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PRECISE LARGE DEVIATIONS FOR DEPENDENT RANDOM VARIABLES WITH APPLICATIONS TO THE COMPOUND RENEWAL RISK MODEL

YANG YANG AND KAIYONG WANG

ABSTRACT. This paper investigates some precise large deviations for the partial sums of extended negatively dependent (END) and non-identically distributed random variables with dominantly varying tails, which slightly extend some corresponding results of Liu [**13**]. Furthermore, we obtain precise large deviations for the random sums, where the random number is a nonnegative integer-valued process. As applications, we derive the asymptotics for the finite-time ruin probability in the END compound renewal risk model.

1. Introduction. Let $\{X_i,\ i \geq 1\}$ be a sequence of random variables (r.v.s) with distributions $F_i = 1 - \overline{F_i}$ and finite mean μ_i, $i \geq 1$, and let $S_n = \sum_{i=1}^n X_i$ be its nth partial sums, $n \geq 1$. In the present paper, we are firstly interested in precise large deviations for these partial sums of $\{X_i,\ i \geq 1\}$ with heavy tails. Many earlier works have been devoted to this field, see, e.g., Heyde [**8–10**], A.V. Nagaev [**16**], S.V. Nagaev [**17**], Cline and Hsing [**5**], Mikosch and A.V. Nagaev [**15**], Tang et al. [**20**], Ng et al. [**18**], Tang [**19**] and Liu [**14**], among others. All of the above-mentioned results are restricted to identically distributed r.v.s, and derived such that, for any fixed $\gamma > 0$, the relation

$$\mathrm{P}(S_n - n\mu_1 > x) \sim n\overline{F_1}(x)$$

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holds uniformly for all $x \geq \gamma n$ as $n \to \infty$. That is,

$$\lim_{n\to\infty} \sup_{x\geq\gamma n} \left| \frac{\mathrm{P}(S_n - n\mu_1 > x)}{n\overline{F_1}(x)} - 1 \right| = 0.$$

Recently, Liu [**13**] derived some precise large deviations for END and non-identically distributed r.v.s with consistently varying tails. Motivated by [**13**], our first main result slightly extends those results to the case of dominantly-varying-tailed r.v.s.

To formulate our results, we introduce some concepts and properties on some heavy-tailed classes and the extended negative dependence structure. Recall that an r.v. ξ (or its distribution) is said to be heavy-tailed if $\mathrm{E}e^{\rho\xi} = \infty$ for all $\rho > 0$, and light-tailed otherwise. One of the most important heavy-tailed subclasses is the class $\mathscr{D}$. By definition, a distribution V is said to have a dominantly varying tail, denoted by $V \in \mathscr{D}$, if $\limsup_{x\to\infty} \overline{V}(xy)/\overline{V}(x) < \infty$ for any $0 < y < 1$. A slightly smaller class is $\mathscr{C}$. A distribution V is said to have a consistently varying tail, denoted by $V \in \mathscr{C}$, if $\lim_{y\nearrow 1} \limsup_{x\to\infty} \overline{V}(xy)/\overline{V}(x) = 1$. A related class is the long-tailed distribution class $\mathscr{L}$. A distribution V is said to have a long tail, denoted by $V \in \mathscr{L}$, if $\lim_{x\to\infty} \overline{V}(x+y)/\overline{V}(x) = 1$ for any $y > 0$. It is well known that the following inclusion relationships hold:

$$\mathscr{C} \subset \mathscr{L} \cap \mathscr{D} \subset \mathscr{L}$$

(see, e.g., [**4, 7**]). Furthermore, for a distribution V, denote the upper and lower Matuszewska index of V, respectively, by

$$J_V^+ = -\lim_{y\to\infty} \frac{\log \overline{V}_*(y)}{\log y} \quad \text{with} \quad \overline{V}_*(y) := \liminf_{x\to\infty} \frac{\overline{V}(xy)}{\overline{V}(x)} \text{ for } y > 1,$$

$$J_V^- = -\lim_{y\to\infty} \frac{\log \overline{V}^*(y)}{\log y} \quad \text{with} \quad \overline{V}^*(y) := \limsup_{x\to\infty} \frac{\overline{V}(xy)}{\overline{V}(x)} \text{ for } y > 1.$$

Define another important parameter $L_V = \lim_{y\searrow 1} \overline{V}_*(y)$. The following assertions are equivalent: (i) $V \in \mathscr{D}$;

(ii) $\overline{V}_*(y) > 0$ for some (or for any) $y > 1$;

(iii) $L_V > 0$;

(iv) $J_V^+ < \infty$.

It also holds that $V \in \mathscr{C}$ if and only if $L_V = 1$. For more details, see [**2**, Chapter 2.1] The following proposition is due to Lemma 3.5 of Tang and Tsitsiashvili [**21**], see also [**19**, Lemma 2.1].

Proposition 1.1. *If $V \in \mathscr{D}$, then it holds for any $p > J_V^+$ that $\lim_{x\to\infty} x^{-p}/\overline{V}(x) = 0$.*

Liu [**13**] introduced a general dependence structure.

Definition 1.1. Say that r.v.s $\{\xi_i,\ i \geq 1\}$ are END, if there exists a positive constant M such that both

$$\mathrm{P}\bigg(\bigcap_{i=1}^{n}\{\xi_i > y_i\}\bigg) \leq M \prod_{i=1}^{n} \mathrm{P}(\xi_i > y_i) \tag{1.1}$$

and

$$P\bigg(\bigcap_{i=1}^{n}\{\xi_i \leq y_i\}\bigg) \leq M \prod_{i=1}^{n} \mathrm{P}(\xi_i \leq y_i) \tag{1.2}$$

hold for each $n \geq 1$ and all $y_1, \ldots, y_n$.

Recall that r.v.s $\{\xi_i,\ i \geq 1\}$ are called Upper Negatively Dependent (UND), if (1.1) holds when $M = 1$; they are called Lower Negatively Dependent (LND), if (1.2) holds when $M = 1$; and they are called Negatively Dependent (ND), if both (1.1) and (1.2) hold when $M = 1$. These negative dependence structures were introduced by Ebrahimi and Ghosh [**6**] and Block et al. [**3**]. Clearly, ND r.v.s must be END r.v.s. And Example 4.1 of [**13**] shows that the END structure also includes some other dependence structure. By direct verification, END r.v.s have the following properties similar to those of ND r.v.s, see [**13**, Lemma 3.1].

Proposition 1.2. (1) *If* r.v.s $\{\xi_i,\ i \geq 1\}$ *are nonnegative and* END*, then for any $n \geq 1$, there exists a positive constant M such that $\mathrm{E}(\prod_{i=1}^{n} \xi_i) \leq M \prod_{i=1}^{n} \mathrm{E}\xi_i$;*

(2) *If* r.v.s $\{\xi_i,\ i \geq 1\}$ *are* END *and* $\{f_i(\cdot),\ i \geq 1\}$ *are either all strictly increasing or all strictly decreasing, then* $\{f_i(\xi_i),\ i \geq 1\}$ *are still* END; *and, for each real number* a, r.v.s $\{\min\{\xi_i, a\},\ i \geq 1\}$ *are still* END.

In this paper, we mainly study the precise large deviations for partial and random sums of END r.v.s with dominantly varying tails. The rest of the paper is organized as follows. In Section 2, we investigate some precise large deviations for partial sums of END and non-identically distributed r.v.s. In Section 3, the precise large deviation results are then extended to random sums $S_{N(t)}$ with identically distributed r.v.s, where $\{N(t),\ t \geq 0\}$ is a nonnegative integer-valued process. As applications, in Section 4, we derive asymptotics for the finite-time ruin probabilities in the END compound renewal risk model with constant interest rate, where the claims at each accident moment are aggregated from a number of END individual claims, and the total amount of premiums is a nonnegative and nondecreasing stochastic process.

Throughout, for two positive functions $u(x)$ and $v(x)$, we write $u(x) \lesssim v(x)$ (equivalently, $v(x) \gtrsim u(x)$) if $\limsup_{x\to\infty} u(x)/v(x) \leq 1$; $u(x) \sim v(x)$ if $\lim_{x\to\infty} u(x)/v(x) = 1$; $u(x) = o(v(x))$ if $\lim_{x\to\infty} u(x)/v(x) = 0$; $u(x) = O(v(x))$ if $\limsup_{x\to\infty} u(x)/v(x) < \infty$; and $u(x) \asymp v(x)$ if $u(x) = O(v(x))$ and $v(x) = O(u(x))$. The indicator function of an event A is denoted by $\mathbf{1}_A$. For real y, denote by $\lceil y \rceil$ the smallest integer greater than or equal to y; and similarly, denote by $\lfloor y \rfloor$ the greatest integer smaller than or equal to y. We denote $a^+ = \max\{a, 0\}$ and $a^- = -\min\{a, 0\}$.

2. Precise large deviations for partial sums. In this section, we investigate the asymptotic behavior of precise large deviations for partial sums, which slightly extends Theorem 2.1 of Liu [**13**] to the class $\mathscr{D}$. Throughout this section, let $\{X_i,\ i \geq 1\}$, X be non-identically distributed r.v.s with distributions $\{F_i,\ i \geq 1\}$, F and finite means $\{\mu_i,\ i \geq 1\}$, μ. In the sequel, C always represents a finite and positive constant whose value may vary in different places. All limit relationships hold uniformly for all $x \geq \gamma n$ as $n \to \infty$ unless stated otherwise. Our first main result is on the basis of the following assumptions.

Assumption 2.1. *For some* $T > 0$,

$$\frac{1}{n}\sum_{i=1}^{n}\overline{F_i}(x) \asymp \overline{F}(x) \tag{2.1}$$

and

$$\frac{1}{n}\sum_{i=1}^{n}F_i(-x) \asymp F(-x) \tag{2.2}$$

hold uniformly for all $x \geq T$ *as* $n \to \infty$.

Assumption 2.2. *For all* $i \geq 1$, $F_i \in \mathscr{D}$, *i.e.*, $L_{F_i} > 0$. *Furthermore, for all* $i \geq 1$ *and any* $\varepsilon > 0$, *there exist some* $w_0 = w_0(\varepsilon) > 1$ *and* $x_0 = x_0(\varepsilon) > 0$, *irrespective to* i, *such that, for all* $1 \leq w \leq w_0$ *and* $x \geq x_0$,

$$\frac{\overline{F_i}(wx)}{\overline{F_i}(x)} \geq L_{F_i} - \varepsilon.$$

Notice that Assumption 2.2 is equivalent to the fact that, for all $i \geq 1$ and any $\varepsilon > 0$, there exist some $0 < v_0 = v_0(\varepsilon) < 1$ and $x_0 = x_0(\varepsilon) > 0$, irrespective to i, such that $\overline{F_i}(vx)/\overline{F_i}(x) \leq L_{F_i}^{-1} + \varepsilon$ for all $v_0 \leq v \leq 1$ and $x \geq x_0$.

Our first main result is formulated as follows.

Theorem 2.1. *Let* $\{X_i,\ i \geq 1\}$ *be* END r.v.s *with* $\mu_i = 0,\ i \geq 1$. *If Assumptions* 2.1 *and* 2.2 *hold, then for any* $\gamma > 0$,

$$\mathrm{P}(S_n > x) \lesssim \sum_{i=1}^{n} L_{F_i}^{-1}\overline{F_i}(x) \tag{2.3}$$

holds uniformly for all $x \geq \gamma n$ *as* $n \to \infty$. *Furthermore, if*

$$F(-x) = o(\overline{F}(x)) \text{ as } x \to \infty, \tag{2.4}$$

and $\mathrm{E}(X_i^-)^r < \infty,\ i \geq 1$, $\mathrm{E}(X^-)^r < \infty$ *for some* $r > 1$, *then for any* $\gamma > 0$,

$$\mathrm{P}(S_n > x) \gtrsim \sum_{i=1}^{n} L_{F_i}\overline{F_i}(x) \tag{2.5}$$

holds uniformly for all $x \geq \gamma n$ *as* $n \to \infty$.

Before proving Theorem 2.1, we firstly give a lemma, which is similar to Lemma 3.3 of [**13**]. We omit the proof.

Lemma 2.1. *Assume that* $\mathrm{E}(X_i^{\pm})^q < \infty$, $i \geq 1$, *and* $\mathrm{E}(X^{\pm})^q < \infty$ *for some* $q \geq 1$. *If Assumption* 2.1 *holds, then there exists some finite constant* $\widehat{\mu}_q^{\pm}$ *such that, for all* $n \geq 1$,

$$\sum_{i=1}^{n} \mathrm{E}(X_i^{\pm})^q \leq n\widehat{\mu}_q^{\pm}.$$

Proof of Theorem 2.1. By $F_i \in \mathscr{D}$, $i \geq 1$, and (2.1), clearly, it holds that $F \in \mathscr{D}$. To prove (2.3), we follow the line of Liu [**13**], whose idea is from [**19**]. For any fixed $0 < v < 1$, we write $\widetilde{X}_i = \min\{X_i, vx\}$, $i \geq 1$, which, by Proposition 1.2 (2), are still END. Denote $\widetilde{S}_n = \sum_{i=1}^n \widetilde{X}_i$, $n \geq 1$. A standard truncation argument gives that

$$\mathrm{P}(S_n > x) \leq \sum_{i=1}^{n} \overline{F_i}(vx) + \mathrm{P}(\widetilde{S}_n > x). \tag{2.6}$$

By $0 < L_{F_i} \leq 1$, $i \geq 1$, and (2.1), we have for all $x \geq \gamma n$ and sufficiently large n,

$$\frac{\mathrm{P}(\widetilde{S}_n > x)}{\sum_{i=1}^n L_{F_i}^{-1}\overline{F_i}(x)} \leq \frac{\mathrm{P}(\widetilde{S}_n > x)}{\sum_{i=1}^n \overline{F_i}(x)} \leq \frac{\mathrm{P}(\widetilde{S}_n > x)}{Cn\overline{F}(x)},$$

which, by the same argument of [**13**, pages 1294–1295], implies

$$\mathrm{P}(\widetilde{S}_n > x) = o\bigg(\sum_{i=1}^{n} L_{F_i}^{-1}\overline{F_i}(x)\bigg). \tag{2.7}$$

By Assumption 2.2, for all $i \geq 1$ and any $\varepsilon > 0$, there exist some $0 < v_0(\varepsilon) < 1$ and $x_0(\varepsilon) > 0$, irrespective to i, such that $\overline{F_i}(vx) \leq (L_{F_i}^{-1} + \varepsilon)\overline{F_i}(x)$ for all $v_0 \leq v \leq 1$ and $x \geq x_0$. Thus, for all $v_0 \leq v \leq 1$, $x \geq \gamma n$ and $n \geq \gamma^{-1}x_0$, we have

$$\sum_{i=1}^{n} \overline{F_i}(vx) \leq \sum_{i=1}^{n} (L_{F_i}^{-1} + \varepsilon)\overline{F_i}(x), \tag{2.8}$$

which, combining the arbitrariness of ε, (2.6) and (2.7), yields (2.3).

We prove (2.5) by slightly complementing the proof of Lemma 3.7 of [**13**]. For every $1 \le i \le n$ and any fixed $w > 1$, let $A_i = \{X_i > wx,\ \max_{1\le j\ne i\le n} X_j \le wx\}$, which are pairwise disjoint sets. By using this partition, we obtain

$$
\begin{aligned}
\mathrm{P}(S_n > x) &\ge \mathrm{P}\Big(S_n > x, \bigcup_{i=1}^{n} A_i\Big) \\
&= \sum_{i=1}^{n} \mathrm{P}(A_i) - \sum_{i=1}^{n} \mathrm{P}(S_n \le x, A_i) \qquad (2.9)\\
&:= I_1 - I_2.
\end{aligned}
$$

By (1.1), we have

$$
\begin{aligned}
I_1 &= \sum_{i=1}^{n} \overline{F_i}(wx) - \sum_{i=1}^{n} \mathrm{P}\Big(\bigcup_{j\ne i}\{X_j > wx, X_i > wx\}\Big) \\
&\ge \sum_{i=1}^{n} \overline{F_i}(wx) - M\Big(\sum_{i=1}^{n} \overline{F_i}(wx)\Big)^2 \qquad (2.10)\\
&= (1 - o(1)) \sum_{i=1}^{n} \overline{F_i}(wx),
\end{aligned}
$$

where the last step holds, because, by (2.1) and $\mathrm{E}X^+ < \infty$,

$$
\begin{aligned}
\sum_{i=1}^{n} \overline{F_i}(wx) &= \frac{1}{n}\sum_{i=1}^{n} n \int_{wx}^{\infty} F_i(\mathrm{d}y) \\
&\le \frac{1}{\gamma w n}\sum_{i=1}^{n} \int_{wx}^{\infty} y F_i(\mathrm{d}y) \\
&= \frac{x}{\gamma n}\sum_{i=1}^{n} \overline{F_i}(wx) + \frac{1}{\gamma w}\int_{wx}^{\infty} \frac{1}{n}\sum_{i=1}^{n} \overline{F_i}(y)\,\mathrm{d}y \\
&\le C\Big(x\overline{F}(wx) + \int_{wx}^{\infty} \overline{F}(y)\,\mathrm{d}y\Big) = o(1).
\end{aligned}
$$

To deal with I_2, for any fixed $0 < \widetilde{v} < 1$, we write $Y_i = -X_i$, $\widetilde{Y}_i = \min\{Y_i, \widetilde{v}x\}$, $i \ge 1$. By Proposition 1.2 (2), $\{Y_i,\ i \ge 1\}$ and

$\{\widetilde{Y}_i,\ i \geq 1\}$ are END r.v.s, respectively. Since A_i, $1 \leq i \leq n$, are pairwise disjoint sets, we have

(2.11)
$$\begin{aligned}
I_2 &\leq \sum_{i=1}^{n} \mathrm{P}\Big(\sum_{j\neq i} Y_j \geq (w-1)x, A_i\Big) \\
&\leq \sum_{i=1}^{n} \mathrm{P}\Big(A_i, \bigcup_{j\neq i}\{Y_j > \widetilde{v}x\}\Big) \\
&\quad + \sum_{i=1}^{n} \mathrm{P}\Big(\sum_{j\neq i} \widetilde{Y}_j \geq (w-1)x\Big) \\
&\leq \sum_{j=1}^{n} \mathrm{P}(Y_j > \widetilde{v}x) + \sum_{i=1}^{n} \mathrm{P}\Big(\sum_{j\neq i} \widetilde{Y}_j \geq (w-1)x\Big) \\
&:= I_{21} + I_{22}.
\end{aligned}$$

By (2.4), (2.11) and $F \in \mathscr{D}$, we get that for sufficiently large n and any $x \geq \gamma n$

$$I_{21} \leq \sum_{j=1}^{n} F_j(-\widetilde{v}x) \leq CnF(-\widetilde{v}x) = o(n\overline{F}(\widetilde{v}x)) = o(n\overline{F}(x)). \tag{2.12}$$

As for I_{22}, for any $1 \leq i \leq n$ and any temporarily fixed $h = h(x,n) > 0$, we obtain from Markov's inequality, Proposition 1.2 and the elementary inequality $\log(1+u) \leq u$, $u \geq 0$,

(2.13)
$$\begin{aligned}
\mathrm{P}\Big(\sum_{j\neq i} \widetilde{Y}_j \geq (w-1)x\Big) \\
&\leq \widetilde{M} e^{-h(w-1)x} \prod_{j\neq i} \mathrm{E} e^{h\widetilde{Y}_j} \\
&\leq \widetilde{M} e^{-h(w-1)x} \prod_{j\neq i} \Big\{ \int_{-\infty}^{\widetilde{v}x} (e^{hy}-1) F_{Y_j}(\mathrm{d}y) \\
&\qquad\qquad + (e^{h\widetilde{v}x}-1)\overline{F_{Y_j}}(\widetilde{v}x) + 1 \Big\}
\end{aligned}$$

$$\leq \widetilde{M} \exp\bigg\{ - h(w-1)x + \sum_{j\neq i} \bigg(\int_{-\infty}^{\widetilde{v}x} (e^{hy}-1) F_{Y_j}(\mathrm{d}y) + (e^{h\widetilde{v}x}-1) F_j(-\widetilde{v}x) \bigg) \bigg\},$$

where $\widetilde{M}$ is a constant. By the same as the argument of (3.9), (3.18) and (3.19) of [**13**], taking $h = 1/(\widetilde{v}x)\log((\widetilde{v}^{q-1}x^q/n\widehat{\mu}_q^-)+1)$, which tends to 0, for any fixed $1 < q < \min\{r, 2\}$, and by (2.2) and (2.13), we have

$$\begin{aligned}
(2.14)\quad & \mathrm{P}\bigg(\sum_{j\neq i} \widetilde{Y}_j \geq (w-1)x \bigg) \\
& \leq \widetilde{M} \exp\bigg\{ - h(w-1)x + o(1)nh + \frac{e^{h\tilde{v}x}-1}{(\widetilde{v}x)^q} n\widehat{\mu}_q^- + (e^{h\tilde{v}x}-1) CnF(-\widetilde{v}x) \bigg\} \\
& \leq \widetilde{M} \exp\bigg\{ - \frac{w-1-o(1)}{\widetilde{v}} \log\bigg(\frac{\widetilde{v}^{q-1}x^q}{\widehat{\mu}_q^-} + 1 \bigg) + \frac{1}{\widetilde{v}} + \frac{C\widetilde{v}^{q-1}x^q}{\widehat{\mu}_q^-} F(-\widetilde{v}x) \bigg\} \\
& \leq \widetilde{M} \exp\bigg\{ \frac{1}{\widetilde{v}} + \frac{C\widetilde{v}^{q-1}x^q}{\widehat{\mu}_q^-} F(-\widetilde{v}x) \bigg\} \bigg(\frac{\gamma\widetilde{v}^{q-1}}{\widehat{\mu}_q^-} x^{q-1} \bigg)^{-(w-1)/2\tilde{v}} \\
& \leq C_0 x^{-[(q-1)(w-1)]/2\tilde{v}},
\end{aligned}$$

where the coefficient C_0 is given by

$$C_0 = \widetilde{M} \sup_{x\geq 0} \exp\bigg\{ \frac{1}{\widetilde{v}} + \frac{C\widetilde{v}^{q-1}x^q}{\widehat{\mu}_q^-} F(-\widetilde{v}x) \bigg\} \bigg(\frac{\gamma\widetilde{v}^{q-1}}{\widehat{\mu}_q^-} \bigg)^{-(w-1)/2\tilde{v}} < \infty.$$

Indeed, by $\mathrm{E}(X^-)^q < \infty$, the following holds:

$$\begin{aligned}
x^q F(-\widetilde{v}x) = x^q \int_{-\infty}^{-\tilde{v}x} F(\mathrm{d}y) &\leq \widetilde{v}^{-q} \int_{-\infty}^{-\tilde{v}x} |y|^q F(\mathrm{d}y) \\
&= \widetilde{v}^{-q}\mathrm{E}(X^-)^q \mathbf{1}_{\{X\leq -\widetilde{v}x\}} \longrightarrow 0 \\
&\text{as } x \to \infty.
\end{aligned}$$

In (2.14), for any fixed $w > 1$, we take a sufficiently small $\widetilde{v} > 0$ such that $(q-1)(w-1)/(2\widetilde{v}) > J_F^+$. Thus, from (2.11), (2.12), (2.14), Proposition 1.1 and (2.1), we obtain

$$\begin{aligned} I_2 &\le o(n\overline{F}(x)) + C_0 n x^{-[(q-1)(w-1)]/2\tilde{v}} \\ &= o(n\overline{F}(x)) = o\biggl(\sum_{i=1}^{n} L_{F_i}^{-1}\overline{F_i}(x)\biggr). \end{aligned} \tag{2.15}$$

Analogously to (2.8), again by Assumption 2.2, for any $\varepsilon > 0$, there exist some $w_0(\varepsilon) > 1$ and $N(\varepsilon)$ such that, for any $1 \le w \le w_0$, all $x \ge \gamma n$ and $n \ge N$, we have

$$\sum_{i=1}^{n} \overline{F_i}(wx) \ge \sum_{i=1}^{n} (L_{F_i} - \varepsilon)\overline{F_i}(x). \tag{2.16}$$

Therefore, the desired (2.14) follows from (2.9), (2.10), (2.15) and (2.16). $\square$

Corollary 2.1. *Let $\{X_i,\ i \ge 1\}$ be* END *r.v.s with common distribution F and mean $\mu = 0$ satisfying $\mathrm{E}(X_1^-)^r < \infty$ for some $r > 1$. If $F \in \mathscr{D}$ and* (2.4) *is satisfied, then for any fixed $\gamma > 0$,*

$$L_F n\overline{F}(x) \lesssim \mathrm{P}(S_n > x) \lesssim L_F^{-1} n\overline{F}(x)$$

holds uniformly for all $x \ge \gamma n$ as $n \to \infty$. In particular, if $F \in \mathscr{C}$, then

$$\mathrm{P}(S_n > x) \sim n\overline{F}(x)$$

holds uniformly for all $x \ge \gamma n$ as $n \to \infty$.

3. Precise large deviations for random sums. In this section, we investigate precise large deviations for random sums $S_{N(t)} = \sum_{i=1}^{N(t)} X_i$, $t \ge 0$, where $\{N(t),\ t \ge 0\}$ is a nonnegative integer-valued process. For a nice review on the precise large deviations for random sums, one can see [**18, 23**]. In this section, all limit relationships hold for t tending to ∞ unless stated otherwise. We always assume that

$\lambda(t) = \mathrm{E}N(t) \to \infty$. In many earlier works, the nonnegative integer-valued process $N(t)$ was restricted to satisfying

$$\frac{N(t)}{\lambda(t)} \xrightarrow{\mathrm{P}} 1, \tag{3.1}$$

and for any $\nu > 0$ and some $\epsilon > 0$,

$$\mathrm{E}(1+\epsilon)^{N(t)}\mathbf{1}_{\{N(t)>(1+\nu)\lambda(t)\}} = o(1). \tag{3.2}$$

As was verified by Klüppelberg and Mikosch [**11**], the homogenous Poisson process satisfies both (3.1) and (3.2). Recently, Kočetova et al. [**12**] proved that the renewal counting process satisfies condition (3.2) if the inter-arrival times are i.i.d. nonnegative and nondegenerate r.v.s with finite mean. However, in more general cases, (3.2) is apparently hard to verify if $N(t)$ is a nonnegative integer-valued process. Later, Ng et al. [**18**] weakened (3.1) and (3.2) to the single condition that, for some $p > J_F^+$ and any $\nu > 0$,

$$\mathrm{E}(N(t))^p\mathbf{1}_{\{N(t)>(1+\nu)\lambda(t)\}} = O(\lambda(t)). \tag{3.3}$$

Many works on precise large deviations for random sums are based on this assumption, see, e.g., [**14, 18, 23**], among others.

Before the statement of our main result of this section, we give a lemma below, which is similar to [**23**, Lemma 2.1] or [**22**, Lemma 3.1]. It can be proved as the similar argument of Lemma 2.3 of [**19**]. We omit the details.

Lemma 3.1. *Let* $\{\xi_i,\ i \geq 1\}$ *be* END *r.v.s with common distribution* V *and mean* $\mu_V = 0$ *satisfying* $\mathrm{E}(X_1^+)^r < \infty$ *for some* $r > 1$. *Then, there exists a positive constant* C *such that, for any* $v > 0$, $x > 0$ *and* $n \geq 1$

$$\mathrm{P}\left(\sum_{i=1}^{n}\xi_i > x\right) \leq n\overline{V}(xv^{-1}) + C(e\mu_V^+ nx^{-1})^v.$$

On the basis of the precise large deviations for partial sums, we can obtain the following result.

Theorem 3.1. *Let* $\{X_i,\ i \geq 1\}$ *be* END *r.v.s with common distribution* F *and mean* $\mu = 0$. $\{N(t),\ t \geq 0\}$ *is a nonnegative integer-valued process, which is independent of* $\{X_i,\ i \geq 1\}$. *Assume that there exists some* $r > 1$ *such that* $\mathrm{E}(X_1^-)^r < \infty$. *If* $F \in \mathscr{D}$, (2.4) *and* (3.1) *are satisfied, then, for any fixed* $\gamma > \mu^+ = \mathrm{E}X_1^+$

$$L_F \lambda(t)\overline{F}(x) \lesssim \mathrm{P}(S_{N(t)} > x) \lesssim L_F^{-1}\lambda(t)\overline{F}(x)$$

holds uniformly for all $x \geq \gamma\lambda(t)$. *In particular, if* $F \in \mathscr{C}$, *then*

$$\mathrm{P}(S_{N(t)} > x) \sim \lambda(t)\overline{F}(x)$$

holds uniformly for all $x \geq \gamma\lambda(t)$.

Proof. By [**18**, Lemma 2.4], (3.3) implies (3.1). Thus, for any $0 < \nu < 1$, we split $P(S_{N(t)} > x)$ into three parts as

$$\begin{aligned} \mathrm{P}(S_{N(t)} > x) = &\Bigg(\sum_{n<(1-\nu)\lambda(t)} + \sum_{(1-\nu)\lambda(t)\leq n\leq(1+\nu)\lambda(t)} \\ &+ \sum_{n>(1+\nu)\lambda(t)} \Bigg) \mathrm{P}(S_n > x)\mathrm{P}(N(t) = n) \\ := &K_1 + K_2 + K_3. \end{aligned} \tag{3.4}$$

We firstly estimate K_1. Since $\{X_i,\ i \geq 1\}$ are END r.v.s, it follows from Proposition 1.2 (2) that $\{X_i^+,\ i \geq 1\}$ are also END, and for any fixed $\gamma > \mu^+$ and any $x \geq \gamma\lambda(t)$,

$$x - \mu^+\lfloor(1-\nu)\lambda(t)\rfloor \geq (\gamma - \mu^+)\lfloor(1-\nu)\lambda(t)\rfloor,$$

from which, together with Corollary 2.1, (3.1) and $F \in \mathscr{D}$, it holds uniformly, for $x \geq \gamma\lambda(t)$ and sufficiently large t, that

$$\begin{aligned} K_1 \leq \mathrm{P}\Bigg(&\sum_{i=1}^{\lfloor(1-\nu)\lambda(t)\rfloor} X_i^+ - \mu^+\lfloor(1-\nu)\lambda(t)\rfloor \\ &> x - \mu^+\lfloor(1-\nu)\lambda(t)\rfloor \Bigg) \mathrm{P}(N(t) < (1-\nu)\lambda(t)) \end{aligned} \tag{3.5}$$

$$\begin{aligned}
&\lesssim L_F^{-1}\lfloor(1-\nu)\lambda(t)\rfloor\overline{F}(x-\mu^+\lfloor(1-\nu)\lambda(t)\rfloor)\mathrm{P}(N(t)\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad < (1-\nu)\lambda(t))\\
&\le L_F^{-1}\lfloor(1-\nu)\lambda(t)\rfloor\overline{F}((1-\mu^+(1-\nu)\gamma^{-1})x)\mathrm{P}(N(t)\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad < (1-\nu)\lambda(t))\\
&= o(\lambda(t)\overline{F}(x)).
\end{aligned}$$

For K_2, again by Corollary 2.1 and (3.1), it holds uniformly for $x \ge \gamma\lambda(t)$ that

$$\begin{aligned}
K_2 &\lesssim L_F^{-1}\overline{F}(x)\sum_{(1-\nu)\lambda(t)\le n\le(1+\nu)\lambda(t)} n\mathrm{P}(N(t)=n)\\
&\le L_F^{-1}\overline{F}(x)(1+\nu)\lambda(t)\mathrm{P}(|N(t)-\lambda(t)|<\nu\lambda(t)).
\end{aligned}$$

Now, firstly letting $t\to\infty$ and then $\nu\searrow 0$, we have that uniformly, for $x\ge\gamma\lambda(t)$,

$$K_2 \lesssim L_F^{-1}\lambda(t)\overline{F}(x). \tag{3.6}$$

Analogously to (3.6), we can also obtain that uniformly, for $x\ge\gamma\lambda(t)$,

$$K_2 \gtrsim L_F\lambda(t)\overline{F}(x). \tag{3.7}$$

Finally, we deal with K_3. For some $p > J_F^+(\ge 1)$ in (3.3), by Hölder inequality, the following holds:

$$\begin{aligned}
\mathrm{E}N(t)\mathbf{1}_{\{N(t)>(1+\nu)\lambda(t)\}} &\le \left(\mathrm{E}(N(t))^p\mathbf{1}_{\{N(t)>(1+\nu)\lambda(t)\}}\right)^{1/p}\\
&= O((\lambda(t))^{1/p}) = o(\lambda(t)).
\end{aligned} \tag{3.8}$$

Taking $v = p > J_F^+$ in Lemma 3.1, and by $F\in\mathscr{D}$, (3.3), (3.8) and Proposition 1.1, we have that, uniformly for $x\ge\gamma\lambda(t)$,

$$\begin{aligned}
K_3 &\le \sum_{n>(1+\nu)\lambda(t)}\left(n\overline{F}(xp^{-1})+C(e\mu^+nx^{-1})^p\right)\mathrm{P}(N(t)=n)\\
&= \overline{F}(xp^{-1})\mathrm{E}N(t)\mathbf{1}_{\{N(t)>(1+\nu)\lambda(t)\}}\\
&\quad + C(e\mu^+x^{-1})^p\mathrm{E}(N(t))^p\mathbf{1}_{\{N(t)>(1+\nu)\lambda(t)\}} = o(\lambda(t)\overline{F}(x)).
\end{aligned} \tag{3.9}$$

Therefore, the desired result follows from (3.4)–(3.7) and (3.9) immediately. □

4. Applications to the END compound renewal risk model. In this section, all limit relationships hold for x tending to ∞, unless stated otherwise. Motivated by [**1**], we study the asymptotic behavior of the finite-time ruin probability in the END compound renewal risk model by using the precise large deviation results in Section 2 for identically distributed r.v.s. In such a model, the claims at each accident moment are aggregated from a number of END individual claims; meanwhile, in the classical risk model, one claim at each accident time appears. The END *compound renewal risk model* satisfies the following three requirements.

Assumption $\mathbf{H}_1$. The individual claim sizes $\{X_i,\ i \geq 1\}$ form a sequence of END nonnegative r.v.s with common distribution F.

Assumption $\mathbf{H}_2$. The inter-arrival times $\{\theta_i,\ i \geq 1\}$ are LND and identically distributed nonnegative r.v.s, which are independent of $\{X_i,\ i \geq 1\}$.

Assumption $\mathbf{H}_3$. Individual claim sizes and the claim number caused by the nth accident at the time $\tau_n = \sum_{i=1}^n \theta_i$ are $\{X_i^{(n)},\ i \geq 1\}$ and N_n, respectively. Here, $\{\{X_i^{(n)},\ i \geq 1\},\ N_n,\ n \geq 1\}$ are independent copies of $\{\{X_i^{(1)},\ i \geq 1\},\ N_1\}$, $\{X_i^{(1)},\ i \geq 1\}$ are END nonnegative r.v.s with common distribution F and finite $\mu > 0$, and $\{N_i,\ i \geq 1\}$ are independent and identically distributed (i.i.d.) nonnegative integer-valued r.v.s with common distribution H. In addition, random sequences $\{\theta_n,\ n \geq 1\}$ and $\{\{X_i^{(n)},\ i \geq 1\},\ N_n,\ n \geq 1\}$ are mutually independent, but for each $n \geq 1$, $\{X_i^{(n)},\ i \geq 1\}$ are not necessarily independent of N_n.

If the individual claim sizes $\{X_i,\ i \geq 1\}$ and the inter-arrival times $\{\theta_i,\ i \geq 1\}$ are both independent r.v.s, respectively, the model is the so-called *independent compound renewal risk model*, which was introduced by Tang et al. [**20**]. If $N_1 = N_2 = \cdots = 1$, then it becomes the classical renewal risk model.

The times of successive accidents $\{\tau_n,\ n \geq 1\}$ constitute a counting process

$$\tau(t) = \sup\{n \geq 0 : \tau_n \leq t\}, \quad t \geq 0,$$

which represents the number of accidents in the interval $[0, t]$ with mean function $\alpha(t) = \mathrm{E}\tau(t)$. The total claim amount at the time τ_n and the total claim amount up to time $t \geq 0$ are, respectively,

$$S_{N_n}^{(n)} = \sum_{i=1}^{N_n} X_i^{(n)} \quad \text{and} \quad \sum_{n=1}^{\tau(t)} S_{N_n}^{(n)} = \sum_{n=1}^{\tau(t)} \sum_{i=1}^{N_n} X_i^{(n)}.$$

The total number of premiums accumulated up to time $t \geq 0$, denoted by $C(t)$ with $C(0) = 0$ and $C(t) < \infty$ almost surely for every $t > 0$, is a nonnegative and nondecreasing stochastic process, which is independent of $\{\theta_i,\ i \geq 1\}$ and $\{X_i^{(n)},\ i \geq 1\}$, $n \geq 1$. Let $\delta > 0$ be the constant interest rate (that is, after time t a capital x becomes $xe^{\delta t}$). For the initial capital reserve x we can define the finite-time ruin probability by

$$\Psi(x, t) = \mathrm{P}\left(\sup_{0 < v \leq t} \left(\sum_{n=1}^{\tau(v)} S_{N_n}^{(n)} e^{-\delta \tau_n} - \int_0^v e^{-\delta s} C(\mathrm{d}s) \right) > x \right).$$

We derive the asymptotics for the finite-time ruin probability in the END compound renewal risk model and formulate it as follows.

Theorem 4.1. *Assume that assumptions* βH_1, $\mathbf{H}_2$ *and* $\mathbf{H}_3$ *are satisfied. If* $F \in \mathscr{D}$, $H \in \mathscr{D}$, $J_H^- > 0$, $x\overline{F}(x) = o(\overline{H}(x))$ *and* $t_1 \in (0, \infty)$ *is a constant such that* $\mathrm{P}(\theta_1 \leq t_1) > 0$, *then, for any fixed* $t \geq t_1$,

(4.1)
$$L_H^4 \int_0^t \overline{H}(\mu^{-1} x e^{\delta s}) \alpha(\mathrm{d}s) \lesssim \Psi(x, t) \lesssim L_H^{-7} \int_0^t \overline{H}(\mu^{-1} x e^{\delta s}) \alpha(\mathrm{d}s).$$

In particular, if $H \in \mathscr{C}$, *then*

$$\Psi(x, t) \sim \int_0^t \overline{H}(\mu^{-1} x e^{\delta s}) \alpha(\mathrm{d}s). \tag{4.2}$$

To prove Theorem 4.1, we need some lemmas. The first lemma gives a result in some dependent classical risk model, where $N_1 = N_2 = \cdots = 1$ (see [**25**, Lemma 5.2] or [**24**, Theorem 2.1].

Lemma 4.1. *Assume that the individual claim sizes* $\{X_i,\ i \geq 1\}$ *are* UND *nonnegative r.v.s with common distribution* F*, and Assumption* H_2 *is satisfied. If* $F \in \mathscr{D}$*,* $J_F^- > 0$ *and* $t_1 \in (0, \infty)$ *is a constant such that* $\mathrm{P}(\theta_1 \leq t_1) > 0$*, then for any* $t \geq t_1$*,*

$$L_F \int_0^t \overline{F}(xe^{\delta s})\alpha(\mathrm{d}s) \lesssim \mathrm{P}\bigg(\sup_{0<v\leq t}\bigg(\sum_{n=1}^{\tau(v)} X_n e^{-\delta\tau_n} - \int_0^v e^{-rs}C(\mathrm{d}s)\bigg) > x\bigg) \lesssim L_F^{-2}\int_0^t \overline{F}(xe^{\delta s})\alpha(\mathrm{d}s).$$

Our second lemma investigates the tailed asymptotic behavior of the random sums by using Corollary 2.1, which will play an important role in proving Theorem 4.1.

Lemma 4.2. *Let* $\{X_i,\ i \geq 1\}$ *be* END *nonnegative r.v.s with common distribution* F *and finite mean* $\mu > 0$*,* N *a nonnegative integer-valued r.v. with distribution* H*. If* $F \in \mathscr{D}$*,* $H \in \mathscr{D}$ *and* $x\overline{F}(x) = o(\overline{H}(x))$*, then*

$$L_H\overline{H}(\mu^{-1}x) \lesssim \mathrm{P}(S_N > x) \lesssim L_H^{-1}\overline{H}(\mu^{-1}x). \tag{4.3}$$

In particular, if $H \in \mathscr{C}$*, then*

$$\mathrm{P}(S_N > x) \sim \overline{H}(\mu^{-1}x).$$

Note that, in this result, $\{X_i,\ i \geq 1\}$ are not necessarily independent of N.

Proof. We firstly prove the right-hand inequality of (4.3). For any fixed $0 < \varepsilon < 1$ and $x > 0$,

$$\begin{aligned}\mathrm{P}(S_N > x) &= \mathrm{P}(S_N > x,\ N \leq (1-\varepsilon)\mu^{-1}x) \\ &\quad + \mathrm{P}(S_N > x, N > (1-\varepsilon)\mu^{-1}x) \\ &\leq \mathrm{P}(S_{\lfloor(1-\varepsilon)\mu^{-1}x\rfloor} > x) + \overline{H}((1-\varepsilon)\mu^{-1}x).\end{aligned} \tag{4.4}$$

Applying Corollary 2.1 with $\gamma = \gamma(\varepsilon) = \varepsilon(1-\varepsilon)^{-1}\mu$ and by $x\overline{F}(x) = o(\overline{H}(x))$, $H \in \mathscr{D}$, we get

$$\begin{aligned} \mathrm{P}(S_{\lfloor(1-\varepsilon)\mu^{-1}x\rfloor} > x) &\le \mathrm{P}(S_{\lfloor(1-\varepsilon)\mu^{-1}x\rfloor} \\ &\qquad - \lfloor(1-\varepsilon)\mu^{-1}x\rfloor\mu > \varepsilon x) \\ &\le L_F^{-1}\lfloor(1-\varepsilon)\mu^{-1}x\rfloor\overline{F}(\varepsilon x) \\ &= o(\overline{H}(x)). \end{aligned} \tag{4.5}$$

By $H \in \mathscr{D}$, we have

$$\lim_{\varepsilon\searrow 0}\limsup_{x\to\infty}\frac{\overline{H}((1-\varepsilon)\mu^{-1}x)}{\overline{H}(\mu^{-1}x)} = L_H^{-1} \tag{4.6}$$

and

$$\lim_{\varepsilon\searrow 0}\liminf_{x\to\infty}\frac{\overline{H}((1+\varepsilon)\mu^{-1}x)}{\overline{H}(\mu^{-1}x)} = L_H.$$

Thus, from (4.4)–(4.6), we obtain the right-hand inequality of (4.3).

Secondly, we prove the left-hand inequality of (4.3). Analogously to (4.4), the lower bound for $\mathrm{P}(S_N > x)$ can be established by

$$\begin{aligned} \mathrm{P}(S_N > x) &\ge \mathrm{P}(S_{\lceil(1+\varepsilon)\mu^{-1}x\rceil} > x, \\ &\qquad\qquad N > (1+\varepsilon)\mu^{-1}x) \\ &\ge \overline{H}((1+\varepsilon)\mu^{-1}x) - \mathrm{P}(S_{\lceil(1+\varepsilon)\mu^{-1}x\rceil} \le x). \end{aligned} \tag{4.7}$$

Since $\{X_i,\ i \ge 1\}$ are END nonnegative r.v.s, by Markov's inequality and Proposition 1.2, we have for any $t > 0$

$$\begin{aligned} \mathrm{P}(S_{\lceil(1+\varepsilon)\mu^{-1}x\rceil} \le x) &\le e^{hx}\mathrm{E}e^{-hS_{\lceil(1+\varepsilon)\mu^{-1}x\rceil}} \\ &\le M\Big(e^{h\mu/1+\varepsilon}\mathrm{E}e^{-hX_1}\Big)^{(1+\varepsilon)\mu^{-1}x}. \end{aligned} \tag{4.8}$$

Note that the function $f(h) := e^{h\mu/1+\varepsilon}\mathrm{E}e^{-hX_1}$ satisfies $f(0) = 1$ and $f'(0) = -\varepsilon(1+\varepsilon)^{-1}\mu < 0$; hence, there exists a sufficiently small number $h_0 > 0$ such that $f(h_0) < 1$. Then, clearly, $\rho :=$

$(f(h_0))^{(1+\varepsilon)\mu^{-1}} \in (0,1)$. Taking $h = h_0$ in (4.8), by Proposition 1.1, we have

$$\mathrm{P}(S_{\lceil (1+\varepsilon)\mu^{-1}x\rceil} \le x) \le M\rho^x = o(\overline{H}(x)). \tag{4.9}$$

The desired result now follows from (4.6), (4.7) and (4.9). □

Proof of Theorem 4.1. Clearly, $\{S_{N_n}^{(n)},\ n \ge 1\}$ are i.i.d. r.v.s with common distribution, denoted by G. Obviously, (4.3) and $H \in \mathscr{D}$ imply $G \in \mathscr{D}$. Furthermore, by (4.3), we have for any $y > 1$,

$$\begin{aligned}\overline{G}^*(y) &= \limsup_{x\to\infty} \frac{\overline{G}(xy)}{\overline{G}(x)}\\ &= \limsup_{x\to\infty} \frac{\overline{G}(xy)}{\overline{H}(\mu^{-1}xy)} \cdot \frac{\overline{H}(\mu^{-1}x)}{\overline{G}(x)} \cdot \frac{\overline{H}(\mu^{-1}xy)}{\overline{H}(\mu^{-1}x)}\\ &\le L_H^{-2}\overline{H}^*(y),\end{aligned}$$

which, by $J_H^- > 0$, implies

$$J_G^- \ge L_H^{-2}\left(-\lim_{y\to\infty}\frac{\log \overline{H}^*(y)}{\log y}\right) = L_H^{-2}J_H^- > 0. \tag{4.10}$$

Analogously, for any $y > 1$, we can prove $\overline{G}_*(y) \ge L_H^2\overline{H}_*(y)$, so, $L_G = \lim_{y\searrow 1}\overline{G}_*(y) \ge L_H^3$. By $G \in \mathscr{D}$ and (4.10), we can apply Lemma 4.1 to obtain, for any $t \ge t_1$,

$$L_G\int_0^t \overline{G}(xe^{\delta s})\alpha(\mathrm{d}s) \lesssim \Psi(x,t) \lesssim L_G^{-2}\int_0^t \overline{G}(xe^{\delta s})\alpha(\mathrm{d}s). \tag{4.11}$$

Using Lemma 4.2, we have

$$\begin{aligned}&\int_0^t \overline{G}(xe^{\delta s})\alpha(\mathrm{d}s) \qquad (4.12)\\ &= \int_0^t\left(L_H^{-1}\overline{H}(\mu^{-1}xe^{\delta s}) + L_H^{-1}\overline{H}(\mu^{-1}xe^{\delta s})\left(\frac{\overline{G}(xe^{\delta s})}{L_H^{-1}\overline{H}(\mu^{-1}xe^{\delta s})} - 1\right)\right)\alpha(\mathrm{d}s)\\ &\lesssim L_H^{-1}\int_0^t \overline{H}(\mu^{-1}xe^{\delta s})\alpha(\mathrm{d}s),\end{aligned}$$

which, combining (4.11) and $L_G \geq L_H^3$, yields the right-hand side of (4.1). In the same manner, we can also get the left-hand side of (4.1). This ends the proof of Theorem 4.1. $\square$

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School of Mathematics and Statistics, Nanjing Audit University, Nanjing, 210029, P.R. China and School of Economics and Management, Southeast University, Nanjing, 210096, P.R. China
Email address: yyangmath@gmail.com, yyang@nau.edu.cn

Department of Information and Computational Science, School of Mathematics and Physics, Suzhou University of Science and Technology, Suzhou, 215009, P.R. China
Email address: beewky@163.com