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Dennis E. Hesseling
Gnomes in the Fog,
The Reception of Brouwer's Intuitionism in the 1920s
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REVIEW

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Intuitionism has quite an eventful story. In the mid thirties, it looked like it had no future. Hilbert had won, albeit so unfairly that Brouwer decided to withdraw from foundational research and turned instead to non academic activities, like local government or fighting for duped shareholders. Today, the situation looks quite different: Wittgenstein and Dummett taught us the philosophical value of intuitionism, and computer scientists appreciate a logic which chooses to stick to effectivity. Hesseling's book relates the story of the very passionate debate which just after the first world war arose from Brouwer's ideas. As the *Grundlagenstreit* counts among the highlights of logic in the last century, every logician had some knowledge of it, but the precise story remained to be told. In order to have a well-defined field of inquiry, the author takes as *terminus a quo* Weyl's siding with intuitionism, and, as *terminus ad quem*, Brouwer's withdrawal in the mid thirties. As the author remarks, this choice contributes to a one sided image of intuitionism and of its founder. It must never be forgotten that Brouwer had a very bad opinion of logic, that his goal was the reconstruction of mathematics upon a safer basis than the basis afforded by Cantorian set theory; he strove for a good theory of the continuum and of real numbers, —as if, said Bishop, he feared that "unless he personally intervene[s] to prevent it, the continuum would turn out to be discrete " (p. 66n). But this positive part of Brouwer's work was still less understood at that time, and Hesseling rightly chooses not to speak about it. This decision strengthens the negative role Brouwer assumed in the debate: he was the man who prohibited powerful modes of reasoning, who saw problems where there were none before, who, as he said once, made life harder, and so it was easy for Hilbert to present his opponent as a "*Verbotsdiktator*" *à la Kronecker*.

The book has two main parts. The central one includes chapters 3 to 5 and is dedicated to the reception of Brouwer's ideas between 1919 and 1933. But, in order to understand the debate, the reader must first be made familiar with Brouwer's point of view and the context in which it appeared. The first part begins, therefore, with an exposition of the various forerunners of intuitionism: Kronecker on one side, the French semi-intuitionists Borel and Poincaré on the other side. The second chapter gives a short presentation of the genesis of Brouwer's intuitionism, from his 1907 dissertation to his 1929 lecture in Vienna and to the "*Annalen*" affair, which puts a brutal end to the debate and was followed by Brouwer's year long quasi retreat from scientific life. Hesseling presents Hilbert's autocratic dismissal of Brouwer as a member of the editorial board of the *Mathematische Annalen* as an immediate consequence of the two men's disagreement concerning German participation in the International Congress of Mathematicians held in Bologna in 1928[1].

Chapter three gives a short overview of the debate, from a quantitative as well as from a qualitative point of view. In the first case, an histogram is given that shows that the peak of publications lies between 1927 and 1932. In the second case, Hesseling tries to identify the themes, the tone and the schools involved.

The kernel of the book, namely the reactions to Brouwer's ideas, is divided according to two headings: existence and constructivity, logic and the excluded middle. Both chapters follow the same pattern: The beginning of the debate, the debate widened, later reactions.

It is generally admitted that the *Grundlagenstreit* started in 1919 with Hermann Weyl's *Über die neue Grundlagenkrise in der Mathematik*. Weyl was the most gifted among Hilbert's students, and his taking sides with Brouwer had a very powerful impact. Weyl's work in topology was inspired by Brouwer and they shared a common sympathy for idealism, both Kantian and Husserlian. The two men met during a summer vacation spent in Engadine in 1919. Some months later, Weyl gave three lectures in Zurich where he addressed the question of mathematical existence. We cannot quantify over infinite collections as we used to do with finite ones. Existential statements are not genuine

[1]For the importance of this congress in the history of mathematics, especially in the area of probability theory, see B. Bru: "Souvenirs de Bologne", *Journal de la Société Française de Statistique*, tome 144, n^0 1-2, 2003, pp. 136-226.

propositions, expressing a state of affairs. They are rather "proposition abstracts" and they stand to propositions as paper money to gold. Weyl wanted to rouse the sleepers, and he achieved his goal.

At that time Hilbert was engaged in his own foundational program. His reactions followed quickly. He seems to have been very affected by the defection of the most gifted of his students but one may wonder why he reacted so angrily, if one remembers his own constructivist leaning. The book analyzes very closely Hilbert's various lectures and papers during that first stage, paying due attention to Bernays's position. The next subsection introduces a new protagonist, A. Fraenkel, and shows that the famous set theorist was also one of the first to see the importance of the debate and to comment about it.

From 1924 onward, the second stage is characterised by the intervention of new participants, especially non German, like the North-American Arnold Dresden, who had translated Brouwer's inaugural lecture into English one year after its Dutch publication in 1912, or Rolin Wavre, a Geneva professor close to Borel. During this second stage, the idea of constructivity is taken for granted and the debate focuses on existence, as shown, for instance, in Wavre's paper. Hilbert's 1925 paper "On the infinite" gives a more formalistic view of mathematics. Until this date, Hilbert held on to the idea that the formalised parts of mathematics were linked to content: formalisation consisted precisely in disregarding this content. With the introduction of ideal elements, it is now admitted that ordinary mathematics also contains meaningless statements, mere *façons de parler*, the dispensability of which it is the function of metamathematics to establish. At the end of this period, Weyl withdrew his full support to Brouwer and acknowledged some legitimacy to the point of view of his former teacher.

After a three page history of classical logic, chapter five begins again with Weyl's *Grundlagenkrise* and notes that his argumentation regarding the excluded middle differs markedly from Brouwer's. In this case, Hilbert showed more comprehension: he conceded that the negation of quantified statements, when applied to infinite collections, was problematic; in order to solve the difficulty, he proposed adding transfinite axioms and proving that the resulting system was consistent. Fraenkel saw clearly where the disagreement lay: Hilbert was content to prove that the application of the excluded middle was without danger, that is to say: free of contradiction. Intuitionists were not satisfied with such an answer: to be innocuous does not mean to be justified and they asked for something more. The debate widened and Hesseling describes briefly Skolem's and Ramsey's interventions. The most important one came this time from Russia, with Kolmogorov's 1925 paper, which unfortunately remained almost totally unnoticed until 1933. Nowadays, it is mostly known as the first formalisation of (parts of) intuitionistic logic. However, Kolmogorov's main goal lay elsewhere: he wanted to show why, as Brouwer had already pointed out, an illegitimate use of the principle of the excluded middle does not lead to a contradiction, and why the illegitimacy had hardly been noticed before. The last part of the chapter relates the history of the formalisation of intuitionistic logic, from Wavre's first essay in 1926 to Heyting, through Lévy, Barzin-Errera and Glivenko.

The last chapter studies the foundational crisis in its context. Political circumstances influenced it, especially in the beginning and at the end of the Weimar Republic. "Revolution", "crisis", such words had immediate and heavy connotations in post first world war Germany and these connotations were used, more or less consciously, as weapons on both sides. With the rise of the Third Reich, Bieberbach gave a racial interpretation of the debate. Hesseling also looks for relationships with art, philosophy and physics. He reminds us aptly that constructivism gained popularity in art before mathematics. As to "philosophical intuitionism", or more broadly "*Lebensphilosophie*", it could have influenced the debate in various ways and we know for instance that Hilbert had institutional quarrels with some "*Lebensphilosoph*" in Göttingen. However the author concludes in most cases that there is little evidence establishing any pertinent connection. Besides a short conclusion, the book also contains useful appendices: a chronology of the debate, a list of public reactions to Brouwer's intuitionism and a very rich bibliography.

Gnomes in the fog has many fine qualities. It is not necessary to stress the crucial role of crisis in science. Logic is no exception and, due to its very nature, the *Grundlagenstreit* is a good place to study this aspect of scientific life. Thanks to Hesseling, we now have a much better historical understanding of it. He has visited many archives, public and private, and brings in a lot of new material. We know the precise circumstances of Weyl's first intervention and Polya's reaction to it. We discover how few people were concerned. The main participants' names have always been well known but who did remember that Fraenkel played an important role too? The debate started in Germany and when it stopped being a purely German affair, it remained a continental one. Ramsey's brief commentary ("Brouwer would refuse to agree that it was raining or it was not raining unless he had looked to see" (p. 250)) is beside the point, but Church's pragmatic stance anticipates what is now the most usual position. The working mathematician was not interested either. Hadamard expressed clearly this lack of interest for foundations when he presented the controversy as useless or, worse, as a return from the positive stage to the metaphysical stage. In the case of J. von Neumann, the author points out that, even as late as 1927, some of the better informed mathematicians misunderstood the intuitionistic view of mathematical existence; some years later, Herbrand was still unable to distinguish between finitism and intuitionism.

Unfortunately, there are also some obvious weaknesses. First of all, much data seem to be given only "for the sake of completeness". However, a more drastic selection of the material would have been desirable in order to focus still more on the most relevant contributions. The reader would have appreciated it, if Hesseling had applied more often the maxim he used in the case of Barzin and Errerra's last publications: "I did not aim at completeness in pointing out the mistakes, misinterpretations and wrong conclusions that are present in their contribution. The reader interested in such an account should consult the original publication"(p. 290). On the other hand, the way the material is organised is not totally convincing either. By giving to Brouwer's view a separate treatment in chapter two, his own direct participation to the debate tends to be underestimated. Following exactly the same pattern in chapters four and five induces too many useless repetitions. The same biographical details are given twice, sometimes without changing any word (compare the presentation of Wavre's 1924 paper, p. 158 and p. 235, or p. 162 and p. 241 (with notes 302 and 143) on Hilbert's 1925 paper). Hesseling is more an historian than a philosopher and he is obviously much more comfortable with facts (*who? when? where?*) than with ideas. However, as shown by Hadamard's reactions, foundational studies belong as much to philosophy as to mathematics proper and we know from other sources than some of the main participants (Brouwer, Weyl, Gödel and, to a lesser degree, Hilbert), had a strong interest in philosophy. Hesseling shows uneasiness with some of the conceptual issues involved. When he comes across the knowledge-dependence of mathematical propositions, which is an important Brouwerian point, he does not succeed in pointing out its relevance (p. 255). Likewise, on p. 265 or p. 283, the question of the decidability of mathematical propositions, which Hilbert took as his starting point with his *Ignoramus sed non ignorabimus*, and which Brouwer contested from the very beginning, does not receive the treatment it deserves.

It would be unfair to put too much stress upon such weaknesses. Hesseling has written a valuable thorough work. Studies in the history of logic are not so frequent; as it stands, the book will be quite useful. The debate between Hilbert and Brouwer remains one of the more momentous episodes of the history of mathematical logic and, from now on, it will be difficult to speak about it without referring to Hesseling's book.

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