parts of the field of logic. No wonder then that logic was sometimes identified even well into the nineteenth century with the theory of knowledge or with the psychology of reasoning (for example, by Wundt, Lotze, and Ueberweg). Indeed, viewed from this perspective, the basic analogy is one of logic and language: basic reasoning skills are learned the same way as basic language skills.

In the first half of the nineteenth century, however, Bernard Bolzano carefully distinguished the abstract entities of "logic" from the corresponding linguistic and psychological entities. Logic, for Bolzano, was more than the study of grammar, semantics, and rhetoric. Logic was primarily the theory of science. The theory contains a formalized language in which it is possible to follow the course of reasoning without knowing the meaning of the terms in which the reasoning is expressed. Logic is thus "pure", uninterpreted, devoid of any semantics. In calling such a system "logic", Bolzano departs from tradition, for the principles of inference and the correctness of reason historically involved meaning. In the middle of the nineteenth century, George Boole began the "mathematization" of logic by developing the algebra of logic in much the same way that Fermat and Descartes developed analytic geometry by showing how to represent geometric constructions algebraically. In the latter part of the nineteenth century and into the twentieth century, Frege, Peano, Whitehead, and Russell completed the process of assimilation by identifying logic as the foundation of mathematics. N-valued calculi or n-valued logics? They are two different names for the same thing. There would be no analogy of logic and geometry.

In our day, the word "logic" is used in contexts that are foreign to the use of past generations. Such names as "lattice-valued logic", "parallel set logic with bio-device model", and "maximum length logic", all employing different systems of "multi-valued logics" are frequently encountered in computer circuit design. It is difficult to determine what these various uses of the word "logic" have in common with past understanding of the word, and, given the inherent vagueness of words, how legitimate or radical a departure their meanings are from traditional use. Given this situation, it is best to seek those semantic features of the word "logic" which occur persistently in its historical usage, for the proper meaning of the word "logic" is best derived from the ongoing tradition of its use. Some common features in the historical use of the word reveal: logic almost always involves a study of the rules of inference and meaning; it further involves the classification of the principles of valid inference in some interpreted system, for ordinary logical laws (whatever they may be) tend to be vague and indefinite when not embodied in or with some system; and finally, the study of logic embraces not only form and validity but also truth and meaning. Thus, a system of uninterpreted axioms whose consequences are not inferable from those same axioms will not be called a "system of logic". Whether an all inclusive set of logical axioms can ever be devised remains uncertain, given Gödel's proof that there will always be mathematical statements which cannot be formally deduced from a given set of axioms by a closed set of rules of inference.

What is geometry? The answer to that question depends to some degree on the role geometry is thought to have in unifying mathematics, and on one's notion of the nature of definition. Can geometry, by definitional fiat, be reduced to an exercise in logical syntax, a purely hypothetical-deductive system? Or must not abstract systems, at the very least, be capable of spatial interpretations to qualify as "geometry"?

The earliest records of the study of geometry come from Babylonia and date from about 3000B.C.; those from Egypt date from about 2000 B.C. These documents show that geometry was viewed as a collection of rules for the measurement of physical objects. These rules were developed empirically and heuristically. Early Egyptian surveying practices mark the high point of geometry as a practical science. Then a gradual transition in the understanding and use of geometry took place. In the sixth century B.C., Thales and Pythagoras replaced the empirical notions about physical objects in a reality called "space" with abstract concepts about idealized objects, such as the rectangle and the circle. They realized that some statements of geometry did not require any empirical verification at all; they could be deduced as logical statements from other statements of geometry. Thus, in the course of time, the propositions of geometry were divided into two classes: those that needed empirical verification and those that did not.

By the time of Euclid, geometry had become an axiomatic study of a unique, homogeneous realm called "space" where idealized objects, considered only from the point of view of their extension, could be freely moved and aptly compared with one another. Superposability became a fundamental condition of absolute space. An appeal to intuition and visual imagery provided justification, if not empirical evidence, for the basic statements of geometry. In Euclid's Elements, theorem followed theorem in an elegant display of logical reasoning. That geometry had become an exercise in logic with the use of idealized geometrical figures is made clear by the attempt made by many generations to derive Euclid's fifth postulate from his first four postulates by means of geometrical reasoning. The fifth postulate, so proved, would then be logically dependent upon the first four postulates and thus become a theorem. Motivation for this persistent effort through many centuries was the knowledge that the converse of the fifth postulate could be proven. Why, then, could the fifth itself not be proven from the other postulates and common notions? The answer to that question challenged the geometrical and logical acumen of scientists for two millenia. The fifth postulate also made an assertion about infinitely remote regions of space. It claims that two lines will not meet even "at infinity". But the ancient geometers were familiar with asymptotic lines that do meet at infinity, although they do not intersect in any finite region of space. Thus, it is not intuitively clear that only one line can be drawn that will not meet the given line even at infinity. The attempt by Saccheri in 1733 to prove the dependence of the parallel postulate, using a logical principle called the "consequentia mirabilis", resulted in a proof for its independence instead, although it was only in 1868 that Beltrami used this fact to develop the concept of the relative consistency proof.

The creation of classical non-Euclidean hyperbolic geometry at the beginning of the nineteenth century by Lobachevskii and Bolyai, whose efforts were preceded by Saccheri, Lambert, Gauss, Schweikart, Wachter, and Taurinus, demonstrated that an alternative system of geometry is obtainable from Euclid's first four postulates. As a result, different axiom sets for non-Euclidean as well as Euclidean geometry were developed, such as those of Pasch, Peano, Hilbert, and Veblen. Beltrami was able to show that the fifth postulate is independent of the first four postulates but consistent with them. He demonstrated that if the Euclidean system is consistent, then so are the non-Euclidean systems. That is, Euclid's first four postulates, together with his fifth postulate, form a system that is consistent, complete, and categorical. But, likewise, Lobachevskii's denial of Euclid's fifth postulate, together with the first four postulates, forms a system that is consistent, complete, and categorical (provided that the standard unit of length is used in all models). In his Erlanger Program of 1872, Felix Klein undertook to unify the many systems of geometry by a projective model in which all models, the hyperbolic and parabolic (to name the more prominent ones), given an appropriate metric, can be deduced from an absolute space provided by the model. Klein thereby turned geometry into the study of congruent transformations in "conceived space". Accordingly, only systems of invariant transformations that have spatial interpretations will be called geometries. Hilbert's Foundation of Geometry sought to take the Erlanger program of unification one step further by providing a "pre-systematic geometry" or meta-geometry as a system of axioms which would be sufficiently general and sufficiently powerful to serve all of the various geometries.

What can be said of the analogy of geometry and logic? Some speak of the analogy as seriously deficient and burdened with crucial disanalogies. It is said that systematic logic is bound to a "pre-systematic logic" or metalogic as a sine qua non, and that one need not employ a pre-systematic geometry to formulate a system of geometry. This "pre-systematic logic" is conceived as a core of logical principles that plays a role in the articulation of all logical systems. Thus, Tarski proved that the classical two-valued logic is the only complete and consistent logic to which every system of alternative logic that satisifies certain specified conditions and remains consistent can be extended. On the contrary, it is asserted that there is no hard core of geometrical principles that plays a role in the formation of all geometrical systems. It is further claimed that the nature of the choice of one logical system over another logical system is different from the nature of the choice of one geometrical system over another.

In support of the views of Vasiliev, it can be stated that there are different logical and geometrical systems that do not "reduce" into one system of logic and one system of geometry. None of these different systems can be grounded in any absolute manner. None enjoys abolute conceptual priority over all others. All systems presuppose some "absolutes" - some necessary and sufficient conditions - that provide a basis for the use of the words "logic" and "geometry" as appropriately designating these different systems. What these absolutes are in the case of logic and geometry remains vague and indefinite. Certainly the Law of the Excluded Middle is not a good candidate for a pre-systematic principle, if it must accompany all logical systems. However, it may be a pre-systematic principle for some logical systems. Certainly, the first four postulates of Euclid are fairly good candidates for pre-systematic geometrical principles for the Euclidean system. These principles are also pre-systematic to Lobachevskii's hyperbolic geometry. However, these principles are not pre-systematic to all systems of geometry. Accordingly, there does not seem to be a crucial disanalogy between logic and geometry based upon the need for pre-systems.

Concerning the choice of systems, logical or geometrical, there seems to be no fundamental difference in the nature of that choice that would vitiate any analogy between choosing logics and choosing geometries. One logic is better than another in a sense analogous to one geometry being better than another. No logic is better than any other logic in any absolute sense; no geometry is better than any other geometry in any absolute sense. This is probably the case because no system or system of systems of logic and geometry can capture completely the complexity and diversity of reality. Seen in this light, one system of geometry (as well as one system of logic), may be "better" than another system of geometry in that it is more convenient, effective, and productive for the task at hand, if there be one. This, of course, does not deny that some systems are more powerful instruments than others. It only affirms that "better" must be taken in a relative sense. No system has absolute priority over another. Insofar as Tarski proves that some systems of logic are demonstrably fragments of some other system, this only shows that the systems containing those fragments contain pre-systematic elements of several alternative systems.

Vasiliev's basic intuition is correct. There can be many logics, as there can be many geometries. As Lobachevskii denied a time-honored principle - the Fifth Postulate of Euclid - to create a new space, so Vasil'ev denied an equally venerated principle - the Law of the Excluded Middle - to facilitate the creation of new systems of logic. The advancement of creative thought and scientific achievement is in their debt. A debt is also owed to Dr. Bazhanov for his informative work on the life and work of Vasil'ev.

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